Total Pages—8

M.Tech/1st/WRE, TE(CE)

Computational and Statistical Methods

Full Marks : 70

Time : 3 hours

Q. No. 1 which is compulsory and answer five out of the rest seven questions

The figures in the right-hand margin indicate marks

1. Answer the following questions : 2 × 10

(*a*) Using Taylor's series method, compute $y(1{\cdot}2)$ correct to three decimal places, from the equation

$$\frac{dy}{dx} = \frac{1}{x^2} - \frac{y}{x}, \quad y(1) = 0.$$

(*b*) Find the numerical solution of the differential equation

$$\frac{dy}{dx} = x^2 - y, \quad y(0) = 1$$

(Turn Over)

for $x = 0{\cdot}3$ taking step size $h = 0{\cdot}1$, using Euler's method.

(*c*) Obtain the finite difference approximation of the equation

$$y'' + f(x)y' + g(x)y = r(x) \text{ at } x = x_i.$$

(*d*) What is the general range of relaxation factor w in the successive over-relaxation (SOR) method for solving a partial differential equation numerically ?

(*e*) If $\text{Var}(x) = 9$, $\text{Var}(y) = 4$ and $\text{Var}(x - y) = \text{Var}(x)$, find the correlation coefficient between x and y.

(*f*) If $x = 4y + 5$ and $y = kx + 4$ be two regression equations of x on y and y on x respectively, then find the interval in which k lies.

(*g*) The mean and standard deviation of marks of 70 students were found to be 65 and 5.2 respectively. Later it was detected that the

marks of one student was wrongly recorded as 85 instead of 58. Obtain the correct standard deviation.

(*h*) Using Tchebycheff's inequality, verify whether there exists a variate X for which

$$P(\mu - 2\sigma \le X \le \mu + 2\sigma) = 0{\cdot}6$$

(*i*) Find the probability that at most 5 defective fuses will be found in a box of 200 fuses if it is experienced that 2% of such fuses are defective.

(*j*) Verify whether the following is a distribution function :

$$F(x) = \begin{cases} 0, & x < -a \\ \frac{1}{2}\left(\frac{x}{a} + 1\right), & -a \le x \le a \\ 1, & x > a \end{cases}$$

2. (*a*) Evaluate $y(0.4)$ by modified Euler's method taking $h = 0.1$, given that

$$\frac{dy}{dx} = \frac{1}{2}(1+x^2)y^2, \ y(0) = 1.$$

Compare the results with the exact solution. 6

(*b*) From the differential equation

$$\frac{dy}{dx} = \frac{1-x^2-y^2}{1-xy}, \ y(0) = 0,$$

Compute the value of $y(0{\cdot}2)$ correct upto 4 decimal places by 4th order Runge-Kutta method. 4

3. (*a*) Using second order finite difference method, solve the boundary value problem $y'' = y' + 1, \ 0 < x < 1, \ y(0) = 1, \ y(1) = 2(e-1)$. Hence, compare with the exact solution $y(x) = 2e^x - x - 1$. 4 + 2

(*b*) Use Taylor's series method to solve the system of differential equations

$$\frac{dx}{dt} = y - t, \quad \frac{dy}{dt} = x + t$$

with $x = 1, \ y = 1$ when $t = 0$, taking $\Delta x = \Delta t = 0{\cdot}1$. 4

4. Solve by Crank-Nicolson's method, the equation

$$\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 4, \ t > 0$$

satisfying the conditions $u(0, t) = u(4, t) = 0, t \geq 0$ and

$$u(x,0) = \begin{cases} 20x, & 0 \leq x \leq 2 \\ 20(4-x), & 2 \leq x \leq 4 \end{cases}$$

compute u for two time steps with $h = 1$ and a convenient value of k. 10

5. (*a*) The heights in inches of the 8 students of college, chosen at the random, were as follows : 62.2, 62.4, 63.1, 63.2, 65.5, 66.2, 66.3 and 66.5. Compute 98% and 95% confidence intervals for the mean and the standard deviation of the population of the heights of the students of the college, assuming it to be normal and find the length of the interval in each case. 5

(*b*) An observer watches six different types of birds

Types	1	2	3	4	5	6
Numbers	6	7	13	17	6	5

Test the consistency of above observation with a proportion 1 : 1 : 2 : 3 : 1 : 1 of birds in any park at 5 % level of significance. 5

6. (*a*) The joint probability density function of the random variables *X* and *Y* is

$$f(x,y) = 3x^2 - 8xy + 6y^2 \quad \text{for } 0 \le x \le 1, 0 \le y \le 1$$

$$= 0, \quad \text{elsewhere}$$

Examine whether *X* and *Y* are independent. 4

(*b*) The probability density function (p.d.f) of a random variate is as follows

$$f(x) = 2\left(\frac{\alpha - x}{\alpha^2}\right), \quad 0 \le x \le \alpha$$

Find the maximum likelihood estimate for the population parameter α. Hence, prove that for a unit random sample x_1 the population parameter α has maximum likelihood estimate $2x_1$. 6

7. (*a*) If the equations of two regression lines obtained is a correlation analysis are $3x + 12y = 19$, $33y + 9x = 46$, determine which one of these is regression equation of x on y. Find the means, correlation coefficient and ratio of standard deviation of x and y. 6

(*b*) Use Tchebycheff's in equality to show that for $n \ge 36$, the probability that in n throws of a fair die the number of sixes lies between $\frac{1}{6}n - \sqrt{n}$, $\frac{1}{6}n + \sqrt{n}$ is at least $\frac{31}{36}$. 4

8. (*a*) If the life of certain tires(in thousands of miles) has the density $f(x) = \theta e^{-\theta x}(x > 0, \theta > 0)$, what mileage can you expect to get on one of these tires ? Let $\theta = 0{\cdot}05$ and find the probability that a tire will last at least 30,000 miles. 4

(*b*) The pressure and volume of a gas are related by the equation $Pv^{\lambda} = k$ (λ and k are constant). Fit this equation for the following data using the principle of least squares. 6

p	0·5	1	1·5	2	2·5	3
v	1·62	1·00	0·75	0·62	0·52	0·46