Lecture Notes On Analogue Communication Techniques (Module 1 & 2)

Topics Covered:

1. **Spectral Analysis of Signals**
2. **Amplitude Modulation Techniques**
3. **Angle Modulation**

Module-I (12 Hours)

Spectral Analysis: Fourier Series: The Sampling Function, The Response of a linear System, Normalized Power in a Fourier expansion, Impulse Response, Power Spectral Density, Effect of Transfer Function on Power Spectral Density, The Fourier Transform, Physical Appreciation of the Fourier Transform, Transform of some useful functions, Scaling, Time-shifting and Frequency shifting properties, Convolution, Parseval's Theorem, Correlation between waveforms, Auto-and cross correlation, Expansion in Orthogonal Functions, Correspondence between signals and Vectors, Distinguishability of Signals.

Module-II (14 Hours)

Amplitude Modulation Systems: A Method of frequency translation, Recovery of base band Signal, Amplitude Modulation, Spectrum of AM Signal, The Balanced Modulator, The Square law Demodulator, DSB-SC, SSBSC and VSB, Their Methods of Generation and Demodulation, Carrier Acquisition, Phase-locked Loop (PLL), Frequency Division Multiplexing. Frequency Modulation Systems: Concept of Instantaneous Frequency, Generalized concept of Angle Modulation, Frequency modulation, Frequency Deviation, Spectrum of FM Signal with Sinusoidal Modulation, Bandwidth of FM Signal Narrowband and wideband FM, Bandwidth required for a Gaussian Modulated WBFM Signal, Generation of FM Signal, FM Demodulator, PLL, Preemphasis and Deemphasis Filters.

Module-III (12 Hours)

Mathematical Representation of Noise: Sources and Types of Noise, Frequency Domain Representation of Noise, Power Spectral Density, Spectral Components of Noise, Response of a Narrow band filter to noise, Effect of a Filter on the Power spectral density of noise, Superposition of Noise, Mixing involving noise, Linear Filtering, Noise Bandwidth, Quadrature Components of noise. Noise in AM Systems: The AM Receiver, Super heterodyne Principle, Calculation of Signal Power and Noise Power in SSB-SC, DSB-SC and DSB, Figure of Merit ,Square law Demodulation, The Envelope Demodulation, Threshold

Module-IV (8 Hours)

Noise in FM System: Mathematical Representation of the operation of the limiter, Discriminator, Calculation of output SNR, comparison of FM and AM, SNR improvement using preemphasis, Multiplexing, Threshold in frequency modulation, The Phase locked Loop.

Text Books:

1. Principles of Communication Systems by Taub & Schilling,2nd Edition.Tata Mc Graw Hill. Selected portion from Chapter1, 3, 4, 8, 9 & 10

2. Communication Systems by Siman Haykin,4th Edition, John Wiley and Sons Inc.

References Books:

1. Modern digital and analog communication system, by B. P. Lathi, 3rd Edition, Oxford University Press.

2. Digital and analog communication systems, by L.W.Couch, 6th Edition, Pearson Education, Pvt. Ltd.

Spectral Analysis of Signals

A signal under study in a communication system is generally expressed as a function of time or as a function of frequency. When the signal is expressed as a function of time, it gives us an idea of how that instantaneous amplitude of the signal is varying with respect to time. Whereas when the same signal is expressed as function of frequency, it gives us an insight of what are the contributions of different frequencies that compose up that particular signal. Basically a signal can be expressed both in time domain and the frequency domain. There are various mathematical tools that aid us to get the frequency domain expression of a signal from the time domain expression and vice-versa. *Fourier Series* is used when the signal in study is a periodic one, whereas *Fourier Transform* may be used for both periodic as well as non-periodic signals.

Fourier Series

Let the signal x(t) be a periodic signal with period T_0. The Fourier series of a signal can be obtained, if the following conditions known as the Dirichlet conditions are satisfied:

1. x(t) is absolutely integrable over its period, i.e.

$$\int_{-\infty}^{\infty} |x(t)| dt = 0$$

2. The number of maxima and minima of x(t) in each period is finite.

3. The number of discontinuities of x(t) in each period is finite.

A periodic function of time say *v(t)* having a fundamental period T_0 can be represented as an infinite sum of sinusoidal waveforms, the summation being called as the *Fourier series* expansion of the signal.

$$v(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{2\pi nt}{T_0}\right) + \sum_{n=1}^{\infty} B_n \sin\left(\frac{2\pi nt}{T_0}\right)$$

Where A_0 is the average value of *v(t)* given by:

$$A_0 = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} v(t)\,dt$$

And the coefficients A_n and B_n are given by

$$A_n = \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} v(t) \cos\left(\frac{2\pi nt}{T_0}\right) dt$$

$$B_n = \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} v(t) \sin\left(\frac{2\pi nt}{T_0}\right) dt$$

Alternate form of Fourier Series is

$$v(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos\left(\frac{2\pi nt}{T_0} - \phi_n\right)$$
$$C_0 = A_0$$
$$C_n = \sqrt{A_n^2 + B_n^2}$$
$$\phi_n = \tan^{-1}\frac{B_n}{A_n}$$

The Fourier series hence expresses a periodic signal as infinite summation of harmonics of fundamental frequency $f_0 = \frac{1}{T_0}$. The coefficients C_n are called spectral amplitudes i.e. C_n is the amplitude of the spectral component $C_n \cos\left(\frac{2\pi nt}{T_0} - \phi_n\right)$ at frequency nf_0. This form gives one sided spectral representation of a signal as shown in1st plot of Figure 1.

Exponential Form of Fourier Series

This form of Fourier series expansion can be expressed as :

$$v(t) = \sum_{n=-\infty}^{\infty} V_n e^{j2\pi nt/T_0}$$
$$V_n = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} v(t) e^{j2\pi nt/T_0} dt$$

The spectral coefficients V_n and V_{-n} have the property that they are complex conjugates of each other $V_n = V_{-n}^{*}$. This form gives two sided spectral representation of a signal as shown in 2nd plot of Figure-1. The coefficients V_n can be related to C_n as :

$$V_0 = C_0$$
$$V_n = \frac{C_n}{2} e^{-j\phi_n}$$

The V_n*'s* are the spectral amplitude of spectral components $V_n e^{j2\pi ntf_0}$.

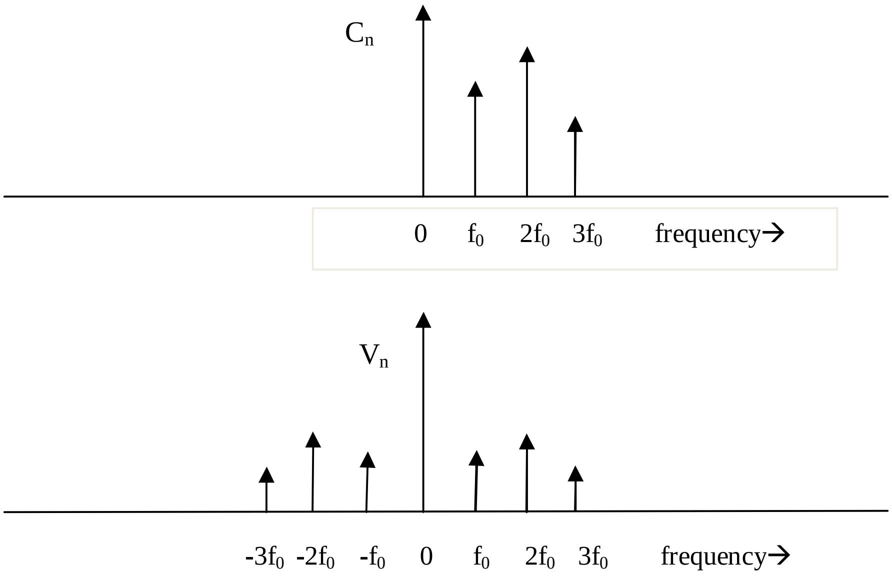


Figure 1 One sided and corresponding two sided spectral amplitude plot

The Sampling Function

The sampling function denoted as *Sa(x)* is defined as:

$$Sa(x) = \frac{\mathrm{Sin}(x)}{x}$$

And a similar function *Sinc(x)* is defined as :

$$Sinc(x) = \frac{Sin(\pi x)}{\pi x}$$

The *Sa(x)* is symmetrical about *x=0*, and is maximum at this point *Sa(x)=1*. It oscillates with an amplitude that decreases with increasing *x*. It crosses zero at equal intervals on *x* at every $x = \pm n\pi$, where *n* is an non-zero integer.

