

**LECTURE NOTES**  
**On**  
**Electrical Machine 1**

**Name of the Department- Electrical Engineering**

**SUBJECT CODE-1302**

**NAME OF THE SUBJECT- ELECTRICAL MACHINE1 (PART 1)**

**SEMESTER- 3<sup>RD</sup>**

**BRANCH- EE&EEE**

**PART1- MODULE1+ MODULE2**

## DETAIL SYLLABUS

Veer Surendra Sai University of Technology, Orissa, Burla, India  
 Department of Electrical Engineering,  
 Syllabus of Bachelor of Technology in Electrical & Electronics Engineering, 2010

### (3<sup>RD</sup> SEMESTER)

#### ELECTRICAL MACHINES-I (3-1-0)

##### MODULE-I (10 HOURS)

Electromechanical Energy conversion, forces and torque in magnetic field systems – energy balance, energy and force in a singly excited magnetic field system, determination of magnetic force, coenergy, multi excited magnetic field systems.

DC Generators – Principle of operation, Action of commutator, constructional features, armature windings, lap and wave windings, simplex and multiplex windings, use of laminated armature, E. M.F. Equation,

Methods of Excitation: separately excited and self excited generators, build up of E.M.F., critical field resistance and critical speed, causes for failure to self excite and remedial measures, Armature reaction: Cross magnetizing and demagnetizing AT/pole, compensating winding, commutation, reactance voltage, methods of improving commutation

Load characteristics of shunt, series and compound generators, parallel operation of DC generators, use of equalizer bar and cross connection of field windings, load sharing.

##### MODULE-II (10 HOURS)

**Transformers:** Single phase transformer, Constructional details, Core, windings, Insulation, principle of operation, emf equation, magnetising current and core losses, no load and on load operation, Phasor diagram, equivalent circuit, losses and efficiency, condition for maximum efficiency, voltage regulation, approximate expression for voltage regulation, open circuit and short circuit tests, Sumpner's test, Inrush of switching currents, harmonics in single phase transformers, magnetizing current wave form, Parallel operation of transformers.

##### MODULE-III (10 HOURS)

DC Motors: Principle of operation, Back E.M.F., Torque equation, characteristics and application of shunt, series and compound motors, Armature reaction and commutation, Starting of DC motor, Principle of operation of 3 point and 4 point starters, drum controller, Constant & Variable losses, calculation of efficiency, condition for maximum efficiency.

Speed control of DC Motors: Armature voltage and field flux control methods, Ward Leonard method.

Methods of Testing: direct, indirect and regenerative testing, brake test, Swinburne's test, Load test, Hopkinson's test, Field's test, Retardation test, separation of stray losses in a DC motor test.

#### **MODULE-IV (10 HOURS)**

Three phase Transformer: Constructional features of three phase transformers – three phase connection of transformers (Dd0, Dd6, Yy0, Yy6, Dy1, Dy11, Yd1, Yd11, zigzag), Scott connection, open delta connection, three phase to six phase connection, oscillating neutral, tertiary winding, three winding transformer, equal and unequal turns ratio, parallel operation, load sharing. Distribution transformers, all day efficiency, Autotransformers, saving of copper, applications, tap- changing transformers, cooling of transformers.

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# MODULE-I

## DC GENERATOR

### TOPICS

Electromechanical Energy conversion, forces and torque in magnetic field systems – energy balance, energy and force in a singly excited magnetic field system, determination of magnetic force, co-energy, multi excited magnetic field systems.

DC Generators – Principle of operation, Action of commutator, constructional features, armature windings, lap and wave windings, simplex and multiplex windings, use of laminated armature, E. M.F. Equation,

Methods of Excitation: separately excited and self-excited generators, build-up of E.M.F., critical field resistance and critical speed, causes for failure to self-excite and remedial measures, Armature reaction: Cross magnetizing and demagnetizing AT/pole, compensating winding, commutation, reactance voltage, methods of improving commutation

Load characteristics of shunt, series and compound generators, parallel operation of DC generators, use of equalizer bar and cross connection of field windings, load sharing.

[Topics are arranged as per above sequence]

## 1.1 Electromechanical-Energy-Conversion Principles

The electromechanical-energy-conversion process takes place through the medium of the electric or magnetic field of the conversion device of which the structures depend on their respective functions.

- Transducers: microphone, pickup, sensor, loudspeaker
- Force producing devices: solenoid, relay, and electromagnet
- Continuous energy conversion equipment: motor, generator

## 1.2 Forces and Torques in Magnetic Field Systems

The Lorentz Force Law gives the force  $F$  on a particle of charge  $q$  in the presence of electric and magnetic fields.

$$F = q(E + v \times B)$$

Where,  $F$  : newtons,  $q$ : coulombs,  $E$ : volts/meter,  $B$  : telsas,  $v$ : meters/second

> In a pure electric-field system,

$$F = qE$$

> In pure magnetic-field systems,  $F = q(v \times B)$

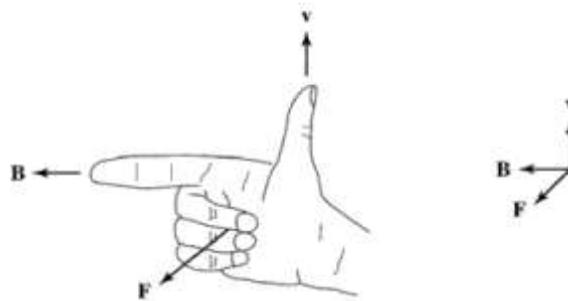


Fig 1.1 Right-hand rule for  $F = q(v \times B)$

> For situations where large numbers of charged particles are in motion,

$$F_v = \rho(E + v \times B)$$

$$J = \rho v$$

$$F_v = J \times B$$

$\rho$  (charge density): coulombs/m<sup>3</sup>,  $F_v$  (force density): newtons/m<sup>3</sup>,  $J = \rho v$  (current density): amperes/m<sup>2</sup>.

Most electromechanical-energy-conversion devices contain magnetic material.

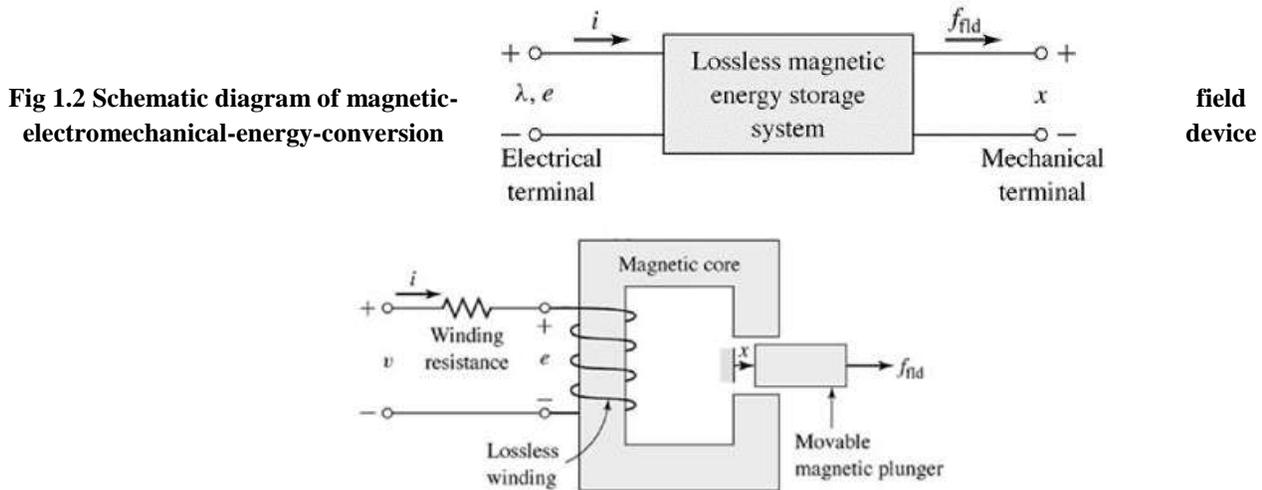
- Forces act directly on the magnetic material of these devices which are constructed of rigid, non-deforming structures.
- The performance of these devices is typically determined by the net force, or torque, acting on the moving component. It is rarely necessary to calculate the details of the internal force distribution.
- Just as a compass needle tries to align with the earth's magnetic field, the two sets of fields associated with the rotor and the stator of rotating machinery attempt to align, and torque is associated with their displacement from alignment.
  - In a motor, the stator magnetic field rotates ahead of that of the rotor, pulling on it and performing work.
  - For a generator, the rotor does the work on the stator.

