Total Pages—5

(Set-2)

B.Tech-4th

Mathematics-IV

Full Marks : 70

Time : 3 hours

Q. No. 1 is compulsory. Answer any five questions from the remaining seven questions

The figures in the right-hand margin indicate marks

1. Answer *all* parts of this question : 2 × 10

(*a*) Prove that in multiplication a bound for the relative error of the results is given approximately by the sum of the bounds of the relative errors of the given numbers.

(*b*) To find the smallest positive solution of $x = \tan x$, if we write $x = \pi + arc \tan x$ then discuss if the fixed iteration will converge.

(*c*) Find the degree of the interpolating polynomial for the data (1, 5), (2, 8), (3, 37), (4, 62), (5, 93).

(*Turn Over*)

(*d*) What do you mean by rate of convergence of an iterative method ?

(*e*) Evaluate $\int_0^1 e^{-x^2} dx$ by Simpson's rule with $2m = 10$ and estimate the error.

(*f*) State Runge-Kutta fourth order method.

(*g*) What is an artificial variable in linear programming ?

(*h*) Define a random variable and give an example of a random variable.

(*i*) Let $f(x, y) = k$, $4 \leq x \leq 10$, $0 \leq y \leq 5$ and zero else. If $f(x, y)$ is a probability density function, then find k.

(*j*) Define regression line of x on y and regression line of y on x.

2\. (*a*) Determine the order of convergence for the Newton-Raphson method. 5

(*b*) Let $x = s$ be a solution of $x = g(x)$ and suppose that g has a continuous derivative

in some interval J containing s. Then if $|g'(x)| \leq K < 1$ in J, prove that the iteration process $x_{n+1} = g(x_n)$, $n = 0, 1, 2, \ldots$ converges for any x_0 in J. 5

3\. (*a*) Setup the Jacobi and Gauss-Seidel iterative schemes (so that the schemes converge) for the following system and iterate two times each with the initial solution $x_1 = 0$, $x_2 = 0$, $x_3 = 0$, $x_4 = 1$. 5

$$4x_1 + x_2 + 2x_3 + 20x_4 = 4$$
$$3x_1 + 5x_2 + 15x_3 - 2x_4 = 7$$
$$10x_1 + x_2 + 3x_3 - 5x_4 = 3$$
$$x_1 + 32x_2 + 3x_3 - 7x_4 = 12.$$

(*b*) Find the error term in Lagrange interpolation for the points $x_0, x_1, \ldots, x_n$. 5

4\. (*a*) Evaluate the integral

$$\int_{-1}^{1} x^2 e^{-x} dx$$

by composite Simpson's $\frac{1}{3}$ rule with spacing $h = 0.25$. 5

(*b*) Find the 2nd derivative $f''(x)$ at 0·3, 0·4 and 0·15 for the following function : 5

x	0·1	0·2	0·3	0·4	0·5	0·6
$f(x)$	0·425	0·475	0·4	0·45	0·525	0.575

5. Solve $y' = 2x^{-1}\sqrt{y - \ln x} + x^{-1}$, $y(1) = 0$ for $1 \le x \le 1 \cdot 8$ by Euler's method with $h = 0 \cdot 1$. Also solve it by classical Runge-Kutta method with $h = 0 \cdot 4$. 10

6. Maximize $Z = 2x_1 + x_2$,
Subject to
$$2x_1 - x_2 \ge 1,$$
$$x_1 - x_2 \le 2,$$
$$x_1 + x_2 \le 4$$
$$x_1, x_2 \ge 0.$$
10

7. (*a*) If the probability of hitting a target is 25% and 4 shots are fired independently, what is the probability that the target will be hit at least once ? 5

(*b*) A company that sells discount accessories for cell phones often ships an excessive number of defective products. The company needs better control of quality. Suppose it has 40 identical car chargers on hand but that 10 are defective. If the company decides to randomly select 15 of these items, what is the probability that 3 of the 15 will be defective? Find the mean and variance of the probability distribution of the number of defective in a sample of 15 randomly chosen for inspection. 5

8. (*a*) Find a 95% confidence interval for the mean μ of a normal population with standard deviation 1.2, using the sample 10, 10, 8, 12, 10, 11, 10, 11. 5

(*b*) Using a sample size $n = 15$ and sample variance $s^2 = 13$ from a normal population, test the hypothesis $\sigma^2 = \sigma_0^2 = 10$ against the alternative $\sigma^2 = \sigma_1^2 = 20$. 5

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