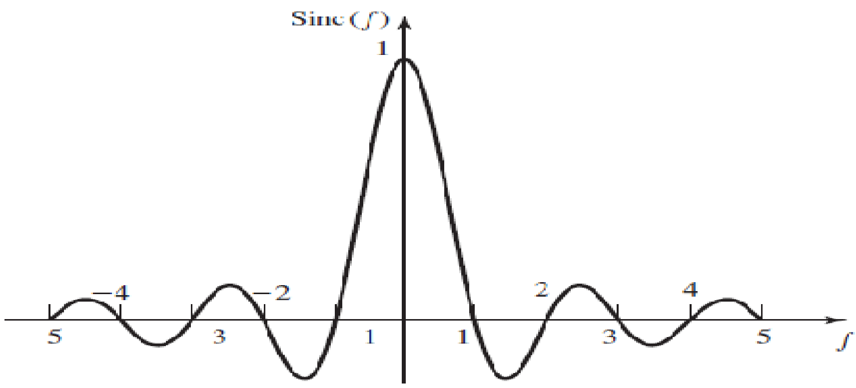


Figure 2 Plot of Sinc(f)

Fourier Transform

The Fourier transform is the extension of the Fourier series to the general class of signals (periodic and nonperiodic). Here, as in Fourier series, the signals are expressed in terms of complex exponentials of various frequencies, but these frequencies are not discrete. Hence, in this case, the signal has a continuous spectrum as opposed to a discrete spectrum. Fourier Transform of a signal *x(t)* can be expressed as:

$$F[\mathrm{x(t)}] = \mathrm{X(f)} = \int_{-\infty}^{\infty} \mathrm{x(t)}\,\mathrm{e}^{-j2\pi ft}\,dt$$

$x(\mathrm{t}) \Leftrightarrow \mathrm{X(f)}$ represents a Fourier Transform pair

The time-domain signal *x(t)* can be obtained from its frequency domain signal *X(f)* by Fourier inverse defined as:

$$x(\mathrm{t}) = \mathrm{F}^{-1}\left[X(\mathrm{f})\right] = \int_{-\infty}^{\infty} X(\mathrm{f}) e^{j2\pi ft} df$$

When frequency is defined in terms of angular frequency ω ,then Fourier transform relation can be expressed as:

$$F[\mathrm{x(t)}] = \mathrm{X}(\omega) = \int_{-\infty}^{\infty} \mathrm{x(t)}\,\mathrm{e}^{-j\omega t}\,dt$$

and

$$x(\mathrm{t}) = \mathrm{F}^{-1}\left[X(\omega)\right] = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$

Properties of Fourier Transform

Let there be signals x(t) and y(t) ,with their Fourier transform pairs:

$$x(\mathrm{t}) \Leftrightarrow \mathrm{X(f)}$$
$$\mathrm{y(t)} \Leftrightarrow \mathrm{Y(f)}\ \text{then,}$$

1. **Linearity Property**
   $ax(\mathrm{t}) + \mathrm{by(t)} \Leftrightarrow \mathrm{aX(f)} + \mathrm{bY(f)}$, where *a* and *b* are the constants

2. **Duality Property**
   $X(\mathrm{t}) \Leftrightarrow x(-f)$ or
   $\mathrm{X(t)} \Leftrightarrow 2\pi\,\mathrm{X}(-\omega)$

3. **Time Shift Property**

$$x(t-t_0) \Leftrightarrow e^{-j2\pi f t_0} X(f)$$

4. **Time Scaling Property**

$$x(at) \Leftrightarrow \frac{1}{|a|} X\left(\frac{f}{a}\right)$$

5. **Convolution Property:** If convolution operation between two signals is defined as:

$$x(t) \otimes y(t) = \int_{-\infty}^{\infty} x(\tau) x(t-\tau)\, d\tau \text{ , then}$$

$$x(t) \otimes y(t) \Leftrightarrow X(f)\, Y(f)$$

6. **Modulation Property**

$$e^{j2\pi f_0 t} x(t) \Leftrightarrow X(f-f_0)$$

7. **Parseval's Property**

$$\int_{-\infty}^{\infty} x(t)\, y^*(t)\, dt = \int_{-\infty}^{\infty} X(f)\, Y^*(f) df$$

8. **Autocorrelation Property:** If the time autocorrelation of signal *x(t)* is expressed as:

$$R_x(\tau) = \int_{-\infty}^{\infty} x(t)\, x*(t-\tau) dt \text{ ,then}$$

$$R_x(\tau) \Leftrightarrow |X(f)|^2$$

9. **Differentiation Property:**

$$\frac{d}{dt} x(t) \Leftrightarrow j2\pi\, f X(f)$$

Response of a linear system

The reason what makes Trigonometric Fourier Series expansion so important is the unique characteristic of the sinusoidal waveform that such a signal always represent a particular frequency. When any linear system is excited by a sinusoidal signal, the response also is a sinusoidal signal of same frequency. In other words, a sinusoidal waveform preserves its wave-shape throughout a linear system. Hence the response-excitation relationship for a linear system can be characterised by, how the response amplitude is related to the excitation amplitude (amplitude ratio) and how the response phase is related to the excitation phase (phase difference) for a particular frequency. Let the input to a linear system be :

$$v_i(t, \omega_n) = V_n e^{j\omega_n t}$$

Then the filter output is related to this input by the *Transfer Function* (characteristic of the Linear Filter): $H(\omega_n) = |H(\omega_n)| e^{-j\theta(\omega_n)}$, such that the filter output is given as

$$v_o(t, \omega_n) = V_n |H(\omega_n)| e^{j(\omega_n t - j\theta(\omega_n))}$$

Normalised Power

While discussing communication systems, rather than the absolute power we are interested in another quantity called Normalised Mean Power. It is an average power normalised across a 1 ohm resistor, averaged over a single time-period for a periodic signal. In general irrespective of , if it is a periodic or non-periodic signal, average normalised power of a signal *v(t)* is expressed as :

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{\frac{-T}{2}}^{\frac{T}{2}} v^2(t)\,dt$$

Energy of signal

For a continuous-time signal, the energy of the signal is expressed as:

$$E = \int_{-\infty}^{\infty} x^2(\mathrm{t})\,dt$$

A signal is called an ***Energy Signal*** if

$$0 < E < \infty$$
$$P = 0$$

A signal is called ***Power Signa***l if

$$0 < P < \infty$$
$$E = \infty$$

Normalised Power of a Fourier Expansion

If a periodic signal can be expressed as a Fourier Series expansion as:

$$v(t) = C_0 + C_1 \cos(2\pi f_0 t) + C_2 \cos(4\pi f_0 t) + ...$$

Then, its normalised average power is given by :

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{\frac{-T}{2}}^{\frac{T}{2}} v^2(t)\,dt$$

Integral of the cross-product terms become zero, since the integral of a product of orthogonal signals over period is zero. Hence the power expression becomes:

$$P = {C_0}^2 + \frac{{C_1}^2}{2} + \frac{{C_2}^2}{2} + ...$$

By generalisation, normalised average power expression for entire Fourier Series becomes:

$$P = C_0^{\,2} + \sum_{n=1}^{\infty} \frac{C_n^{\,2}}{2} + \ldots$$

In terms of trigonometric Fourier coefficients A_n's, B_n's, the power expression can be written as:

$$P = A_0^{\,2} + \sum_{n=1}^{\infty} A_n^{\,2} + \sum_{n=1}^{\infty} B_n^{\,2}$$