### The Energy Method

- > Based on the principle of conservation of energy: energy is neither created nor destroyed; it is merely changed in form.
- > Fig. 1.2 shows a magnetic-field-based electromechanical-energy-conversion device.
  - A lossless magnetic-energy-storage system with two terminals
  - The electric terminal has two terminal variables:  $e$  (voltage),  $i$  (current).
  - The mechanical terminal has two terminal variables:  $f_{fld}$  (force),  $x$  (position)
  - The loss mechanism is separated from the energy-storage mechanism.
- Electrical losses: ohmic losses...
- Mechanical losses: friction, windage...

> Fig. 1.3: a simple force-producing device with a single coil forming the electric terminal, and a movable plunger serving as the mechanical terminal.

- The interaction between the electric and mechanical terminals, i.e. the electromechanical energy conversion, occurs through the medium of the magnetic stored energy.



**Fig. 1.3 Schematic diagram of simple force-producing device**

-  $W_{fd}$  : the stored energy in the magnetic field

$$\frac{dW_{fd}}{dt} = ei - f_{fd} \frac{dx}{dt}$$

$$e = \frac{d\lambda}{dt}$$

$$dW_{fd} = id\lambda - f_{fd}dx$$

- From the above equation force can be solved as a function of the flux  $\lambda$  and the mechanical terminal position  $x$ .

- The above equations form the basis for the energy method

### **1.3 Energy Balance**

Consider the electromechanical systems whose predominant energy-storage mechanism is in magnetic fields. For motor action, the energy transfer can be accounted as

$$\begin{pmatrix} \text{Energy input} \\ \text{from electric} \\ \text{sources} \end{pmatrix} = \begin{pmatrix} \text{Mechanical} \\ \text{energy} \\ \text{output} \end{pmatrix} + \begin{pmatrix} \text{Increase in energy} \\ \text{stored in magnetic} \\ \text{field} \end{pmatrix} + \begin{pmatrix} \text{Energy} \\ \text{converted} \\ \text{into heat} \end{pmatrix}$$

The ability to identify a lossless-energy-storage system is the essence of the energy method.

- > This is done mathematically as part of the modeling process.
- > For the lossless magnetic-energy-storage system of Fig. 1.2 can be rearranged and gives

$$dW_{\text{elec}} = dW_{\text{mech}} + dW_{\text{fld}}$$

where

$dW_{\text{elec}} = id\lambda =$  differential electric energy input

$dW_{\text{mech}} = f_{\text{fld}}dx =$  differential mechanical energy output

$dW_{\text{fld}} =$  differential change in magnetic stored energy

> Here  $e$  is the voltage induced in the electric terminals by the changing magnetic stored energy. It is through this reaction voltage that the external electric circuit supplies power to the coupling magnetic field and hence to the mechanical output terminals.

$$dW_{\text{elec}} = ei dt$$

> The basic energy-conversion process is one involving the coupling field and its action and reaction on the electric and mechanical systems.

> Combining above two equation –

$$dW_{\text{elec}} = ei dt = dW_{\text{mech}} + dW_{\text{fld}}$$

### **1.4 Energy in Singly-Excited Magnetic Field Systems**

In energy-conversion systems the magnetic circuits have air gaps between the stationary and moving members in which considerable energy is stored in the magnetic field.

> This field acts as the energy-conversion medium, and its energy is the reservoir between the

electric and mechanical system.

Fig. 1.4 shows an electromagnetic relay schematically. The predominant energy storage occurs in the air gap, and the properties of the magnetic circuit are determined by the dimensions of the air gap.

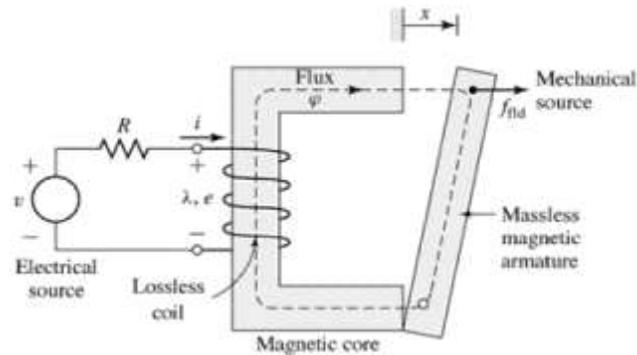


Fig.1.4 Schematic of an electromagnetic relay

$$\lambda = L(x)i$$

$$dW_{\text{mech}} = f_{\text{fld}} dx$$

$$dW_{\text{fld}} = id\lambda - f_{\text{fld}} dx$$

$W_{\text{fld}}$  is uniquely specified by the values of  $\lambda$  and  $x$ . Therefore,  $\lambda$  and  $x$  are referred to as state variables.

Since the magnetic energy storage is lossless, it is conservative system.  $W_{\text{fld}}$  is the same regardless of how  $\lambda$  and  $x$  are brought to their final values. Fig 1.5 shows where tow separate the paths.

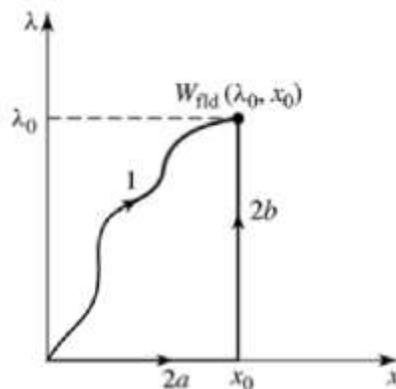


Fig. 1.5 Integration paths for  $W_{\text{fld}}$

On path 2a,  $d\lambda=0$  and  $f_{\text{fld}}=0$ . Thus  $df_{\text{fld}}=0$  on path 2a. On path 2b,  $dx=0$ . Therefore the following equation can be written

$$W_{\text{fld}}(\lambda_0, x_0) = \int_0^{\lambda_0} i(\lambda, x_0) d\lambda$$

For a linear system in which  $\lambda$  is proportional to  $i$  the equation will change and can be written as-

$$W_{\text{fld}}(\lambda, x) = \int_0^{\lambda} i(\lambda', x) d\lambda' = \int_0^{\lambda} \frac{\lambda'}{L(x)} d\lambda' = \frac{1}{2} \frac{\lambda^2}{L(x)}$$

$V$ : the volume of the magnetic field

$$W_{\text{fld}} = \int_V \left( \int_0^B H \cdot dB' \right) dV$$

If  $B = \mu H$ , then

$$W_{\text{fld}} = \int_V \left( \frac{B^2}{2\mu} \right) dV$$

### 1.5 Determination of Magnetic Force and Torque from Energy

The magnetic stored energy is a state function, determined uniquely by the values of the  $W_{\text{fld}}$  independent state variables  $\lambda$  and  $x$ .

$$\begin{aligned} dW_{\text{fld}}(\lambda, x) &= id\lambda - f_{\text{fld}}dx \\ dW_{\text{fld}}(\lambda, x) &= \left. \frac{\partial W_{\text{fld}}}{\partial \lambda} \right|_x d\lambda + \left. \frac{\partial W_{\text{fld}}}{\partial x} \right|_{\lambda} dx \\ i &= \left. \frac{\partial W_{\text{fld}}(\lambda, x)}{\partial \lambda} \right|_x \\ f_{\text{fld}} &= - \left. \frac{\partial W_{\text{fld}}(\lambda, x)}{\partial x} \right|_{\lambda} \end{aligned}$$

### **1.6 Energy and Co-energy:**

It is that energy from which the force can be obtained directly as a function of the current. The selection of energy or co-energy as the state function is purely a matter of convenience.

