## VEER SURENDRA SAI UNIVERSITY OF TECHNOLOGY, BURLA

Mid Semester Examination, September-2015

B.Tech $3^{rd}$ semester Sub:Mathematics-III

Full Marks: 20 Time: 02.00 Hours

**Answer four questions including Q. No. 1 which is compulsory.**

The figures in the right hand margin indicate marks.

1. Answer the following questions: [1×5]
   (a) For what value of $c, u = sin\ ct\ sin\ x$ satisfy one dimensional wave equation?
   (b) Write the Laplace equation in cylindrical form.
   (c) Find the polar form of $\frac{1-i}{1+i}$.
   (d) Determine the region in the complex plane represented by $|z+2-5i| \leq \frac{1}{2}$.
   (e) Check the analyticity of $f(z) = z^6$.

2. (a)Find the deflection in the string of length $L = \pi$ [2.5×2]
   when $c^2 = 1$, the initial velocity is zero and the initial deflection is $k(sin\ x - \frac{1}{3}sin\ 2x)$.
   (b) Transform the following into normal form and solve:
   $$u_{xx} - 4u_{xy} + 3u_{yy} = 0.$$

3. (a)Show that the only solution of Laplace equation depending only on $r = \sqrt{x^2+y^2}$ is [2.5×2]
   $$alnr + b.$$
   (b) Solve by Laplace transforms:
   $$u_x + 2xu_t = 2x, u(x,0) = 1, u(0,t) = 1.$$

4. (a) Solve: [2.5×2]
   $$z^4 - (3+6i)z^2 - 8 + 6i = 0$$
   (b) Test weather $u = x^2 - y^2$ is harmonic. If your answer is yes find the corresponding conjugate Harmonic function.

5. (a) Show that $f(z) = |z|^2$ is nowhere differentiable except at origin. Also state your idea about analyticity of the function. [2.5×2]
   (b) For what value of $a$, $u = e^{ax}cos\ 5x$ is harmonic? Find the corresponding analytic function.

6. (a) Prove that every differentiable function is continuous but it's converse is not necessarily true. [2.5×2]
   (b) Solve: $u_{xy} = u_x$.

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Total Pages—5

(Set-$Q_1$)

**B.Tech-3rd(All Br.)**

**Mathematics-III**

*Full Marks* : 70

*Time* : 3 hours

**Q. No. 1** is compulsory and answer any **five** questions from the remaining seven questions

*The figures in the right-hand margin indicate marks*

1. Answer *all* parts of this question : 2 × 10

(*a*) Solve the equation :

$$\frac{\partial^2 u}{\partial y\, \partial x} = 4x \sin(3xy).$$

(*b*) Form the partial differential equation by eliminating the arbitrary function from $f(x^2 + y^2, z - xy) = 0$.

(*c*) Solve $y^2 u_x - x^2 u_y = 0$ by separation of variables.

( *Turn Over* )

(*d*) Transform the equation $u_{xx} + u_{xy} - 2u_{yy} = 0$ to normal form and solve it.

(*e*) Compute $(1 + i)^n + (1 - i)^n, n \in \mathbb{Z}$.

(*f*) Find the real and imaginary parts of $(1 + i)^8$.

(g) Determine all the roots of the complex number $\sqrt[3]{8i}$.

(*h*) Determine the location and order of the zeros of the function $(\sin z - 1)^5$.

(*i*) Find the Maclaurin series of

$$f(z) = \frac{1}{1+z^2}.$$

(*j*) Compute the residue of

$$f(z) = \frac{e^{-iz}}{z^2+1}$$

at $z = i$.

2. A string is stretched and fastered to two points $l$ apart. Motion is started by displacing the string

(*Continued*)

in the form $y = a\sin\left(\frac{\pi x}{l}\right)$ from which it is released at time $t = 0$. Show that the displacement of any point at a distance $x$ from one end at time $t$ is given by $y(x,t) = a\sin\left(\frac{\pi x}{l}\right)\cos\left(\frac{\pi ct}{l}\right)$. 10

3. An insulated rod of length $l$ has its ends $A$ and $B$ maintained at $0\,^\circ$C and $100\,^\circ$C respectively until steady state conditions prevail. If $B$ is suddenly reduced to $0\,^\circ$C and maintained at $0\,^\circ$C, find the temperature at a distance $x$ from $A$ at time $t$. 10

4. The function $u(x, y)$ satisfies the Laplace's equation in rectangular coordinates $(x, y)$ and for the points within the rectangle $x = 0$, $x = a$, $y = 0$, $y = b$, it satisfies the conditions $u(0, y) = u(a, y) = u(x, b) = 0$ and $u(x, 0) = x(a - x)$; $0 < x < a$. Find $u(x, y)$. 10

5. (a) Find the analytic function $f(z) = u(z) + iv(z)$, given that $u(x, y) = x^3 - 3xy^2$ and $z = x + iy$. 5

(b) Find the radius of convergence of the power series

$$\sum_{n=0}^{\infty} n!z^{n!}.$$ 5

6. (a) Find the value of the integral

$$\int_C e^{4z} dz$$

where $C$ is the shortest path from $8-3i$ to $8-(3+\pi)i$. 5

(b) Evaluate

$$\int_\gamma \frac{\cos z}{z} dz$$

where $\gamma(t) = e^{it}, 0 \le t \le 2\pi$. 5

7. (a) Locate and classify the singularities of the function

$$f(z) = \frac{z^2 - 2z}{(z+1)^2 (z^2+4)}$$

1/2

and find the residue at the singularities. 5

(b) Find the Laurent series expansion of the function

$$f(z) = \frac{z^2 - 2z + 5}{(z-2)(z^2+1)}$$

1/2

in the annulus $1 < |z| < 2$. 5

**8.** Evaluate the following integrals : 5 + 5

Y (*i*) $\int_{-\infty}^{\infty} \frac{dx}{(1+x^2)^3}$

(*ii*) $\int_0^{2\pi} \frac{\sin\theta}{3+\cos\theta} d\theta.$

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