In terms of complex exponential Fourier series coefficients V_n's, the power expressions becomes:

$$P = \sum_{n=-\infty}^{\infty} V_n V_n^{*}$$

Energy Spectral Density(*ESD*)

It can be proved that energy ***E*** of a signal *x(t)* is given by :

$$E = \int_{-\infty}^{\infty} x^2(\mathrm{t})dt = \int_{-\infty}^{\infty} \left|X(\mathrm{f})\right|^2 df \rightarrow \textit{Parseval's Theorem for energy signals}$$

So, $E = \int_{-\infty}^{\infty} \psi(\mathrm{f})df$, where $\psi(\mathrm{f}) = \left|X(\mathrm{f})\right|^2 \rightarrow$ *Energy Spectral Density*

The above expression says that $\psi(\mathrm{f})$ integrated over all of the frequencies, gives the total energy of the signal. Hence *Energy Spectral Density* (***ESD***) quantifies the energy contribution from every frequency component in the signal, and is a function of frequency.

Power Spectral Density(*PSD*)

It can be proved that the average normalised power ***P*** of a signal *x(t)*,such that $x_\tau(\mathrm{t})$ is a truncated version of *x(t)* such that $x_\tau(\mathrm{t}) = \begin{cases} x(\mathrm{t}); \frac{-\tau}{2} < t < \frac{\tau}{2} \\ 0; elsewhere \end{cases}$ is given by :

$$P = \lim_{\tau\to\infty} \frac{1}{\tau} \int_{\frac{-\tau}{2}}^{\frac{\tau}{2}} x^2(t)dt = \lim_{\tau\to\infty} \frac{1}{\tau} \int_{\frac{-\tau}{2}}^{\frac{\tau}{2}} \left|X_\tau(t)\right|^2 dt \rightarrow \textit{Parseval's Theorem for power signals}$$

So, $P = \int_{-\infty}^{\infty} \mathrm{S}(\mathrm{f})df$, where $S(\mathrm{f}) = \lim_{\tau\to\infty} \frac{\left|X_\tau(\mathrm{f})\right|^2}{\tau} \rightarrow$ *Power Spectral Density*

The above expression says that *S(f)*integrated over all of the frequencies, gives the total *normalised power* of the signal. Hence *Power Spectral Density* (***PSD***) quantifies the power contribution from every frequency component in the signal, and is a function of frequency.

Expansion in Orthogonal Functions

Let there be a set of functions $g_1(\mathrm{x}), \mathrm{g}_2(\mathrm{x}), \mathrm{g}_3(\mathrm{x}), \ldots, \mathrm{g}_n(\mathrm{x})$, defined over the interval $x_1 < x < x_2$ and any two functions of the set have a special relation:

$$\int_{x_1}^{x_2} g_i(\mathrm{x})\,\mathrm{g}_j(\mathrm{x})\,\mathrm{dx} = 0 \ .$$

The set of functions showing the above property are said to be *orthogonal functions* in the interval $x_1 < x < x_2$. We can then write a function $f(\mathrm{x})$ in the same interval $x_1 < x < x_2$, as a linear sum of such $\mathrm{g}_n(\mathrm{x})$'s as:

$f(\mathrm{x}) = C_1 g_1(\mathrm{x}) + C_2\,\mathrm{g}_2(\mathrm{x}) + C_3\,\mathrm{g}_3(\mathrm{x}) + \ldots + C_n\,\mathrm{g}_n(\mathrm{x})$, where C_n's are the numerical coefficients

The numerical value of any coefficient C_n can be found out as:

$$C_n = \frac{\int_{x_1}^{x_2} f(\mathrm{x})\,\mathrm{g}_n(\mathrm{x})dx}{\int_{x_1}^{x_2} g_n^{\,2}(\mathrm{x})\,\mathrm{dx}}$$

In a special case when the functions $\mathrm{g}_n(\mathrm{x})$ in the set are chosen such that $\int_{x_1}^{x_2} g_n^{\,2}(\mathrm{x})\,\mathrm{dx}$ =1, then such a set is called as a set of *orthonormal functions*, that is the functions are orthogonal to each other and each one is a normalised function too.

Amplitude Modulation Systems

In communication systems, we often need to design and analyse systems in which many independent message can be transmitted simultaneously through the same channel. It is possible with a technique called *frequency multiplexing*, in which each message is translated in frequency to occupy a different range of spectrum. This involves an auxiliary signal called *carrier* which determines the amount of frequency translation. It requires either the amplitude, frequency or phase of the carrier be instantaneously varied as according to the instantaneous value of the message signal. The resulting signal then is called a modulated signal. When the amplitude of the carrier is changed as according to the instantaneous value of the message/baseband signal, it results in *Amplitude Modulation.* The systems implanting such modulation are called as Amplitude modulation systems.

Frequency Translation

Frequency translation involves translating the signal from one region in frequency to another region. A signal band-limited in frequency lying in the frequencies from f_1 to f_2, after frequency translation can be translated to a new range of frequencies from $f_1^{'}$ to $f_2^{'}$. The information in the original message signal at baseband frequencies can be recovered back even from the frequency-translated signal. There are so many benefits which are satisfied by the frequency translation techniques:

1. Frequency Multiplexing: In a case when there are more than one sources which produce band-limited signals that lie in the same frequency band. Such signals if transmitted as such simultaneously through a channel, they will interfere with each other and cannot be recovered back at the intended receiver. But if each signal is translated in frequency such that they encompass different ranges of frequencies, not interfering with other signal spectrums, then each signal can be separated back at the receiver with the use of proper band-pass filters. The output of filters then can be suitably processed to get back the original message signal.
2. Practicability of antenna: In a wireless medium, antennas are used to radiate and to receive the signals. The antenna operates effectively, only when the dimension of the antenna is of the order of magnitude of the wavelength of the signal concerned. At baseband low frequencies, wavelength is large and so is the dimension of antenna required is impracticable. By frequency translation, the signal can be shifted in frequency to higher range of frequencies. Hence the corresponding wavelength is small to the extend that the dimension of antenna required is quite small and practical.
3. Narrow banding: For a band-limited signal, an antenna dimension suitable for use at one end of the frequency range may fall too short or too large for use at another end of the frequency range. This happens when the ratio of the highest to lowest frequency contained in the signal is large (wideband signal). This ratio can be reduced to close around one by translating the signal to a higher frequency range, the resulting signal being called as a narrow-banded signal. Narrowband signal works effectively well with the same antenna dimension for both the higher end frequency as well as lower end frequency of the band-limited signal.
4. Common Processing: In order to process different signals occupying different spectral ranges but similar in general character, it may always be necessary to adjust the frequency range of operation of the apparatus. But this may be avoided, if by keeping the frequency range of operation of the apparatus constant, every time the signal of interest is translated down to the operation frequency range of the apparatus.

Amplitude Modulation Types:

1. Double-sideband with carrier (DSB+C)
2. Double-sideband suppressed carrier (DSB-SC)
3. Single-sideband suppressed carrier (SSB-SC)
4. Vestigial sideband (VSB)

Double-sideband with carrier (DSB+C)

Let there be a sinusoidal carrier signal $c(t) = A\mathrm{Cos}(2\pi f_c t)$, of frequency f_c . With the concept of amplitude modulation, the instantaneous amplitude of the carrier signal will be modulated (changed) proportionally according to the instantaneous amplitude of the baseband or modulating signal *x(t)*. So the expression for the Amplitude Modulated (AM) wave becomes:

$$s(t) = \left[A + x(t)\right] Cos(2\pi f_c t) = E(t)\mathrm{Cos}(2\pi f_c t)$$

$$E(t) = A + x(t)$$

The time varying amplitude *E(t)* of the AM wave is called as the envelope of the AM wave. The envelope of the AM wave has the same shape as the message signal or baseband signal.