The co-energy  $W'_{\text{fld}}(i, x)$  is defined as a function of  $I$  and  $x$  such that

$$\begin{aligned}
 W'_{\text{fld}}(i, x) &= i\lambda - W_{\text{fld}}(\lambda, x) \\
 d(i\lambda) &= i d\lambda + \lambda di \\
 dW'_{\text{fld}}(i, x) &= d(i\lambda) - dW_{\text{fld}}(\lambda, x) \\
 \boxed{dW'_{\text{fld}}(i, x) &= \lambda di + f_{\text{fld}} dx}
 \end{aligned}$$

From the above equation co-energy  $W'_{\text{fld}}(i, x)$  can be seen to be a state function of the two independent variables  $i$  and  $x$ .

$$\begin{aligned}
 dW'_{\text{fld}}(i, x) &= \left. \frac{\partial W'_{\text{fld}}}{\partial i} \right|_x di + \left. \frac{\partial W'_{\text{fld}}}{\partial x} \right|_i dx \\
 \lambda &= \left. \frac{\partial W'_{\text{fld}}(i, x)}{\partial i} \right|_x \\
 \boxed{f_{\text{fld}} &= \left. \frac{\partial W'_{\text{fld}}(i, x)}{\partial x} \right|_i}
 \end{aligned}$$

For a system with a rotating mechanical displacement,

$$\begin{aligned}
 W'_{\text{fld}}(i, \theta) &= \int_0^i \lambda(i', \theta) di' \\
 T_{\text{fld}} &= \left. \frac{\partial W'_{\text{fld}}(i, \theta)}{\partial \theta} \right|_i
 \end{aligned}$$

If the system is magnetically linear,

$$\begin{aligned}
 W'_{\text{fld}}(i, \theta) &= \frac{1}{2} L(\theta) i^2 \\
 T_{\text{fld}} &= \frac{i^2}{2} \frac{dL(\theta)}{d\theta}
 \end{aligned}$$

In field-theory terms, for soft magnetic materials

$$\begin{aligned}
 W'_{\text{fld}} &= \int_V \left( \int_0^{H_0} B \cdot dH \right) dV \\
 W'_{\text{fld}} &= \int_V \frac{\mu H^2}{2} dV
 \end{aligned}$$

For permanent-magnet (hard) materials

$$W'_{fld} = \int_V \left( \int_{H_c}^{H_0} B \cdot dH \right) dV$$

For a magnetically-linear system, the energy and co-energy (densities) are numerically equal:

$$\frac{1}{2} \lambda^2 / L = \frac{1}{2} Li^2, \quad \frac{1}{2} B^2 / \mu = \frac{1}{2} \mu H^2$$

For a nonlinear system in which  $\lambda$  and  $i$  or  $B$  and  $H$  are not linearly proportional, the two functions are not even numerically equal.

$$W_{fld} + W'_{fld} = \lambda i$$

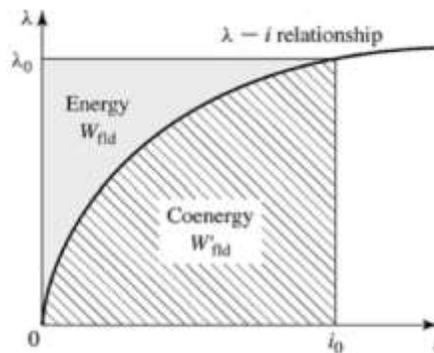


Fig.1.6 Graphical interpretation of energy and co-energy in a singly-excited system

## 1.7 Multiply-Excited Magnetic Field Systems

Many electromechanical devices have multiple electrical terminals.

- > Measurement systems: torque proportional to two electric signals; power as the product of voltage and current.
- > Energy conversion devices: multiply-excited magnetic field system.
- > A simple system with two electrical terminals and one mechanical terminal:

Three independent variables:  $\{\theta, \lambda_1, \lambda_2\}$ ,  $\{\theta, i_1, i_2\}$ ,  $\{\theta, \lambda_1, i_2\}$ , or  $\{\theta, i_1, \lambda_2\}$ .

$$dW_{fld}(\lambda_1, \lambda_2, \theta) = i_1 d\lambda_1 + i_2 d\lambda_2 - T_{fld} d\theta$$

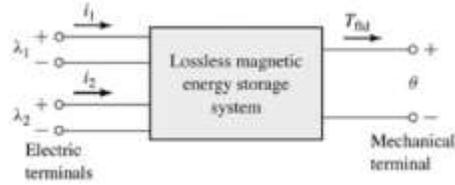


Fig. 1.7 Multiply-excited magnetic energy storage system

$$i_1 = \left. \frac{\partial W_{\text{fld}}(\lambda_1, \lambda_2, \theta)}{\partial \lambda_1} \right|_{\lambda_2, \theta}$$

$$i_2 = \left. \frac{\partial W_{\text{fld}}(\lambda_1, \lambda_2, \theta)}{\partial \lambda_2} \right|_{\lambda_1, \theta}$$

$$T_{\text{fld}} = - \left. \frac{\partial W_{\text{fld}}(\lambda_1, \lambda_2, \theta)}{\partial \theta} \right|_{\lambda_1, \lambda_2}$$

To find  $W_{\text{fld}}$ , use the path of integration as shown in Fig 1.8.

$$W_{\text{fld}}(\lambda_{10}, \lambda_{20}, \theta_0) = \int_0^{\lambda_{20}} i_2(\lambda_1 = 0, \lambda_2, \theta = \theta_0) d\lambda_2 + \int_0^{\lambda_{10}} i_1(\lambda_1, \lambda_2 = \lambda_{20}, \theta = \theta_0) d\lambda_1$$

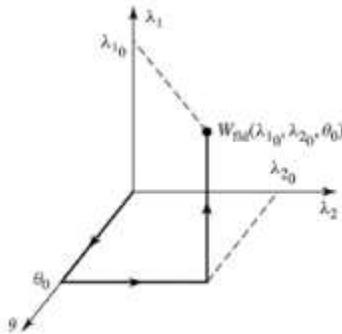


Fig. 1.8 Integration path to obtain  $W_{\text{fld}}(\lambda_{10}, \lambda_{20}, \theta_0)$

> In a magnetically-linear system,

$$\lambda_1 = L_{11}i_1 + L_{12}i_2$$

$$\lambda_2 = L_{21}i_1 + L_{22}i_2$$

$$L_{12} = L_{21}$$

Note that  $L_{ij} = L_{ij}(\theta)$

$$i_1 = \frac{L_{22}\lambda_1 - L_{12}\lambda_2}{D}$$

$$i_2 = \frac{-L_{21}\lambda_1 + L_{11}\lambda_2}{D}$$

$$D = L_{11}L_{22} - L_{12}L_{21}$$

The energy for this linear system is

$$W_{\text{fld}}(\lambda_{1_0}, \lambda_{2_0}, \theta_0) = \int_0^{\lambda_{2_0}} \frac{L_{11}(\theta_0)\lambda_2}{D(\theta_0)} d\lambda_2 + \int_0^{\lambda_{1_0}} \frac{(L_{22}(\theta_0)\lambda_1 - L_{12}(\theta_0)\lambda_{2_0})}{D(\theta_0)} d\lambda_1$$

$$= \frac{1}{2D(\theta_0)} L_{11}(\theta_0)\lambda_{2_0}^2 + \frac{1}{2D(\theta_0)} L_{22}(\theta_0)\lambda_{1_0}^2 - \frac{L_{12}(\theta_0)}{D(\theta_0)} \lambda_{1_0} \lambda_{2_0}$$

Co-energy function for a system with two windings can be defined as

$$W'_{\text{fld}}(i_1, i_2, \theta) = \lambda_1 i_1 + \lambda_2 i_2 - W_{\text{fld}}$$

$$dW'_{\text{fld}}(i_1, i_2, \theta) = \lambda_1 di_1 + \lambda_2 di_2 + T_{\text{fld}} d\theta$$

$$\lambda_1 = \left. \frac{\partial W'_{\text{fld}}(i_1, i_2, \theta)}{\partial i_1} \right|_{i_2, \theta}$$

$$\lambda_2 = \left. \frac{\partial W'_{\text{fld}}(i_1, i_2, \theta)}{\partial i_2} \right|_{i_1, \theta}$$

$$T_{\text{fld}} = \left. \frac{\partial W'_{\text{fld}}(i_1, i_2, \theta)}{\partial \theta} \right|_{i_1, i_2}$$

$$W'_{\text{fld}}(i_1, i_2, \theta_0) = \int_0^{i_{2_0}} \lambda_2(i_1 = 0, i_2, \theta = \theta_0) di_2 + \int_0^{i_{1_0}} \lambda_1(i_1, i_2 = i_{2_0}, \theta = \theta_0) di_1$$

For a linear system

$$W'_{\text{fld}}(i_1, i_2, \theta_0) = \frac{1}{2} L_{11}(\theta) i_1^2 + \frac{1}{2} L_{22}(\theta) i_2^2 + L_{12}(\theta) i_1 i_2$$

$$T_{\text{fld}} = \left. \frac{\partial W'_{\text{fld}}(i_1, i_2, \theta_0)}{\partial \theta} \right|_{i_1, i_2} = \frac{i_1^2}{2} \frac{dL_{11}(\theta)}{d\theta} + \frac{i_2^2}{2} \frac{dL_{22}(\theta)}{d\theta} + i_1 i_2 \frac{dL_{12}(\theta)}{d\theta}$$

- Note that the co-energy function is a relatively simple function of displacement.
- The use of a co-energy function of the terminal currents simplifies the determination of torque or force.

