VEER SURENDRA SAI UNIVERSITY OF TECHNOLOGY, BURLA

Mid Semester Examination, September-2015

B. Tech 3rd semester

Sub: Mathematics-III

Full Marks: 20

Time: 02.00 Hours

Answer four questions including Q. No. 1 which is compulsory.

The figures in the right hand margin indicate marks.

1. Answer the following questions:

1×5

 $[2.5 \times 2]$

- (a) For what value of $c, u = \sin ct \sin x$ satisfy one dimensional wave equation?
 - (b) Write the Laplace equation in cylindrical form.
 - (e) Find the polar form of $\frac{1-i}{1+i}$.
 - (d) Determine the region in the complex plane represented by $|z+2-5i| \leq \frac{1}{2}$.
 - (e) Check the analyticity of $f(z) = z^6$.
- $[2.5 \times 2]$ 2) (a) Find the deflection in the string of length $L = \pi$ when $c^2 = 1$, the initial velocity is zero and the initial deflection is $k(\sin x - \frac{1}{3}\sin 2x)$.
 - (b) Transform the following into normal form and solve:

$$u_{xx} - 4u_{xy} + 3u_{yy} = 0.$$

- 3. (a) Show that the only solution of Laplace equation depending only on $r = \sqrt{x^2 + y^2}$ is $[2.5 \times 2]$ alnr + b
 - (b) Solve by Laplace transforms:

$$u_x + 2xu_t = 2x, u(x, 0) = 1, u(0, t) = 1.$$

4. (a) Solve:

$$z^4 - (3+6i)z^2 - 8 + 6i = 0$$

(b) Test weather $u = x^2 - y^2$ is harmonic. If your answer is yes find the corresponding conjugate Harmonic function.

5) (a) Show that $f(z) = |z|^2$ is nowhere differentiable except at origin. Also state your idea about analyticity of the function

(b) For what value of a, $u = e^{ax}\cos 5x$ is harmonic? Find the corresponding analytic function.

6. (a) Prove that every differentiable function is continuous but it's converse is not necessarily [2.5×2]

(b) Solve: $u_{xy} = u_x$.

(Set-Q₁)

B.Tech-3rd(All Br.) Mathematics-III

Full Marks: 70

Time: 3 hours

Q. No. 1 is compulsory and answer any five questions from the remaining seven questions

The figures in the right-hand margin indicate marks

1. Answer all parts of this question: 2×10

(a) Solve the equation:

$$\frac{\partial^2 u}{\partial y \, \partial x} = 4x \sin(3xy).$$

- (b) Form the partial differential equation by eliminating the arbitrary function from $f(x^2 + y^2, z xy) = 0$.
- (c) Solve $y^2u_x x^2u_y = 0$ by separation of variables.

(Turn Over)

- (d) Transform the equation $u_{xx} + u_{xy} 2u_{yy} = 0$ to normal form and solve it.
- (e) Compute $(1+i)^n + (1-i)^n$, $n \in \mathbb{Z}$.
- (f) Find the real and imaginary parts of $(1+i)^8$.
- (g) Determine all the roots of the complex number $\sqrt[3]{8i}$.
- (h) Determine the location and order of the zeros of the function $(\sin z 1)^5$.
- (i) Find the Maclaurin series of

$$f(z) = \frac{1}{1+z^2}.$$

(i) Compute the residue of

$$f(z) = \frac{e^{-iz}}{z^2 + 1}$$

at z = i.

2. A string is stretched and fastered to two points l apart. Motion is started by displacing the string

B.Tech-3rd(All Br.)/Mathematics-III (Set-Q₁)

(Continued)

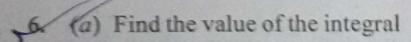
in the form $y = a \sin\left(\frac{\pi x}{l}\right)$ from which it is released at time t = 0. Show that the displacement of any point at a distance x from one end at time t = 0 is given by $y(x,t) = a \sin\left(\frac{\pi x}{l}\right) \cos\left(\frac{\pi ct}{l}\right)$.

- An insulated rod of length *l* has its ends *A* and *B* maintained at 0 °C and 100 °C respectively until steady state conditions prevail. If *B* is suddenly reduced to 0 °C and maintained at 0 °C, find the temperature at a distance *x* from *A* at time *t*.
- 4. The function u(x, y) satisfies the Laplace's equation in rectangular coordinates (x, y) and for the points within the rectangle x = 0, x = a, y = 0, y = b, it satisfies the conditions u(0, y) = u(a, y) = u(x, b) = 0 and u(x, 0) = x(a x); 0 < x < a. Find u(x, y).
- 5. (a) Find the analytic function f(z) = u(z) + iv(z), given that $u(x, y) = x^3 3xy^2$ and z = x + iy.
 - (b) Find the radius of convergence of the power series

$$\sum_{n=0}^{\infty} n! z^{n!}.$$

B. Tech-3rd(All Br.)/Mathematics-III (Set-Q1)

(Turn Over)



$$\int e^{4z}dz$$

where C is the shortest path from 8-3i to $8-(3+\pi)i$.

(b) Evaluate

Locate and classify the singularities of the function

$$f(z) = \frac{z^2 - 2z}{(z+1)^2(z^2+4)}$$

and find the residue at the singularities.

(b) Find the Laurent series expansion of the function

$$f(z) = \frac{z^2 - 2z + 5}{(z - 2)(z^2 + 1)}$$

in the annulus 1 < |z| < 2.

3

B. Tech-Ird(All Br.) Mathematics-III (Set-Q.)

(Continued)

(5)

Evaluate the following integrals:

$$.5 + 5$$

(ii)
$$\int_{-\infty}^{\infty} \frac{dx}{(1+x^2)^3}$$
(iii)
$$\int_{0}^{2\pi} \frac{\sin \theta}{3+\cos \theta} d\theta.$$

(ii)
$$\int_0^{2\pi} \frac{\sin \theta}{3 + \cos \theta} d\theta.$$