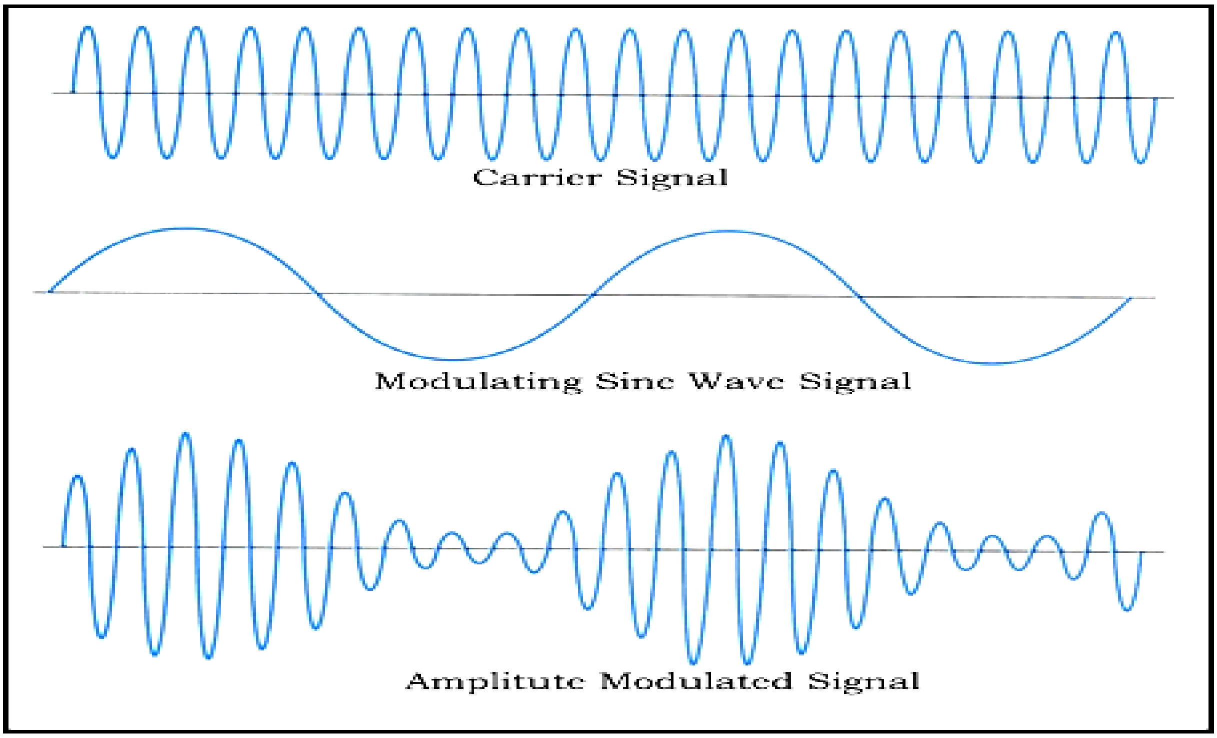


Figure 3 Amplitude modulation time-domain plot

Modulation Index (m_a): It is defined as the measure of extent of amplitude variation about unmodulated maximum carrier amplitude. It is also called as depth of modulation, degree of modulation or modulation factor.

$$m_a = \frac{|x(t)|_{max}}{A}$$

On the basis of modulation index, AM signal can be from any of these cases:

I. $m_a > 1$: Here the maximum amplitude of baseband signal exceeds maximum carrier amplitude, $|x(t)|_{max} > A$. In this case, the baseband signal is not preserved in the AM envelope, hence baseband signal recovered from the envelope will be distorted.

II. $m_a \leq 1$: Here the maximum amplitude of baseband signal is less than carrier amplitude $|x(t)|_{max} \leq A$. The baseband signal is preserved in the AM envelope.

Spectrum of Double-sideband with carrier (DSB+C)

Let *x(t)* be a bandlimited baseband signal with maximum frequency content f_m. Let this signal modulate a carrier $c(t) = \mathrm{A Cos}(2\pi \, f_c t)$ .Then the expression for AM wave in time-domain is given by:

$$\begin{aligned} s(t) &= [A + x(t)]Cos(2\pi \, f_c t) \\ &= \mathrm{ACos}(2\pi \, f_c t) + x(t)\,\mathrm{Cos}(2\pi \, f_c t) \end{aligned}$$

Taking the Fourier transform of the two terms in the above expression will give us the spectrum of the DSB+C AM signal.

$$\mathrm{ACos}(2\pi \, f_c t) \Leftrightarrow \frac{1}{2}[\delta(f + f_c) + \delta(f - f_c)]$$

$$x(t)\,\mathrm{Cos}(2\pi \, f_c t) \Leftrightarrow \frac{1}{2}[X(f + f_c) + X(f - f_c)]$$

So, first transform pair points out two impulses at $f = \pm f_c$, showing the presence of carrier signal in the modulated waveform. Along with that, the second transform pair shows that the AM signal spectrum contains the spectrum of original baseband signal shifted in frequency in both negative and positive direction by amount f_c . The portion of AM spectrum lying from f_c to $f_c + f_m$ in positive frequency and from $-f_c$ to $-f_c - f_m$ in negative frequency represent the *Upper Sideband(USB)*. The portion of AM spectrum lying from $f_c - f_m$ to f_c in positive frequency and from $-f_c + f_m$ to $-f_c$ in negative frequency represent the *Lower Sideband(LSB)*. Total AM signal spectrum spans a frequency from $f_c - f_m$ to $f_c + f_m$, hence has a bandwidth of $2 f_m$.

Power Content in AM Wave

By the general expression of AM wave:

$$s(t) = \mathrm{ACos}(2\pi f_c t) + x(t)\mathrm{Cos}(2\pi f_c t)$$

Hence, total average normalised power of an AM wave comprises of the carrier power corresponding to first term and sideband power corresponding to second term of the above expression.

$$P_{total} = P_{carrier} + P_{sideband}$$

$$P_{carrier} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} A^2 Cos^2(2\pi\, \mathrm{f}_c \mathrm{t}) dt = \frac{A^2}{2}$$

$$P_{sideband} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(\mathrm{t}) Cos^2(2\pi\, \mathrm{f}_c \mathrm{t}) dt = \frac{1}{2} \overline{x^2(\mathrm{t})}$$

In the case of single-tone modulating signal where $x(\mathrm{t}) = \mathrm{V}_m Cos(2\pi \mathrm{f}_m \mathrm{t})$:

$$P_{carrier} = \frac{A^2}{2}$$

$$P_{sideband} = \frac{V_m{}^2}{4}$$

$$P_{total} = P_{carrier} + P_{sideband} = \frac{A^2}{2} + \frac{V_m{}^2}{4}$$

$$\Rightarrow P_{total} = P_{carrier} \left[1 + \frac{m_a{}^2}{2} \right]$$

Where, m_a is the modulation index given as $m_a = \frac{V_m}{A}$.