- Systems with more than two electrical terminals are handled in analogous fashion.

## DC Generators

### 1.8 Principle of operation of DC Generator

A D.C generator as shown in figure below the armature be driven by a prime mover in the clock wise direction and the stator field is excited to produce the field poles as shown. There will be induced voltage in each armature conductor. The direction of the induced voltage can be determined by applying *Fleming's right hand rule*. All the conductors under the influence of North Pole will have  $\otimes$  directed induced voltage, while the conductors under the influence of South Pole will have  $\odot$  induced voltage in them. For a loaded generator the direction of the armature current will be same as that of the induced voltages. Thus  $\otimes$  and  $\odot$  also represent the direction of the currents in the conductors. We know, a current carrying conductor placed in a magnetic field experiences force, the direction of which can be obtained by applying *Fleming's left hand rule*. Applying this rule to the armature conductors in fig 1.9, the rotor experiences a torque ( $T_e$ ) in the counter clockwise direction (i.e., opposite to the direction of rotation) known as back torque. For steady speed operation back torque is equal to the machines input torque ( $T_{pm}$ ) i.e. the torque supplied by prime mover.

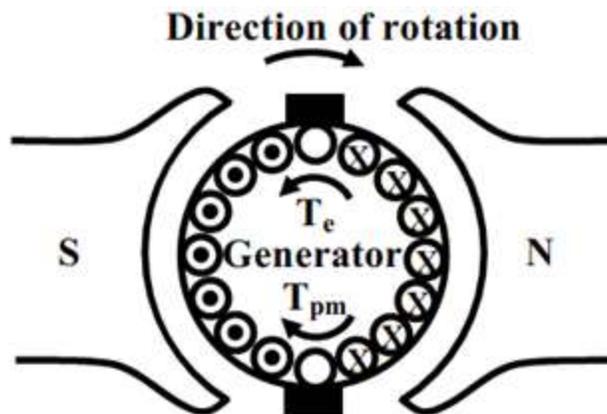


Fig. 1.9 Action of DC generator

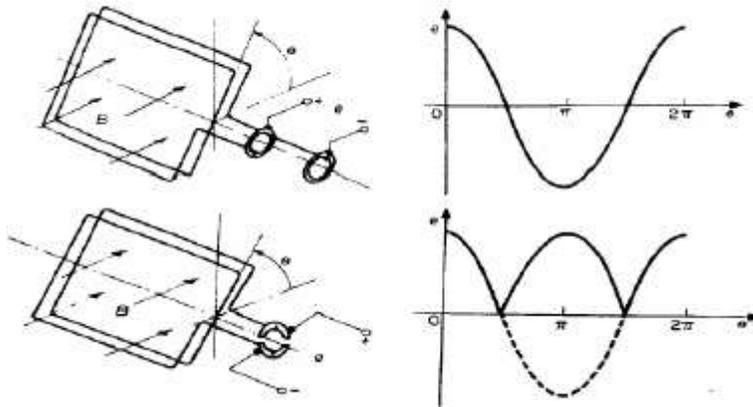
### 1.9 Action of Commutator

In DC machines the current in each wire of the armature is actually alternating, and hence a device is required to convert the alternating current generated in the DC generator by electromagnetic induction into direct current, or at the armature of a DC motor to convert the input direct current into alternating

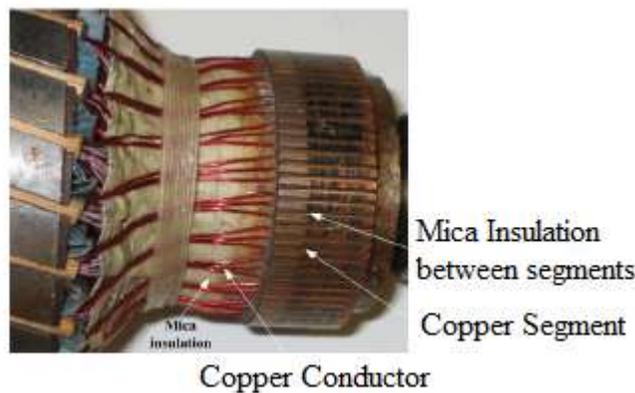
current at appropriate times, as illustrated in Fig. 1.10.

DC generator: induced AC *emf* is converted to DC voltage;

DC motor: input direct current is converted to alternating current in the armature at appropriate times to produce a unidirectional torque. The commutator consists of insulated copper segments mounted on an insulated tube. Armature coils are connected in series through the commutator segments. Two brushes are pressed to the commutator to permit current flow. The brushes are placed in the neutral zone, where the magnetic field is close to zero, to reduce arcing. The *commutator* switches the current from one rotor coil to the adjacent coil. The switching requires the interruption of the coil current. The sudden interruption of an inductive current generates high voltages. The high voltage produces flashover and arcing between the commutator segment and the brush.



**Fig. 1.10** Action of Commutator



**Fig. 1.11** Mechanical view of commutator

## 1.10 Constructional Features

The stator of the dc machine has poles, which are excited by dc current to produce magnetic fields. In the neutral zone, in the middle between the poles, commutating poles or interpoles are placed to reduce sparking of the commutator due to armature reaction. The commutating poles are supplied by dc current. Compensating windings are mounted on the main poles. Field poles are mounted on an iron core that provides a closed magnetic circuit. The motor housing supports the iron core, the brushes and the bearings. The rotor has a ring-shaped laminated iron core with slots. Coils with several turns are placed in the slots. The distance between the two legs of the coil is about 180 electric degrees for full pitch. The coils are connected in series through the commutator segments. The ends of each coil are connected to a commutator segment.

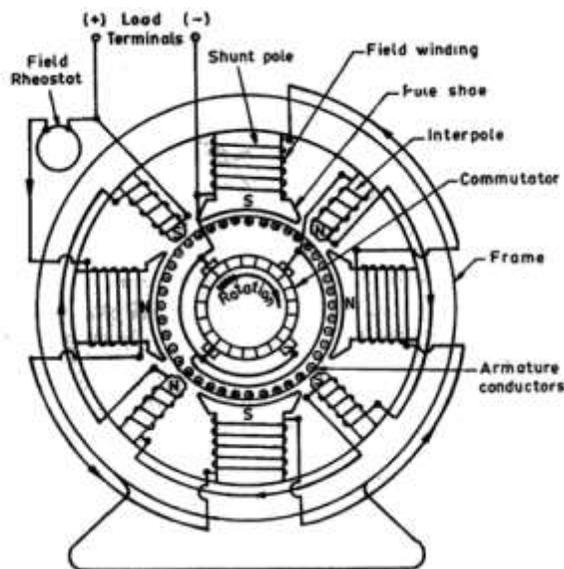
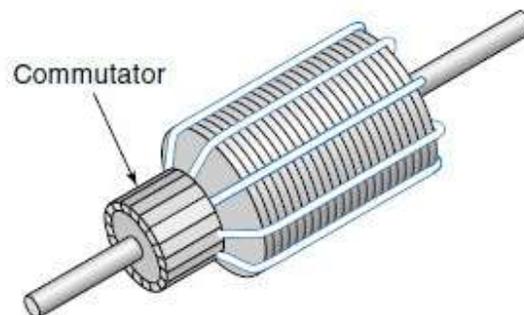


Fig. 1.12 Cross sectional view of DC Machine



**Fig. 1.13 Armature of DC Machine****1.11 Armature Winding**

DC machines armature consists of armature conductors. The conductors distributed in slots provided on the periphery of the armature is called armature winding. Depending on the way in which the coils are interconnected at the commutator end of the armature, the windings can be classified as lap and wave windings. Further they can be classified as simplex and multiplex.

