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**B.Tech./1st
Mathematics - I**

Full Marks : 70

Time : 3 hours

Answer any six questions including Q. No. 1
which is compulsory

Q. No. 1 carries 20 marks and all other questions
carry ten marks each.

1. Answer all parts of this question :

(a) Express the matrix

$$\begin{pmatrix} 6 & 7 & 5 \\ 9 & -1 & 4 \\ -5 & 3 & 1 \end{pmatrix}$$

as sum of a symmetric and skew symmetric-
matrix.

(Turn Over)

(2)

- (b) Find the eigenvalues and eigenvectors of the following matrix

$$\begin{pmatrix} -5 & 2 \\ 2 & -2 \end{pmatrix}$$

- (c) Solve the initial value problem

$$y'x + y = 0, y(1) = 1.$$

- (d) Solve

$$y' + 2y = y^2$$

- (e) State Roll's theorem.

- (f) Find a general solution of

$$y'' + y = 0$$

- (g) Are the following set of vectors linear dependent?

$$[2 \ -4 \ 6], [1 \ 9 \ -3], [3 \ 5 \ 3].$$

- (h) Define skew symmetric and orthogonal matrices.

(3)

- (i) Find a general solution of the differential equation.

$$(D^2 - D - 2)y = 0$$

- (j) Find the radius of convergence of the series

$$\sum_{n=0}^{\infty} \frac{2^n}{n!} x^n$$

2. (a) Find the rank of the matrix

$$\begin{pmatrix} 3 & 0 & 2 & 2 \\ -6 & 42 & 24 & 54 \\ 21 & -21 & 0 & -15 \end{pmatrix}$$

- (b) Find a basis of eigenvectors that form a unitary system

$$\begin{pmatrix} i & 1 \\ -1 & i \end{pmatrix}$$

3. (a) Diagonalize the following matrix

$$\begin{pmatrix} 5 & 10 & -10 \\ 10 & 5 & -20 \\ 5 & -5 & -10 \end{pmatrix}$$

(4)

- (b) Find out what type of conic section the following quadratic form represents and transform it to principal axis

$$Q = 7x_1^2 + 6x_1x_2 + 7x_2^2 = 200.$$

4. (a) Solve the differential equation by variation of parameters

$$y'' + y = \sec x.$$

- (b) Solve the differential equation

$$y' + xy = xy^{-1}$$

5. (a) Find the power series solution of

$$y'' - 3y' + 2y = 0$$

- (b) Prove the Rodrigues formula for Legendre polynomial.

6. (a) Prove that

$$J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$$

(5)

- (b) Prove that

$$\left[x^n J_n(x) \right]' = x^n J_{n-1}(x)$$

7. (a) Write the statement of Maclaurin's theorem and using it find the series expansion of $\sin x$.

- (b) Find the $Lf'(0)$ and $Rf'(0)$ for

$$f(x) = \begin{cases} \frac{x(e^{1/x} - 1)}{(e^{1/x} + 1)} & \text{when } x \neq 0 \\ 0 & \text{when } x = 0 \end{cases}$$

8. (a) Show that the function

$$f(x) = \begin{cases} 1 & \text{when } x \text{ is rational} \\ -1 & \text{when } x \text{ is irrational} \end{cases}$$

is discontinuous at every point.

- (b) Show that $\lim_{x \rightarrow 3} \frac{1}{(x-3)^4} = \infty$ by using definition.