Total Pages—7

(Set-Q_1)

B.Tech-1st (ALL)

Mathematics-I

Full Marks : 70

Time : 3 hours

Answer any **six** questions including **Q. No. 1** which is compulsory

The figures in the right-hand margin indicate marks

1. Answer the following questions : 2 × 10

(*a*) Show that every Cauchy sequence is bounded.

(*b*) Using the definition of limit, prove that

$$\lim_{n \to \infty} \frac{n+1}{2n-1} = \frac{1}{2}.$$

(*c*) Determine the values of x for which the series

$$\sum_{n=1}^{\infty} \frac{\sin nx}{n^2} \text{ converges.}$$

(*Turn Over*)

(d) Let

$$c_0 + \frac{c_1}{2} + \frac{c_2}{3} + \dots + \frac{c_n}{n+1} = 0$$

where $c_k \in \mathbb{R}$. Show that the equation $c_0 + c_1 x + c_2 x^2 + \dots + c_n x^n = 0$ has at least one root in $[0, 1]$.

(e) Explain the geometrical meaning by means of example : Two linear equation in two variables with infinitely many solutions.

(f) Explain the geometrical meaning by means of example : Two linear equations in two variables with no solutions.

(g) Find the echelon form and rank of the matrix :

$$\begin{pmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{pmatrix}$$

(h) Find the value of k for which

$$\begin{pmatrix} 1 & 0 & k \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

is diagonalizable.

(i) Prove that the order of convergence of Newton-Raphson method is 2.

(j) Approximate

$$\int_0^1 \frac{1}{1+x}\,dx$$

by trapezoidal rule using 10 subintervals.

2. (a) Show that

$$\frac{x}{1+x} < \log(1+x) < x,$$

for all $x > 0$.

(b) Let $f : \mathbb{R} - \{0\} \to [-1, 1]$ defined by $f(x) = \sin\frac{1}{x}$, then show that $\lim_{x \to 0} f(x)$ not exists. 10

3. (*a*) Find the points, where the following function is continuous :

$$f(x)=\begin{cases}1+x, & \infty<x<0\\ 1+[x]+\sin x, & 0\le x<\pi/2\\ 3, & x\ge\pi/2.\end{cases}$$

(*b*) If f is continuous on $[a, b]$ and

$$\int_a^b |f(x)|\,dx=0,$$

then show that $f(x)=0$ on $[a, b]$. 10

4. (*a*) Let A be an $n\times n$ matrix. If the system $A^2x=0$ has a non-trivial solution, then show that the system $Ax=0$ has also a non-trivial solution.

(*b*) Prove or disprove : If two matrices are of the same order have the same rank, then they must be row equivalent. 10

5. (*a*) Examine whether the matrix

$$\begin{pmatrix}1 & 2 & 2\\ 0 & 2 & 1\\ -1 & 2 & 2\end{pmatrix}$$

is diagonalizable. If yes, find a matrix P such that $P^{-1}AP$ is a diagonal matrix.

(*b*) Find the values of λ and μ for which the system of equations

$$x+2y+z=6$$
$$x+4y+3z=10$$
$$x+4y+\lambda z=\mu$$

has a (*i*) unique solution, (*ii*) no solution, (*iii*) infinitely many solutions. 10

6. (*a*) Define the asymptotic error constant and the rate of convergence. Find the error constant and the rate of convergence of the secant method. Perform four iterations to find the smallest positive root of the equation

$$x^3-5x+1=0$$

which lies between (0, 1) by the above method.

(*b*) Find a polynomial of degree 2 such that $f(0)=1, f(1)=3$ and $f(2)=7$ by using backward difference interpolation. 10

7. (*a*) Determine the coefficients W_0, W_1, W_2 such that the rule

$$I_n(f) = W_0 f(-1) + W_1 f(-1/3) + W_0 f(1/2),$$

approximately integrating

$$I(f) = \int_{-1}^{1} f(x)\,dx$$

is exact for polynomials of degree ≤ 2.

(*b*) Find a real root of the equation $x^3 - x^2 - 1 = 0$ in the interval [0, 1] by using the method of interation with an accuracy 10^{-3}. 10

8. (*a*) Calculate $y(0\cdot02)$ for

$$\frac{dy}{dx} = x + y, \; y(0) = 1$$

by Euler method with $h = 0\cdot01$. Compare the results with the exact solution.

(*b*) Use Runge-Kutta's method of order 4 to approximate the solutions of the following initial-value problem :

$$y' = y - t^2 + 1 \quad 0 \leq t \leq 1, y(0) = 0\cdot5,$$

with $h = 0\cdot5$. 10