**1.11.1 Coil Span/Coil Pitch:**

It represents the span of the coil. For full pitched winding, the span is  $180^\circ$  electrical or number of slots per pole. Coil pitch can be represented in terms of electrical degrees, slots or conductor. A full pitched coil leads to maximum voltage per coil.

**1.11.2 Back Pitch ( $Y_b$ ):**

It is the distance measured in between the two coil sides of the same coil at the back end of the armature, the commutator end being the front end of armature. It can be represented in terms of number of slots or coil sides. Back pitch also represents the span of coil.

**1.11.3 Front Pitch ( $Y_f$ ):**

The distance between the two coil sides of two different coils connected in series at the front end of the armature is called front pitch.

**1.12 Lap Winding**

Lap winding is suitable for low voltage high current machines because of more number of parallel paths. The number of parallel path in lap winding is equal to number of poles.

$$A=P$$

Equalizing rings are connected in lap winding.

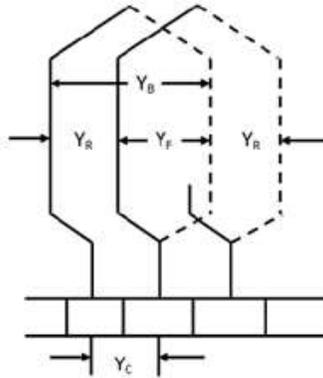


Fig. 1.14 Winding diagram of lap winding

### 1.13 Wave Winding

Wave winding is used for high voltage low current machines. In case of wave winding, the number of parallel path ( $A$ ) = 2 irrespective of number of poles. Each path will have conductors connected in series.

Equalizing rings are not required in wave winding.

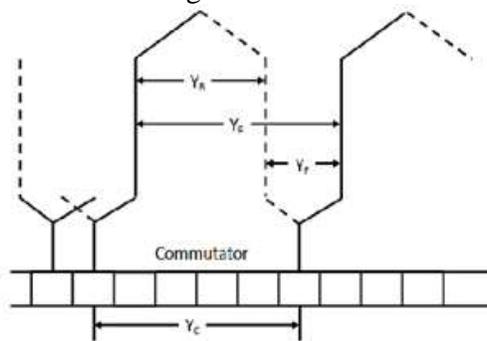


Fig. 1.15 Winding diagram of wave winding

### 1.14 Simplex and Multiplex Winding

Fig 1.14 and Fig. 1.15 shows simplex lap and simplex wave winding.

The degree of multiplicity of a multiplex winding indicates the relative number of parallel paths with respect to the number of parallel paths in the corresponding simplex winding. For example a duplex lap or wave winding is a lap or wave winding having twice as many as parallel paths as a simplex lap or wave winding respectively. The winding can be triplex or quadruplex winding in similar manner.

### 1.15 Use of Laminated Armature

The armature winding of DC machine should be laminated to reduce eddy current losses. The armature body (rotor) rotates in the field magnetic field. Thus in the core of the armature voltage induced which in

turn causes current to flow in the body. This current is known as eddy current. This current causes loss and thus heat will be generated. This loss depends on the amount of current flow. To reduce the amount of current flow the resistance of the body should be increased. Thus using lamination the resistance of the path through which current flows will be increased. The amount of eddy current will be reduced and thus eddy current loss can be minimized.

### **1.16 EMF Equation**

Let  $\phi$  = flux per pole in weber

$Z$  = number of armature conductors = Number of slots X conductors per slot.

$P$  = Number of poles;  $A$  = Number of parallel paths in armature.

$A = P$  for lap wound armature;  $A = 2$  for wave wound armature

$N$  = speed of armature in rpm;  $E$  = induced emf in each parallel path.

Average emf generated/conductor in one revolution =  $\frac{d\phi}{dt}$

Flux cut by a conductor in one revolution =  $d\phi = P\phi$  weber.

Since Number of revolutions/second =  $\frac{N}{60}$

Time taken for one revolution =  $dt = \frac{60}{N}$  seconds

EMF generated/conductor =  $\frac{d\phi}{dt} = \frac{P\phi}{\frac{60}{N}} = \frac{P\phi N}{60}$

Since each path has  $\frac{Z}{A}$  conductors in series,

EMF generated in each path is  $E = \frac{P\phi N}{60} \times \frac{Z}{A}$

$$E = \frac{P\phi ZN}{60A}$$

### **1.17 Methods of excitation**

DC machines are excited in two ways-

#### **1.17.1 Separate excitation:**

When the field winding is connected to an external source to produce field flux. According to the type of excitation this machines are called separately excited dc machine.

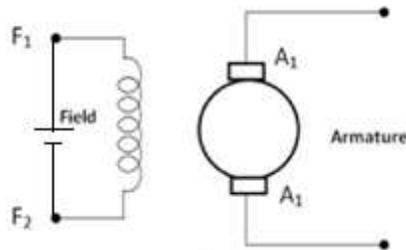


Fig. 1.16 Schematic diagram of separately excited dc machine

### 1.17.2 Self-excitation:

When the field winding is connected with the armature to produce field flux. A self-excited machine requires residual magnetism for operation. According to the type of excitation this machines are called self-excited dc machine.

Depending on the type of field winding connection DC machines can be classified as:

#### 1.17.2.1 Shunt machine:

The field winding consisting of large number of turns of thin wire is usually excited in parallel with armature circuit and hence the name shunt field winding. This winding will be having more resistance and hence carries less current.

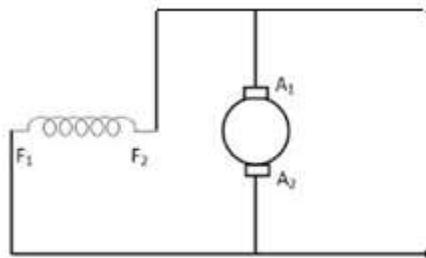


Fig. 1.17 Schematic diagram of dc shunt machine

#### 1.17.2.2 Series machine:

The field winding has a few turns of thick wire and is connected in series with armature.

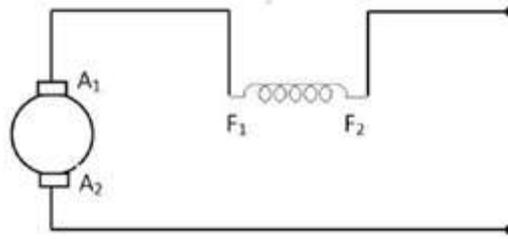


Fig.1.18 Schematic diagram of dc series machine

### 1.17.2.3 Compound machine:

Compound wound machine comprises of both series and shunt windings and can be either short shunt or long shunt, cumulative, differential or flat compounded.

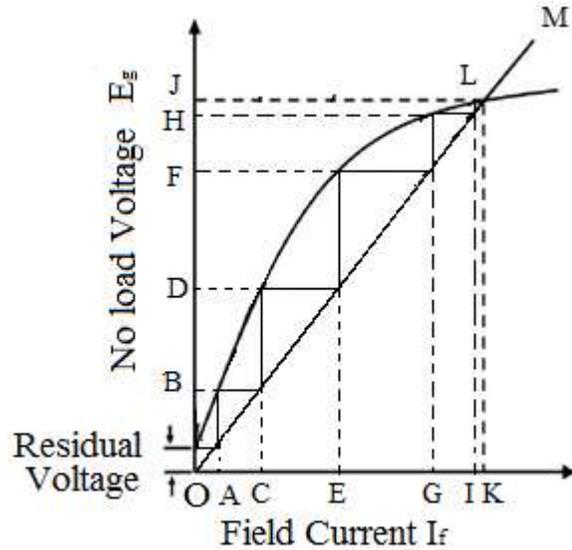


Fig. 1.19 A Schematic diagram of short -shunt compound machine

Fig.1.19 B Schematic diagram of long-shunt compound machine

### 1.18 Build-up of E.M.F

When the armature is rotating with armature open circuited, an emf is induced in the armature because of the residual flux. When the field winding is connected with the armature, a current flows through the field winding ( in case of shunt field winding, field current flows even on No-load and in case of series field winding only with load) and produces additional flux. This additional flux along with the residual flux generates higher voltage. This higher voltage circulates more current to generate further higher voltage. This is a cumulative process till the saturation is attained.