Net Modulation Index for Multi-tone Modulation: If modulating signal is a multitone signal expressed in the form:

$$x(\mathrm{t}) = \mathrm{V}_1 Cos(2\pi \mathrm{f}_1 \mathrm{t}) + \mathrm{V}_2 Cos(2\pi \mathrm{f}_2 \mathrm{t}) + \mathrm{V}_3 Cos(2\pi \mathrm{f}_3 \mathrm{t}) + ... + \mathrm{V}_n Cos(2\pi \mathrm{f}_n \mathrm{t})$$

Then, $P_{total} = P_{carrier} \left[1 + \frac{m_1{}^2}{2} + \frac{m_2{}^2}{2} + \frac{m_3{}^2}{2} ... \frac{m_n{}^2}{2} \right]$

Where $m_1 = \frac{V_1}{A}, m_2 = \frac{V_2}{A}, m_3 = \frac{V_2}{A}, ..., m_n = \frac{V_n}{A}$

Generation of DSB+C AM by Square Law Modulation

Square law diode modulation makes use of non-linear current-voltage characteristics of diode. This method is suited for low voltage levels as the current-voltage characteristic of diode is highly non-linear in the low voltage region. So the diode is biased to operate in this non-linear region for this application. A DC battery V_c is connected across the diode to get such a operating point on the characteristic. When the carrier and modulating signal are applied at the input of diode, different frequency terms appear at the output of the diode. These when applied across a tuned circuit tuned to carrier frequency and a narrow bandwidth just to allow the two pass-bands, the output has the carrier and the sidebands only which is essentially the DSB+C AM signal.

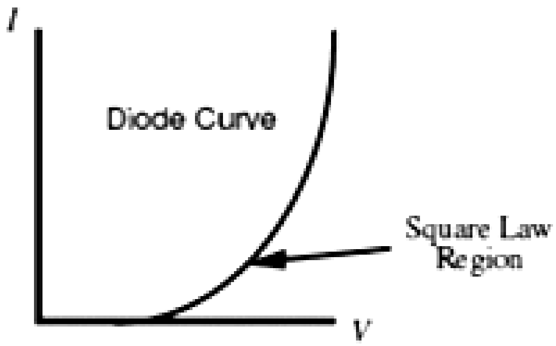


Figure 4 Current-voltage characteristic of diode

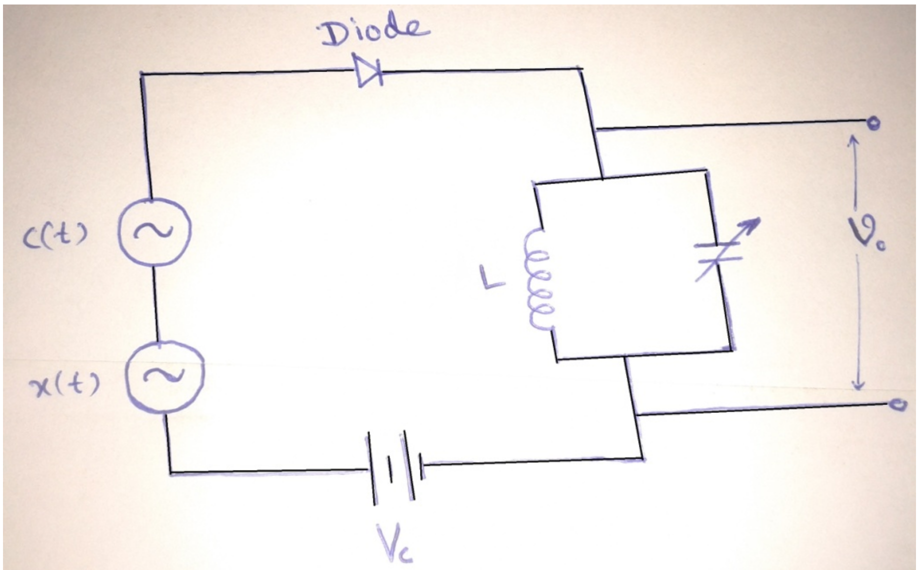


Figure 5 Square Law Diode Modulator

The non-linear current voltage relationship can be written in general as:

$$i = av + bv^2$$

In this application $v = c(\mathrm{t}) + \mathrm{x(t)}$

So

$$i = a[\mathrm{ACos}(2\pi\, \mathrm{f}_c \mathrm{t}) + \mathrm{x(t)}] + \mathrm{b}[\mathrm{ACos}(2\pi\, \mathrm{f}_c \mathrm{t}) + \mathrm{x(t)}]^2$$

$$\Rightarrow i = a\,\mathrm{ACos}(2\pi\, \mathrm{f}_c \mathrm{t}) + a\,\mathrm{x(t)} + bA^2\,\mathrm{Cos}^2(2\pi\, \mathrm{f}_c \mathrm{t}) + b\,\mathrm{x}^2(\mathrm{t}) + 2bA\,\mathrm{x(t)}\,\mathrm{Cos}(2\pi\, \mathrm{f}_c \mathrm{t})$$

$$\Rightarrow i = \boxed{a\,\mathrm{ACos}(2\pi\, \mathrm{f}_c \mathrm{t})} + a\,\mathrm{x(t)} + \frac{bA^2}{2}\mathrm{Cos}(2\pi(2\,\mathrm{f}_c)\,\mathrm{t}) + \frac{bA^2}{2} + b\,\mathrm{x}^2(\mathrm{t}) + \boxed{2bA\,\mathrm{x(t)}\,\mathrm{Cos}(2\pi\, \mathrm{f}_c \mathrm{t})}$$

Out of the above frequency terms, only the boxed terms have the frequencies in the passband of the tuned circuit, and hence will be at the output of the tuned circuit. There is carrier frequency term and the sideband term which comprise essentially a DSB+C AM signal.

Demodulation of DSB+C by Square Law Detector

It can be used to detect modulated signals of small magnitude, so that the operating point may be chosen in the non-linear portion of the V-I characteristic of diode. A DC supply voltage is used to get a fixed operating point in the non-linear region of diode characteristics. The output diode current is hence

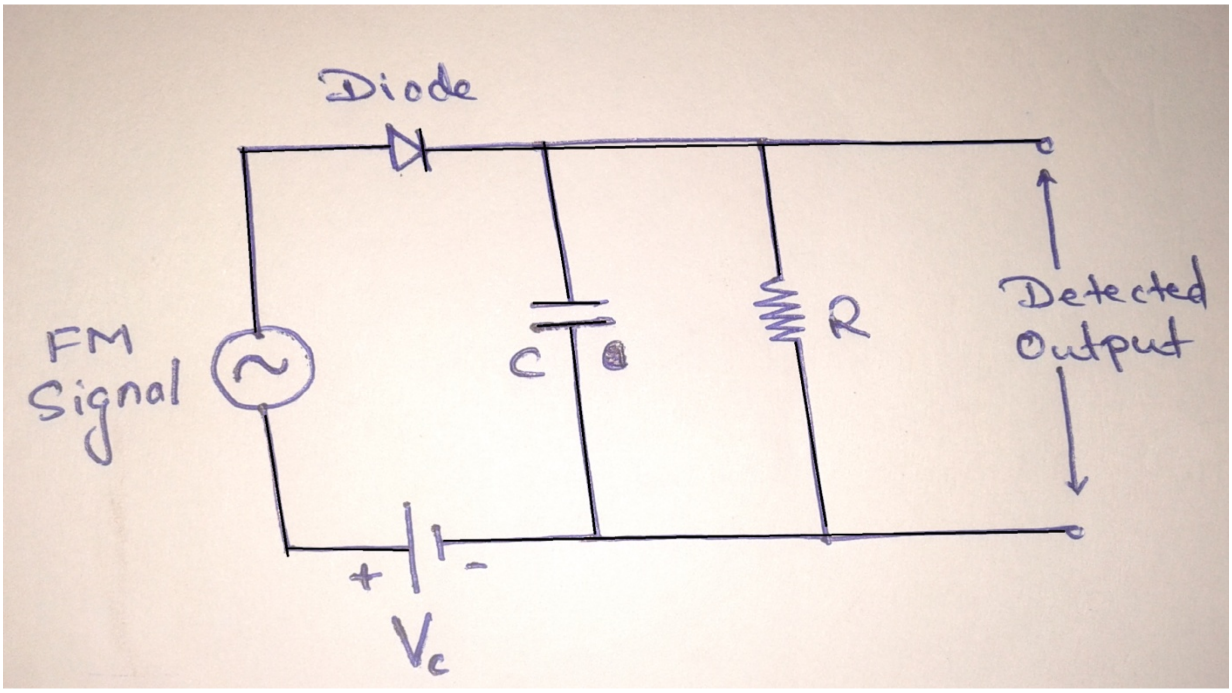


Figure 6 Square Law Detector

given by the non-linear expression:

$$i = a v_{FM}(\mathrm{t}) + \mathrm{b} v^2_{FM}(\mathrm{t})$$

Where $v_{FM}(\mathrm{t}) = [\mathrm{A} + x(\mathrm{t})]\mathrm{Cos}(2\pi \mathrm{f}_c \mathrm{t})$

This current will have terms at baseband frequencies as well as spectral components at higher frequencies. The low pass filter comprised of the RC circuit is designed to have cut-off frequency as the highest modulating frequency of the band limited baseband signal. It will allow only the baseband frequency range, so the output of the filter will be the demodulated baseband signal.

