

(Set-1)

B.Tech-4th
Mathematics-IV

Full Marks : 70

Time : 3 hours

Answer any six questions including Q. No. 1

The figures in the right-hand margin indicate marks

1. Answer all the questions : 2 × 10

(i) Find the sum of 0.123×10^3 and 0.456×10^2 .
Write the result in three-digit mantissa form.

(ii) What will be the error when a third degree polynomial is integrated by Simpson's $\frac{1}{3}$ rule? Justify your answer.

(iii) Find the value of $\nabla^2 y_5$ from the data given below :

x	1	2	3	4	5
y	2	5	10	17	26

(Turn Over)

- (iv) Define basic feasible solution and degenerate basic feasible solution.
- (v) What are the limitations of Lagrange interpolation?
- (vi) Define Karl-Pearson's coefficient of correlation of two random variables X & Y . What do you mean by range of coefficient correlation?
- (vii) Define Null hypothesis.
- (viii) Six dice are thrown 729 times. How many times do you expect at least three dice to show a five or a six?
- (ix) In a poisson distribution if $2P(X=1) = P(X=2)$, then find standard deviation.
- (x) The probability distribution of a continuous random variable X is given by
- $$f(x) = \begin{cases} 6x(1-x), & 0 \leq x \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$
- Check that $f(x)$ is a probability density function. Hence, evaluate $P(0 \leq X \leq \frac{1}{2})$.

2. (a) Find the real root of the equation $x \log_{10} x - 1.2 = 0$, correct to three decimal places by using Newton-Raphson method. 5
- (b) Find the real root of the equation $xe^x - \cos x = 0$ by Regula-Falsi method correct to three decimal places. 5
3. (a) Using Gauss-Seidal method to solve the system of linear equations $2x + y = 3$, $x + 3y + z = 5$; $y + 2z = 3$. 5
- (b) Find the approximate value of $\log_e 5$ by calculating to 4 decimal places, by Simpson's $\frac{1}{3}$ rule, $\int_0^5 \frac{dx}{4x+5}$, dividing the range into 10 equal parts. 5
4. (a) Using Runge-Kutta method of order 4 solve for $y(0.1)$, $y(0.2)$ and $y(0.3)$ given that $y' = xy + x^2$, $y(0) = 1$. 5
- (b) Given that $\sin 45^\circ = 0.7071$, $\sin 50^\circ = 0.7660$, $\sin 55^\circ = 0.8192$, $\sin 60^\circ = 0.8660$ find $\sin 52^\circ$, using Newton's forward interpolation formula. 5

5. (a) Using simplex method solve the following LPP: 5

$$\text{Max. } Z = 3x_1 + 2x_2$$

$$\text{subject to } \begin{aligned} x_1 + x_2 &\leq 4, \\ -x_1 + x_2 &\geq -2, \\ x_1, x_2 &\geq 0. \end{aligned}$$

- (b) Using penalty method solve the following LPP: 5

$$\text{Minimize } Z = 4x_1 + x_2$$

$$\text{subject to } \begin{aligned} 3x_1 + x_2 &= 3, \\ 4x_1 + 3x_2 &\leq 6, \\ x_1 + 2x_2 &\leq 3, \\ x_1, x_2 &\geq 0. \end{aligned}$$

6. (a) Find the mean and variance of a binomial distribution. 5

- (b) From the following data, obtain the regression equation of y on x . 5

x	6	2	10	4	8
y	9	11	5	8	7

7. (a) A car hire firm has two cars, which it hires out day by day. The number of demands for a car on each day is distributed as a poisson distribution with mean 1.5. Calculate the proportion of days on which (i) neither car is used (ii) some demand is refused. 5

- (b) Define normal distribution. Write down the four properties of normal probability curve. 5

8. (a) For sample I, $\sum_{i=1}^{1000} x_i = 49000$,

$$\sum_{i=1}^{1000} (x_i - \bar{x})^2 = 784000. \text{ For sample II,}$$

$$\sum_{i=1}^{1500} x_i = 70500, \quad \sum_{i=1}^{1500} (x_i - \bar{x})^2 = 2400000.$$

Discuss the significance of the difference of the sample means. 5

(b) Write the short notes on : (a) 5

(i) Confidence interval

(ii) Estimation of parameters.