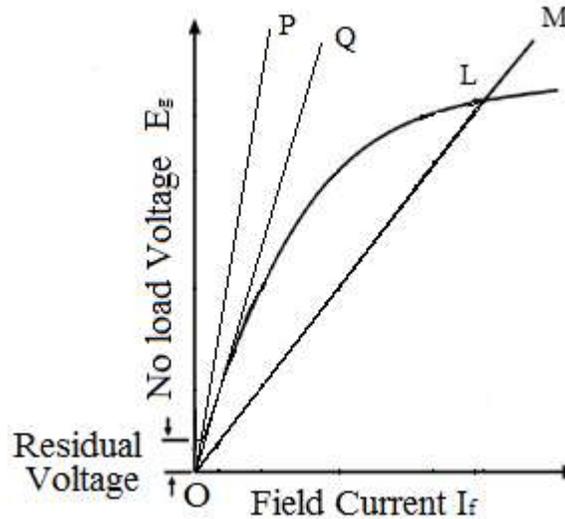


**Fig. 1.20 Process of voltage build-up in DC generator**

Here OM is the field resistance curve in Fig. 1.20. Initially there will be residual voltage which will create OA field current. This field current will increase the existing magnetic field and the induced voltage will increase up-to OB. This OB voltage will further applied to the field winding and increase the field current to OC. This process will continue upto the point L where the emf curve intersect with field resistance and finally the induced voltage will be OJ. This way voltage builds-up in dc generator.

### **1.19 Critical Resistance:-**

The voltage to which it builds is decided by the resistance of the field winding as shown in the figure 1.21. If field circuit resistance is increased such that the resistance line does not cut OCC like 'OP' in the figure1.21, then the machine will fail to build up voltage to the rated value. The slope of the air gap line drawn as a tangent (OQ) to the initial linear portion of the curve represents the maximum resistance that the field circuit can have beyond which the machine fails to build up voltage. This value of field circuit resistance is called critical field resistance. The field circuit is generally designed to have a resistance value less than this so that the machine builds up the voltage to the rated value.

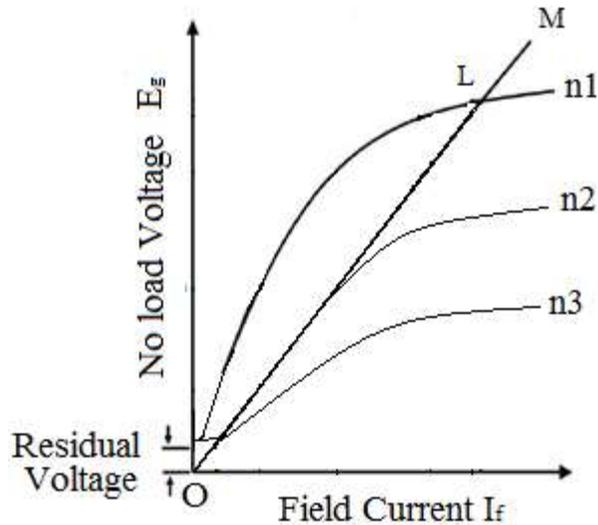


**Fig. 1.21 Field current vs No-load voltage for different field resistances**

Critical field resistance is defined as the maximum field circuit resistance for a given speed with which the shunt generator would excite. The shunt generator will build up voltage only if field circuit resistance is less than critical field resistance.

### **1.20 Critical Speed:**

Voltage of a dc generator is proportional to its speed. Thus when speed will be reduced then the induced voltage will reduced. There can be such situation occur when the speed will be so low that the existing field winding resistance voltage bulid up will not occur. The speed of the generator can be lowered upto a certain level. This minimum value of the speed of the generator for which the generator can excite is called critical speed. It can also define as that speed of a generator for which the existing field resistance of generator becomes its critical field resistance.



connected to the dual flux.

**Fig. 1.22 Field current vs No-load voltage for different speed**

In the above figure it is showing that when speed of the generator changes from  $n_1$  to  $n_2$  and then  $n_3$  emf production changes accordingly. Here  $n_1 > n_2 > n_3$ . For speed  $n_3$  voltage build-up is not possible. The speed  $n_2$  is the critical speed. As shown in the figure at speed  $n_2$  generator field resistance become its critical field resistance.

### **1.21 Causes for failure to self-excite and its remedial**

- i. The field poles may not have residual magnetism. Then the generator will fail to excite.

Then to restore residual magnetism field winding should be connected to an external dc voltage source. This is called flashing of field.

- ii. When the direction of rotation is not proper such that flux produced by the field current reinforces the residual magnetism.

The rotation of the machine has to be reversed.

- iii. The field winding resistance is more than critical resistance then the machine will fail to excite.

The field winding resistance should be less than critical field resistance.

- iv. When the speed of the machine is less than critical speed.

The machine's speed should be more than critical speed.

- v. If the field winding connections are such that newly generated field flux is working in opposite to the existing residual magnetism. Then the generator will fail to excite.

Then the field winding connection should be reversed.

## 1.22 Armature Reaction

The action of magnetic field set up by armature current on the distribution of flux under main poles of a DC machine is called armature reaction.

When the armature of a DC machine carries current, the distributed armature winding produces its own mmf. The machine air gap is now acted upon by the resultant mmf distribution caused by the interaction of field ampere turns ( $AT_f$ ) and armature ampere turns ( $AT_a$ ). As a result the air gap flux density gets distorted.

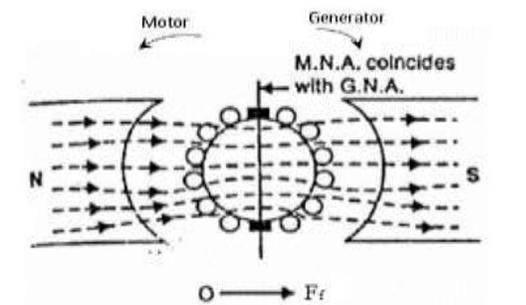


Fig. a

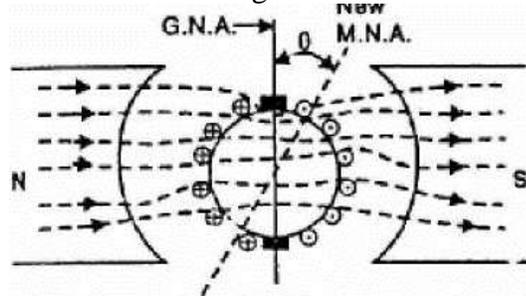


Fig. c

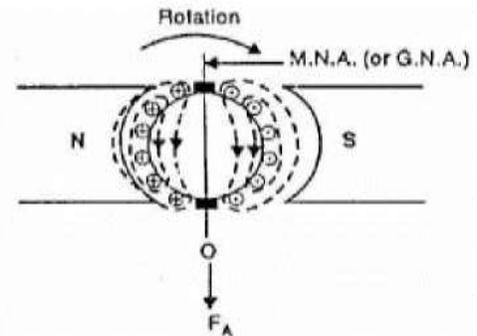


Fig. b

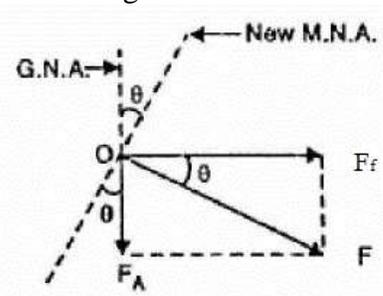


Fig. d

Figure a shows a two pole machine with single equivalent conductor in each slot and the main field mmf ( $F_f$ ) acting alone. The axis of the main poles is called the direct axis (d-axis) and the interpolar axis is called quadrature axis (q-axis). It can be seen from the Figure b that armature mmf ( $F_a$ ) is along the

interpolar axis.  $F_a$  which is at  $90^\circ$  to the main field axis is known as cross magnetizing mmf.