Linear Diode Detector or Envelope Detector

This is essentially just a half-wave rectifier which charges a capacitor to a voltage to the peak voltage of the incoming AM waveform. When the input wave's amplitude increases, the capacitor voltage is increased via the rectifying diode quickly, due a very small RC time-constant (negligible *R*) of the charging path. When the input's amplitude falls, the capacitor voltage is reduced by being discharged by a 'bleed' resistor *R* which causes a considerable RC time constant in the discharge path making discharge process a slower one as compared to charging. The voltage across *C* does not fall appreciably during the small period of negative half-cycle, and by the time next positive half cycle appears. This cycle again charges the capacitor *C* to peak value of carrier voltage and thus this process repeats on. Hence the output voltage across capacitor *C* is a spiky envelope of the AM wave, which is same as the amplitude variation of the modulating signal.

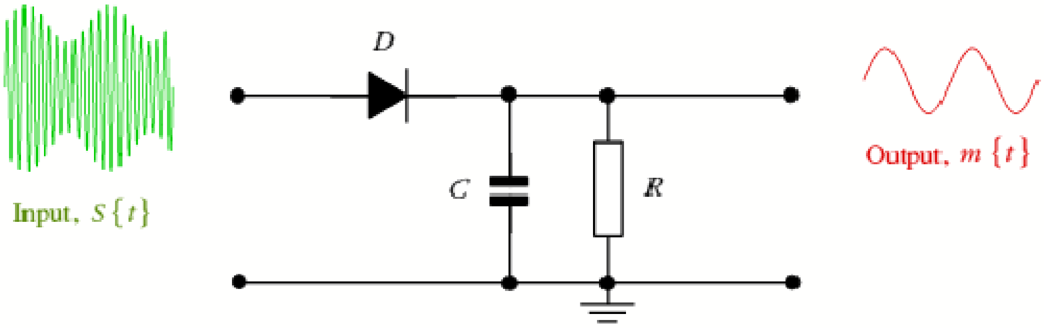


Figure 7 Envelope Detector

Double Sideband Suppressed Carrier(DSB-SC)

If the carrier is suppressed and only the sidebands are transmitted, this will be a way to saving transmitter power. This will not affect the information content of the AM signal as the carrier component of AM signal do not carry any information about the baseband signal variation. A DSB+C AM signal is given by:

$$s_{DSB+C}(t) = A\text{Cos}(2\pi f_c t) + x(t)\text{Cos}(2\pi f_c t)$$

So, the expression for DSB-SC where the carrier has been suppressed can be given as:

$$s_{DSB-SC}(t) = x(t)\text{Cos}(2\pi f_c t)$$

Therefore, a DSB-SC signal is obtained by simply multiplying modulating signal *x(t)* with the carrier signal. This is accomplished by a **product modulator** or **mixer**.

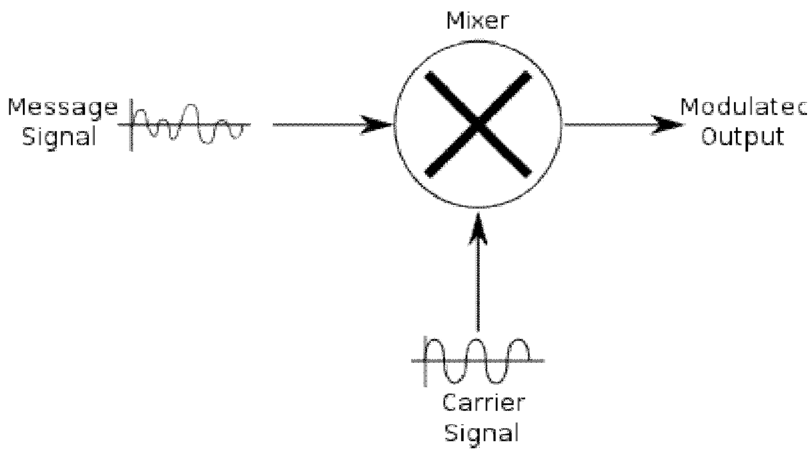


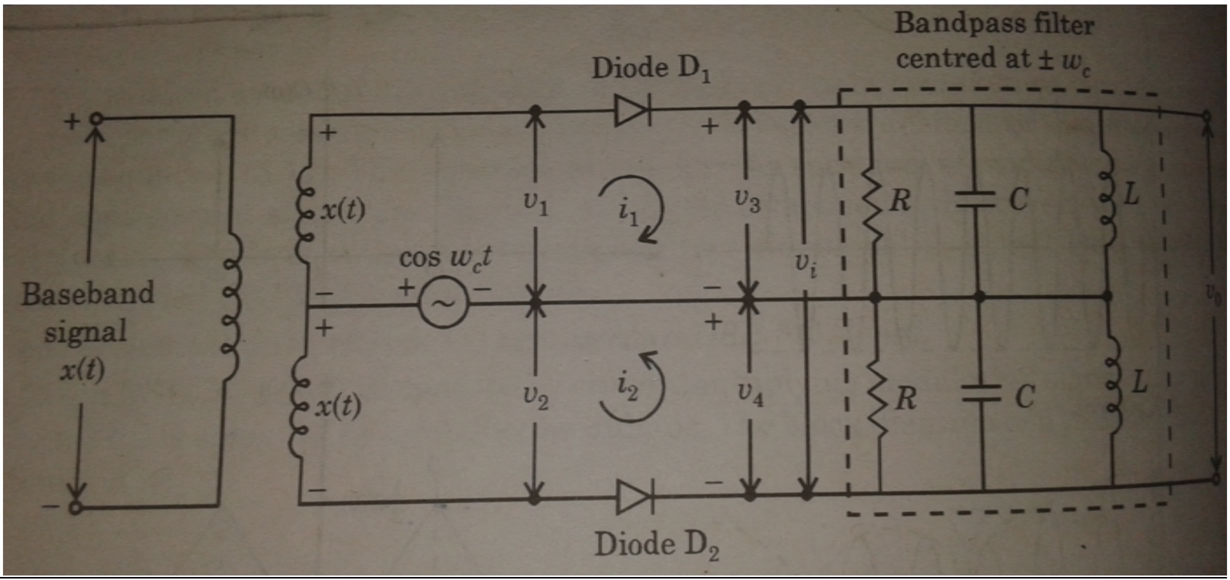
Figure 8 Product Modulator

Difference from the the DSB+C being only the absence of carrier component, and since DSBSC has still both the sidebands, spectral span of this DSBSC wave is still $f_c - f_m$ to $f_c + f_m$, hence has a bandwidth of $2f_m$.

Generation of DSB-SC Signal

A circuit which can produce an output which is the product of two signals input to it is called a product modulator. Such an output when the inputs are the modulating signals and the carrier signal is a DSBSC signal. One such product modulator is a balanced modulator.

