

(Set-R₁)

B.Tech - 2nd
Mathematics - II

Full Marks : 70

Time : 3 hours

**Q. No. 1 is compulsory and answer any five
from the remaining seven questions**

The figures in the right-hand margin indicate marks

1. Answer *all* parts of this questions : 2 × 10

(a) Define a singular solution and find a singular solution to $(y')^2 - xy' + y = 0$.

(b) Plot the direction field and in it plot the particular solution satisfying the initial condition for $9yy' + 4x = 0$, $y(3) = -4$.

(c) Decide if the functions x , $|x|$, x^2 are linearly dependent or independent in the interval $-1 \leq x \leq 1$.

(d) Find the radius of convergence of the series

$$\sum_{m=0}^{\infty} \frac{(-1)^m}{8^m} x^{3m}.$$

(Turn Over)

(e) If $y' = Ay$ has eigenvalues -4 and 3 , what type of critical point is $(0, 0)$.

(f) Find the solution of

$$u_{xy} - u = 0,$$

by separation of variables.

(g) Define a function of exponential order. Discuss if $f(t) = \sqrt{t}$ is of exponential order.

(h) Sketch the function $tu(t-1)$ and find its Laplace transform.

(i) Describe the physical assumptions for modeling the vibrating string.

(j) Describe d'Alemberts solution of the wave equation.

2. Solve :

5 + 5

(i) $x \frac{dy}{dx} = (y-x)^3 + y, \quad y(1) = 3/2$

(ii) $\frac{dy}{dx} = \frac{1-2y-4x}{1+y+2x}$

3. (a) Solve the initial value problem

$$y'' + 2y' + 5y = 6 \sin 2x + 7 \cos 2x,$$

$$y(0) = 1, y'(0) = 0$$

by finding the particular solution by undetermined coefficient method. 5

- (b) Solve :

$$y'' + y = \csc x,$$

by finding the particular solution by variation of parameters method. 5

4. (a) Find a power series solution in powers of x of the equation

$$y'' - 4xy' + (4x^2 - 2)y = 0. \quad 5$$

- (b) Find a general solution for the system

$$\frac{dy_1}{dt} = 2y_1 + 2y_2 + e^t, \quad \frac{dy_2}{dt} = -2y_1 - 3y_2 + e^t. \quad 5$$

5. (a) Obtain the Laplace transform of

$$f(t) = \frac{\sin t}{t},$$

by expanding f in powers of t .

5

- (b) Obtain the inverse Laplace transform of

$$F(s) = \frac{s^2 + 2w^2}{s(s^2 + 4w^2)}.$$

5

6. (a) Solve

$$\frac{d^2 y}{dt^2} - y = -\frac{1}{2}\delta(t), t > 0 \text{ as } t \rightarrow \infty, y(-t) = y(t)$$

using Laplace transform.

5

- (b) Solve for $x(t)$ from

$$te^{-at} = \int_0^t x(t)x(t-\tau)d\tau, a > 0.$$

5

7. (a) Find the temperature in a thin metal rod of length l , with both ends insulated and with

initial temperature in the rod $\sin \frac{\pi x}{l}$.

5

(5)

- (b) A string is stretched and fastened to two points l apart. Motion is started by displacing the string into the form $y = b \sin \frac{\pi x}{l}$ and released from rest in that position at time $t = 0$. Find the displacement at any time $t > 0$ by Laplace transform method. 5

8. (a) Solve the two dimensional temperature distribution problem

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,$$

which satisfies the conditions $u(0, y) =$

$$u(l, y) = u(x, 0) = 0, \text{ and } u(x, a) = \sin \frac{n\pi x}{l}. \quad 5$$

- (b) Find the deflection of the square membrane with $a = b = 1$ and $c = 1$ if the initial velocity is zero and initial deflection is $f(x, y) = 0.1 \sin 3\pi x \sin 4\pi y$. 5
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