Figure c shows the practical condition in which a DC machine operates when both the Field flux and armature flux are existing. Because of both fluxes are acting simultaneously, there is a shift in brush axis and crowding of flux lines at the trailing pole tip and flux lines are weakened or thinned at the leading pole tip. (The pole tip which is first met in the direction of rotation by the armature conductor is leading pole tip and the other is trailing pole tip).

If the iron in the magnetic circuit is assumed unsaturated, the net flux/pole remains unaffected by the armature reaction though the air gap flux density distribution gets distorted. If the main pole excitation is such that the iron is in the saturated region of magnetization (practical case) the increase in flux density at one end of the poles caused by armature reaction is less than the decrease at the other end, so that there is a net reduction in the flux/pole. This is called the demagnetizing effect. Thus it can be summarized that the nature of armature reaction in a DC machine is

1. Cross magnetizing with its axis along the q-axis.
2. It causes no change in flux/pole if the iron is unsaturated but causes reduction in flux/pole in the presence of iron saturation. This is termed as demagnetizing effect. The resultant mmf 'F' is shown in figure d.

### **1.22.1 Cross Magnetizing Ampere Turns/pole(AT<sub>c</sub>)**

If the brush is shifted by an angle  $\theta$  as shown in figure 1.23 then the conductors lying in between the angles BOC and DOA are carrying the current in such a way that the direction of the flux is downwards i.e., at right angles to the main flux. This results in the distortion in the main flux. Hence, these conductors are called cross magnetizing or distorting ampere conductors.

$$\text{Total armature conductors/pole} = \frac{Z}{P}$$

$$\text{Demagnetizing conductors / pole} = Z \frac{2\theta}{360}$$

Therefore cross magnetizing conductors/pole =  $\frac{Z}{P} - Z \frac{2\theta}{360}$

Cross magnetizing ampere turns/pole  $AT_c = \frac{ZI_a}{a} \left( \frac{1}{2P} - \frac{\theta}{360} \right)$

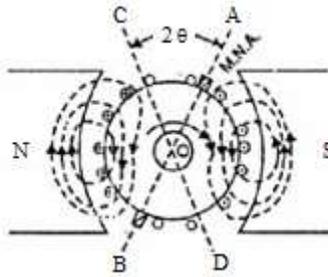


Fig. 1.23 Cross-magnetizing ampere conductors

### 1.22.2 Demagnetizing Ampere Turns /pole (AT<sub>d</sub>)

The exact conductors which produce demagnetizing effect are shown in Fig 1.24, Where the brush axis is given a forward lead of  $\theta$  so as to lie along the new axis of M.N.A. The flux produced by the current carrying conductors lying in between the angles AOC and BOD is such that, it opposes the main flux and hence they are called as demagnetizing armature conductors.

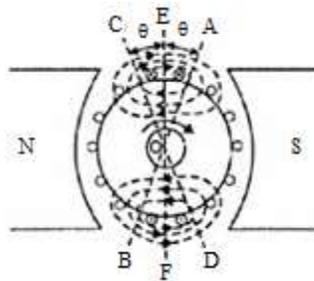


Fig. 1.24 Demagnetizing ampere conductors

$Z$  = total no of armature conductors

Current in each armature conductors =  $\frac{I_a}{a}$

$\theta$  = Forward lead in mechanical or angular deg.

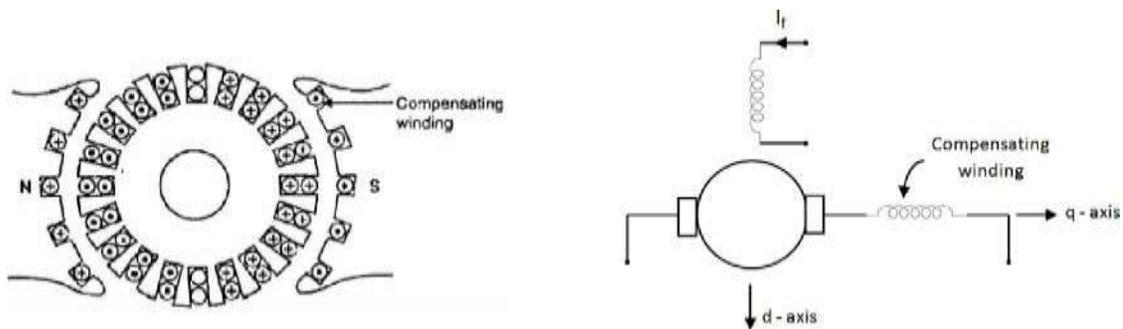
Total no of armature conductors in between angles AOC & BOD =  $\frac{4\theta}{360} Z$

Demagnetizing amp turns/poles  $AT_d = \frac{Z\theta I_a}{360a}$

### 1.23 Compensating Winding

Due to armature reaction flux density wave get distorted and reduced. Due to distortion of flux wave the peak flux density increases to such a high value that it creates high induced emf. If this emf is higher than the breakdown voltage across adjacent segments, a spark over could result which can easily spread over the whole commutator, and there will be a ring of fire, resulting in the complete short circuit of the armature.

To protect armature from such adverse condition armature reaction must be neutralized. To neutralize the armature reaction ampere-turns by compensating winding placed in the slots cut out in pole face such that the axis of the winding coincides with the brush axis as shown figure 1.25.



**Fig. 1.25 Compensating conductors in field poles and the connection of compensating conductors with armature**  
The compensating windings neutralize the armature mmf directly under the pole which is the major portion because in the interpole region the air gap will be large. Compensating windings are connected in series with armature so that it will create mmf proportional to armature mmf.

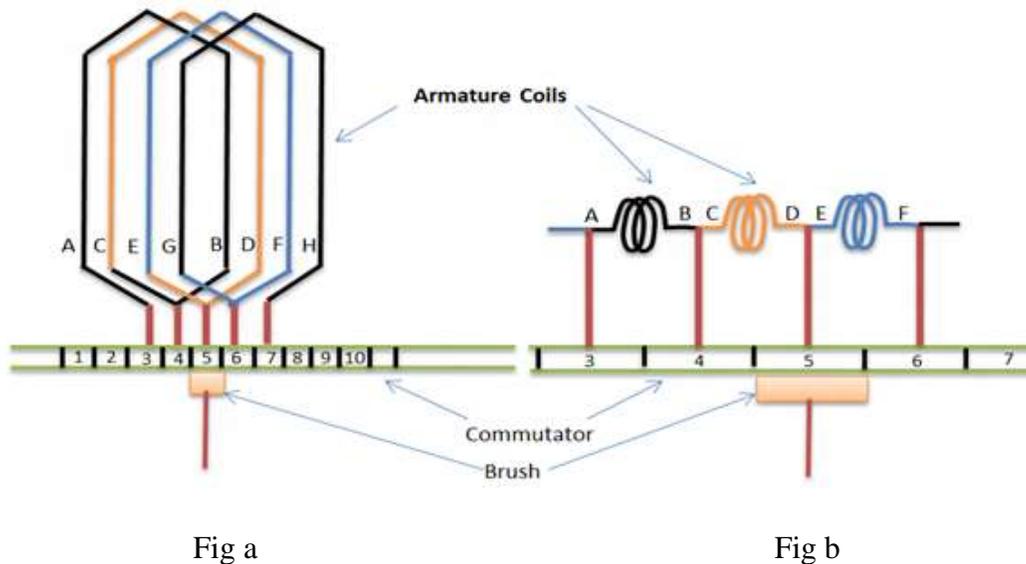
The number of ampere-turns required in the compensating windings is given by

$$AT_c = \text{Total Armature Ampere Turns} \times \frac{\text{Pole Arc}}{\text{Pole Pitch}}$$

### 1.24 Commutation

The process of reversal of current in the short circuited armature coil is called 'Commutation'. This process of reversal takes place when coil is passing through the interpolar axis (q-axis), the coil is short circuited through commutator segments and brush.