Balanced modulator:



$$v_1 = Cos(2\pi f_c t) + x(t)$$
$$v_2 = Cos(2\pi f_c t) - x(t)$$

For diode D_1, the nonlinear v-i relationship becomes:

$$i_1 = av_1 + bv_1^2 = a[Cos(2\pi f_c t) + x(t)] + b[Cos(2\pi f_c t) + x(t)]^2$$

Similarly, for diode D_2,

$$i_2 = av_2 + bv_2^2 = a[Cos(2\pi f_c t) - x(t)] + b[Cos(2\pi f_c t) - x(t)]^2$$

Now, $v_i = v_3 - v_4 = (i_1 - i_2)R$
$\Rightarrow v_i = 2R[ax(t) + 2bx(t)Cos(2\pi f_c t)]$ (substituting for i_1 and i_2)

This voltage is input to the bandpass filter centre frequency f_c and bandwidth $2f_m$. Hence it allows the component corresponding to the second term of the v_i, which is our desired DSB-SC signal.

Demodulation of DSBSC signal

Synchronous Detection: DSB-SC signal is generated at the transmitter by frequency up-translating the baseband spectrum by the carrier frequency f_c. Hence the original baseband signal can be recovered by frequency down-translating the received modulated signal by the same amount. Recovery can be achieved by multiplying the received signal by synchronous carrier signal and then low-pass filtering.

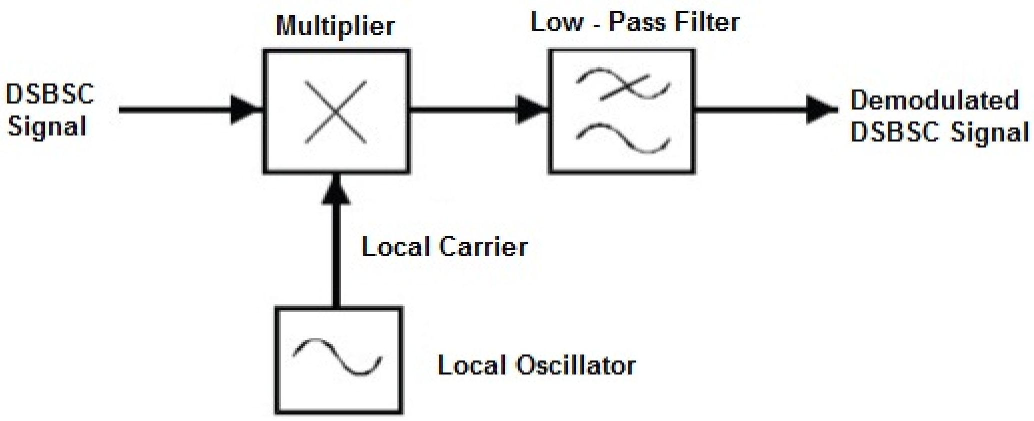


Figure 9 Synchronous Detection of DSBSC

Let the received DSB-SC signal is :

$$r(t) = x(t)Cos(2\pi f_c t)$$

So after carrier multiplication, the resulting signal:

$$e(t) = x(t)\,Cos(2\pi\, f_c t).Cos(2\pi\, f_c t)$$
$$\Rightarrow e(t) = x(t)\,Cos^2(2\pi\, f_c t)$$
$$\Rightarrow e(t) = \frac{1}{2}x(t)\left[1 + Cos(2\pi(2f_c)\,t)\right]$$
$$\Rightarrow e(t) = \frac{1}{2}x(t) + \frac{1}{2}x(t)\,Cos(2\pi(2f_c)\,t)$$

The low-pass filter having cut-off frequency f_m will only allow the baseband term $\frac{1}{2}x(t)$, which is in the pass-band of the filter and is the demodulated signal.

Single Sideband Suppressed Carrier (SSB-SC) Modulation

The lower and upper sidebands are uniquely related to each other by virtue of their symmetry about carrier frequency. If an amplitude and phase spectrum of either of the sidebands is known, the other sideband can be obtained from it. This means as far as the transmission of information is concerned, only one sideband is necessary. So bandwidth can be saved if only one of the sidebands is transmitted, and such a AM signal even without the carrier is called as Single Sideband Suppressed Carrier signal. It takes half as much bandwidth as DSB-SC or DSB+C modulation scheme.

For the case of single-tone baseband signal, the DSB-SC signal will have two sidebands :

The lower side-band: $Cos(2\pi(f_c - f_m)t) = Cos(2\pi f_m t)Cos(2\pi f_c t) + Sin(2\pi f_m t)Sin(2\pi f_c t)$

And the upper side-band: $Cos(2\pi(f_c + f_m)t) = Cos(2\pi f_m t)Cos(2\pi f_c t) - Sin(2\pi f_m t)Sin(2\pi f_c t)$

If any one of these sidebands is transmitted, it will be a SSB-SC waveform:

$$s(t)_{SSB} = Cos(2\pi f_m t)Cos(2\pi f_c t) \pm Sin(2\pi f_m t)Sin(2\pi f_c t)$$

Where (+) sign represents for the lower sideband, and (-) sign stands for the upper sideband. The modulating signal here is $x(t) = Cos(2\pi f_m t)$, so let $x_h(t) = Sin(2\pi f_m t)$ be the Hilbert Transform of $x(t)$. The Hilbert Transform is obtained by simply giving $\left(-\frac{\pi}{2}\right)$ to a signal. So the expression for SSB-SC signal can be written as:

$$s(t)_{SSB} = x(t)Cos(2\pi f_c t) \pm x_h(t)Sin(2\pi f_c t)$$

Where $x_h(t)$ is a signal obtained by shifting the phase of every component present in $x(t)$ by $\left(-\frac{\pi}{2}\right)$.

Generation of SSB-SC signal

Frequency Discrimination Method:

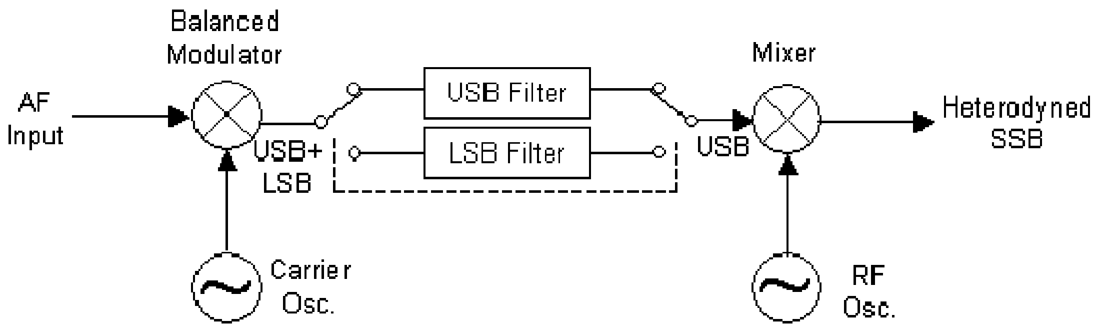


Figure 10 Frequency Discrimination Method of SSB-SC Generation

The filter method of SSB generation produces double sideband suppressed carrier signals (using a balanced modulator), one of which is then filtered to leave USB or LSB. It uses two filters that have different passband centre frequencies for USB and LSB respectively. The resultant SSB signal is then mixed (heterodyned) to shift its frequency higher.

Limitations:

I. This method can be used with practical filters only if the baseband signal is restricted at its lower edge due to which the upper and lower sidebands do not overlap with each other. Hence it is used for speech signal communication where lowest spectral component is 70 Hz and it may be taken as 300 Hz without affecting the intelligibility of the speech signal.
II. The design of band-pass filter becomes quite difficult if the carrier frequency is quite higher than the bandwidth of the baseband signal.

