(Set-1)

B.Tech-2nd

Mathematics-II

Full Marks : 70

Time : 3 hours

Q.No.1 is compulsory and answer any **five** questions from the remaining seven questions.

The figures in the right-hand margin indicate marks

1. Answer *all* parts of this questions : 2 × 10

   (*a*) Determine the radius of convergence of the series $\sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$.

   (*b*) Define the unit step function and graph the function $f(t) = u(t-2)5 \sin t$.

   (*c*) Describe the level surfaces of the scalar field $f = 4x^2 + y^2 + 9z^2$.

(d) To which function the sin series of the function $f(x) = \cos x, 0 < x < \pi$ approximates in the interval $(-\pi, 0)$. Show the graph also.

(e) Prove that for any twice continuously differentiable scalar function f, curl (grad f) = 0.

(f) Find the divergence and curl of F, if $F = \text{grad}(x^3 + y^3 + z^3)$.

(g) Find out the greatest rate of increase of $f = x^2 + yz^2$ at the point $(1, -1, 3)$.

(h) Provide one example to show that partial derivative may exists at a point but the function is not differentiable at that point

(i) Discuss if the vectors $(4, 2, 9), (3, 2, 1)$ and $(-4, 6, 9)$ are linearly independent.

(j) Find $a \times b - b \times a$ if $a = (-3, 2, 0)$ and $b = (6, -7, 2)$.

2. (a) Find a solution of

$$(a^2 - x^2)y'' - 2xy' + 12y = 0$$

by series solution method. 5

(b) Solve the equation

$$xy'' + 2(1 - x)y' + (x - 2)y = 0$$

by Frobenious method. 5

3. Solve the following equations by Laplace transformation method :

(a) $3 \sin 2x = y(x) + \int_0^x (x - t) y(t)\, dt.$ 5

(b) $y'' + 5y' + 6y = 5e^{3t}, y(0) = y'(0) = 0.$ 5

4. (a) Find the Laplace transform of

$$f(t) = 2e^{-t} \cos^2 \frac{t}{2}.$$ 5

(b) Find the Inverse Laplace transform of

$$\frac{s^2}{(s^2 + w^2)^2}.$$ 5

5. (a) Find the Fourier series of the function $f(x) = x^2, (-\pi < x < \pi)$. 5

(b) Find the Fourier series of the periodic function $f(x) = \pi x^3/2, \quad (-1 < x < 1)$, $p = 2L = 2$. 5

6. (a) Sketch the function, state whether it is odd or even and find its Fourier series of

$$f(x) = \begin{cases} k & \text{if } -\pi/2 < x < \pi/2, \\ 0 & \text{if } \pi/2 < x < 3\pi/2. \end{cases}$$ 5

(b) Solve the system of equations by Laplace transform method

$y'_1 = -2y_1 + 3y_2, y'_2 = 4y_1 - y_2, y_1(0) = 4, y_2(0) = 3.$ 5

7. (a) Show that the repeated limit exists but the double limit does not when $(x, y) \to (0, 0)$, for

$$f(x) = \begin{cases} \dfrac{x^2 - y^2}{x^2 + y^2} & \text{if } x \neq y, \\ 0 & \text{if } x = y. \end{cases}$$ 5

(b) If $z = f\left[\dfrac{ny - mz}{nx - lz}\right]$, then prove that

$$(nx - lz)\frac{\partial z}{\partial x} + (ny - mz)\frac{\partial z}{\partial y} = 0.$$ 5

8. (a) Expand

$f(x, y) = 21 + x - 20y + 4x^2 + xy + 6y^2$

in Taylors series about the point the $(-1, 2)$. 5

(b) Find the local maximum and the local minimum values of the function

$f(x, y) = 2(x^2 - y^2) - x^4 + y^4$. 5

VEER SURENDRA SAI UNIVERSITY OF TECHNOLOGY, ODISHA

DEPARTMENT OF MATHEMATICS

2^{nd} SEM B.TECH. MID SEMETSER EXAMINATION

MATHEMATICS – II

Full Marks – 20 Time – 2Hrs

(Answers four questions including Question no. 1)

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1) a) Evaluate gradient of the function $\varphi(x,y,z) = \sqrt{x^2+y^2+z^2}$ at the point *(2,2,2)*. [1 × 5]

b) Compute the divergence of the function $F = \sinh(x-z)\,\mathbf{i} + 2y\,\mathbf{j} + (z-y^2)\mathbf{k}$.

c) Find the curl of the function $F = 2xy\,\mathbf{i} + xe^{y}\mathbf{j} + 2z\,\mathbf{k}$.

d) Find $\nabla \times (\nabla\varphi)$ if $\varphi(x,y,z) = e^{(x+y+z)}$.

e) State the Green's theorem in plane.

2) a) Determine the maximum and minimum rate of change of the function $\varphi(x,y,z) = 2xy + xe^{z}$ at the point (-2,1,6). [2.5]

b) Find the equation of the tangent plane and normal line to the surface $z = x^2 - y^2$ at the point (1,1,0). [2.5]

3) a) Let $f(x,y,z)$ and $g(x,y,z)$ be scalar fields. Prove that $\nabla\cdot(\nabla f \times \nabla g) = 0$. [2.5 × 2]

b) Let F and G be vector fields. Prove that $\nabla\cdot(F\times G) = G\cdot(\nabla\times F) - F\cdot(\nabla\times G)$.

4) a) Evaluate the line integral $\int_C x\,dx - dy + z\,dz$, where C is given by $x(t) = t$, $y(t) = t,\ z(t) = t^3$ for $0 \le t \le 1$. [2.5]

b) Find the work done by $F = x^2\mathbf{i} - 2yz\,\mathbf{j} + z\,\mathbf{k}$ in moving an object along the line segment from (1,1,1) to (4,4,4). [2.5]

5) a) Use Green's theorem to evaluate $\oint_C F\cdot dr$, where $F = (x^2-y)\mathbf{i} + (\cos(2y) - e^{3y} + 4x)\,\mathbf{j}$, with C any square with sides of length 5. [2.5 × 2]

b) Evaluate $\iint_S x\,ds$, where S is part of the plane $x + 4y + z = 10$ in the first octant.

------------ BEST OF LUCK -----------------