Phase-Shift Method:

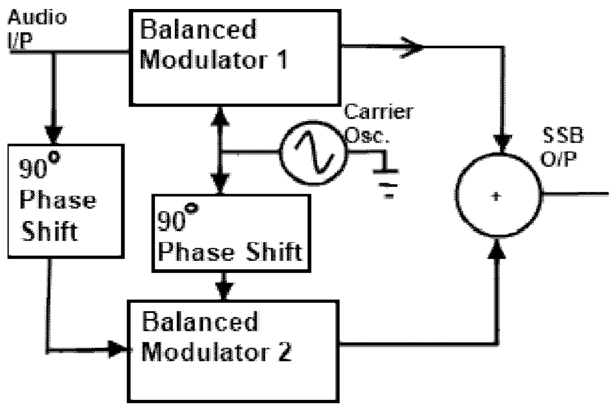
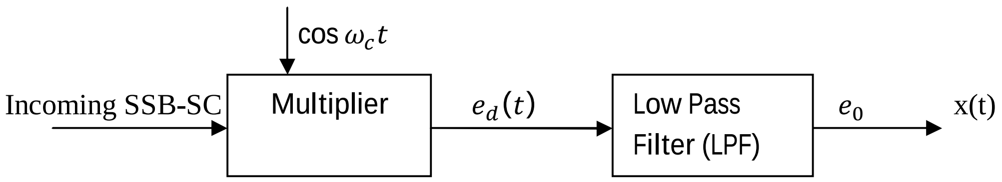


Figure 11 Phase shift method of SSB-SC generation

The phase shifting method of SSB generation uses a phase shift technique that causes one of the side bands to be cancelled out. It uses two balanced modulators instead of one. The balanced modulators effectively eliminate the carrier. The carrier oscillator is applied directly to the upper balanced modulator along with the audio modulating signal. Then both the carrier and modulating signal are shifted in phase by 90o and applied to the second, lower, balanced modulator. The two balanced modulator output are then added together algebraically. The phase shifting action causes one side band to be cancelled out when the two balanced modulator outputs are combined.

Demodulation of SSB-SC Signals:

The baseband or modulating signal x(t) can be recovered from the SSB-SC signal by using synchronous detection technique. With the help of synchronous detection method, the spectrum of an SSB-SC signal centered about $\omega = \pm\omega_c$, is retranslated to the basedand spectrum which is centered about $\omega = 0$. The process of synchronous detection involves multiplication of the received SSB-SC signal with a locally generated carrier.



The output of the multiplier will be

$$e_d(t) = s(t)_{SSB}.\cos \omega_c t$$

or $$e_d(t) = [x(t)\cos \omega_c t \pm x_h(t)\sin \omega_c t]\cos \omega_c t$$

or $$e_d(t) = x(t)\cos^2 \omega_c t \pm x_h(t)\sin \omega_c t \cos \omega_c t$$

or $$e_d(t) = \frac{1}{2}x(t)[1 + \cos(2\omega_c t)] \pm \frac{1}{2}x_h(t)\sin 2\omega_c t$$

or $$e_d(t) = \frac{1}{2}x(t) + \frac{1}{2}[x(t)\cos(2\omega_c t)] \pm x_h(t)\sin 2\omega_c t]$$

When $e_d(t)$ is passed through a low-pass filter, the terms centre at $\pm\omega_c$ are filtered out and the output of detector is only the baseband part i.e. $\frac{1}{2}x(t)$.

Vestigial Sideband Modulation(VSB)

SSB modulation is suited for transmission of voice signals due to the energy gap that exists in the frequency range from zero to few hundred hertz. But when signals like video signals which contain significant frequency components even at very low frequencies, the USB and LSB tend to meet at the carrier frequency. In such a case one of the sidebands is very difficult to be isolated with the help of practical filters. This problem is overcome by the Vestigial Sideband Modulation. In this modulation technique along with one of the sidebands, a gradual cut of the other sideband is also allowed which comes due to the use of practical filter. This cut of the other sideband is called as the *vestige*. Bandwidth of VSB signal is given by :

$$BW=(f_c+f_v)-(f_c-f_m)=f_m+f_v$$

Where $f_m \rightarrow$ bandwidth of bandlimited message signal

$f_v \rightarrow$ width of the vestige in frequency

Angle Modulation

Angle modulation may be defined as the process in which the total phase angle of a carrier wave is varied in accordance with the instantaneous value of the modulating or message signal, while amplitude of the carrier remain unchanged. Let the carrier signal be expressed as:

$$c(\mathrm{t}) = \mathrm{ACos}(2\pi\, \mathrm{f}_c \mathrm{t} + \theta)$$

Where $\phi = 2\pi\, \mathrm{f}_c \mathrm{t} + \theta \rightarrow$ Total phase angle

$\theta \rightarrow$ phase offset

$f_c \rightarrow$ carrier frequency

So in-order to modulate the total phase angle according to the baseband signal, it can be done by either changing the instantaneous carrier frequency according to the modulating signal- the case of *Frequency Modulation*, or by changing the instantaneous phase offset angle according to the modulating signal- the case of *Phase Modulation*. An angle-modulated signal in general can be written as

$$u(t) = ACos(\phi(t))$$

where, $\phi(\mathrm{t})$ is the total phase of the signal, and its instantaneous frequency $f_i(\mathrm{t})$ is given by

$$f_i(t) = \frac{1}{2\pi}\frac{d}{dt}\phi(t)$$

Since *u(t)* is a band-pass signal, it can be represented as

$$u(t) = ACos\left(2\pi f_c t + \theta(t)\right)$$

and, therefore,

$$f_i(t) = f_c + \frac{1}{2\pi}\frac{d}{dt}\theta(t)$$

If *m(t)* is the message signal, then in a PM system we have

$$\theta(t) = k_p m(t)$$

and in an FM system we have

$$f_i(t) - f_c = k_f m(t) = \frac{1}{2\pi}\frac{d}{dt}\theta(t)$$

where k_p and k_f are phase and frequency deviation constants. From the above relationships we have:

$$\theta(t) = \begin{Bmatrix} k_p m(t) \to PM \\ 2\pi k_f \int m(\mathrm{t})\,\mathrm{dt} \to \mathrm{FM} \end{Bmatrix}$$

The maximum phase deviation in a PM system is given by:

$$\Delta\theta_{\max} = k_p \left|m(t)\right|_{\max}$$

And the maximum frequency deviation in FM is given by:

$$\Delta f_{\max} = k_f \left|m(t)\right|_{\max}$$
$$\Delta\omega_{\max} = 2\pi k_f \left|m(t)\right|_{\max}$$

Single Tone Frequency Modulation

The general expression for FM signal is $s(t) = ACos\left(\omega_c t + k_f \int m(\mathrm{t})\,\mathrm{dt}\right)$

So for the single tone case let $m(t) = VCos(\omega_m t)$

Then
$$s(\mathrm{t}) = ACos\left(\omega_c t + \frac{k_f V}{\omega_m} Sin(\omega_m t)\right)$$
$$\Rightarrow s(\mathrm{t}) = ACos\left(\omega_c t + m_f Sin(\omega_m t)\right)$$

Here $m_f = \frac{k_f V}{\omega_m} = \frac{\Delta\omega}{\omega_m} \to$ Modulation Index

Types of Frequency Modulation

High frequency deviation =>High Bandwidth=> High modulation index=>Wideband FM

Small frequency deviation => Small Bandwidth=> Small modulation index=>Narrowband FM

Carson’s Rule

It provides a rule of thumb to calculate the bandwidth of a single-tone FM signal.

$$Bandwidth = 2\left(\Delta f + f_m\right) = 2\left(1 + m_f\right) f_m$$

If baseband signal is any arbitrary signal having large number of frequency components, this rule can be modified by replacing m_f by deviation ratio D.

$$D = \frac{Peak\ Frequency\ deviation\ corresponding\ maximum\ possible\ amplitude\ of\ m(t)}{Maximum\ frquency\ component\ present\ in\ the\ modulating\ signal\ m(t)}$$

Then the bandwidth of FM signal is given as:

$$Bandwidth = 2\left(1 + D\right) f_{\max}$$