The process of commutation of coil 'CD' is shown Fig. 1.26. In sub figure 'c' coil 'CD' carries 20A current from left to right and is about to be short circuited in figure 'd' brush has moved by a small width and the brush current supplied by the coil are as shown. In figure 'e' coil 'CD' carries no current as the brush is at the middle of the short circuit period and the brush current is supplied by coil 'AB' and coil 'EF'. In sub figure 'f' the coil 'CD' which was carrying current from left to right carries current from right to left. In sub fig 'g' spark is shown which is due to the reactance voltage. As the coil is embedded in the armature slots, which has high permeability, the coil possess appreciable amount of self inductance. The current is changed from +20 to -20. So due to self inductance and variation in the current from +20 to -20, a voltage is induced in the coil which is given by  $L \frac{di}{dt}$ . This emf opposes the change in current in coil 'CD' thus sparking occurs.



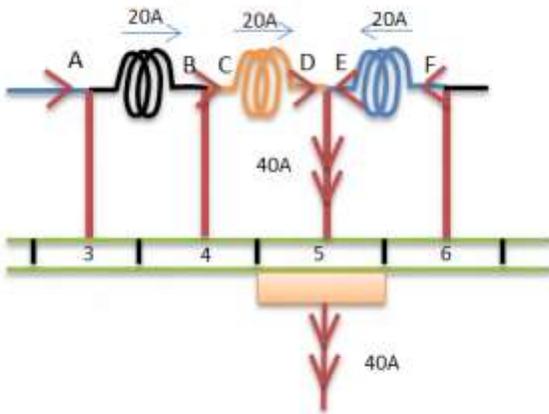


Fig.c

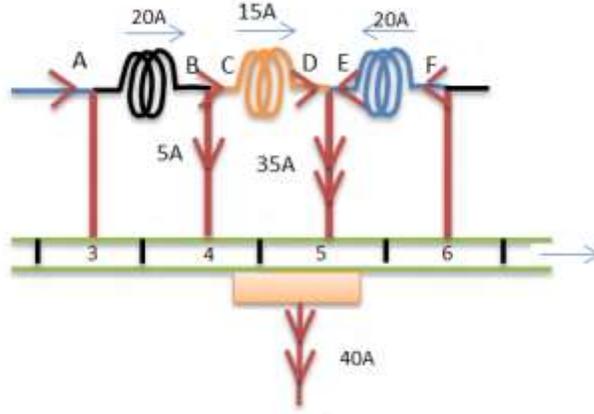


Fig.d

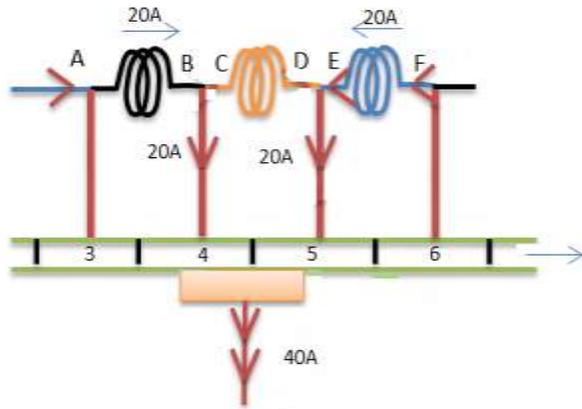


Fig.e

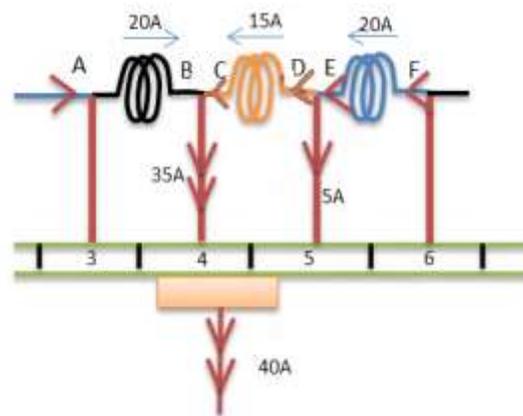


Fig.f

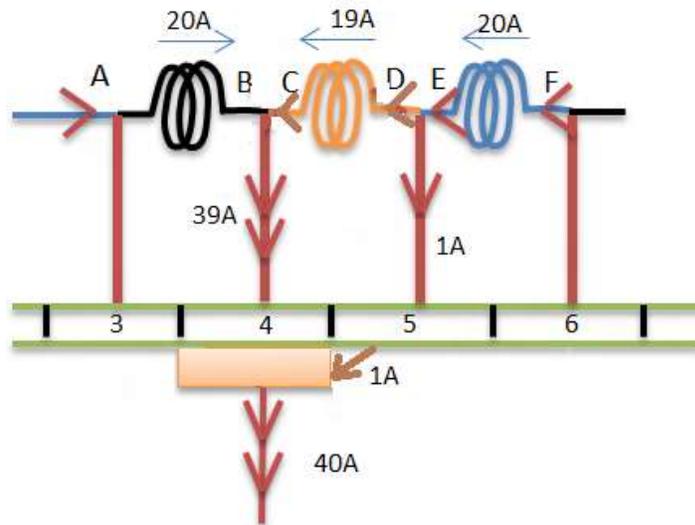


Fig.g

Fig.1.26 a-g shows the process of commutation

### **1.25 Reactance Voltage**

During commutation sparking occurs in the commutator segment and brush due to presence of reactance voltage. This voltage is generated due to change of current in the commutating coil for its self-inductance and also due to mutual inductance of the adjacent coils. This voltage is called reactance voltage and according to Lenz's law this induced voltage oppose its cause of production. Here the cause of production is the change in current in the coil under commutation. Thus the commutation becomes poorer.

Reactance voltage = co-efficient of self-inductance X rate of change of current =  $L \frac{di}{dt}$

Time of short circuit =  $T_c$  = (time required by commutator to move a distance equal to the circumferential thickness of brush) – (one mica insulating strip) = Time of commutation

Let  $W_b$  = brush width in cm

$W_i$  = width of mica insulation in cm

$V_c$  = peripheral velocity of commutator segments in cm/sec.

Then  $T_c = \frac{W_b - W_i}{V_c}$  sec

Total change in current =  $I - (-I) = 2I$

Therefore self-induced or reactance voltage =  $L \frac{2I}{T_c}$  for linear commutation

=  $1.11L \frac{2I}{T_c}$  for sinusoidal commutation

If brush width is given in terms of commutator segments, then commutator velocity should be converted in terms of commutator segments/seconds.

### **1.26 Method of Improving Commutation**

Commutation can be improved in two ways by (i) Resistance commutation

(ii) E.M.F commutation.

### 1.26.1 Resistance Commutation

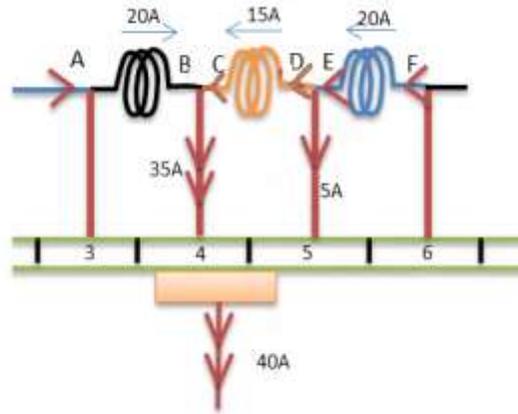


Fig. 1.27

In this method the resistance of the brushes are increased by changing them from copper brush to carbon brush. From the above figure 1.27 it is seen that when current '20A' from coil 'EF' reaches the commutator segment '5', it has two parallel paths opened to it. The first path is straight from bar '5' to the brush and the other is via short circuited coil 'CD' to bar '4' and then to brush. If copper brushes are used the current will follow the first path because of its low contact resistance. But when carbon brushes having high resistance are used, then current '20A' will prefer the second path because the resistance  $r_1$  of first path will increase due to reducing area of contact with bar '5' and the resistance  $r_2$  of second path decreases due to increasing area of contact with bar '4'. Hence carbon brushes help in obtaining sparkless commutation. Also, carbon brushes lubricate and polish commutator. But, because of high resistance the brush contact drop increases and the commutator has to be made larger to dissipate the heat due to loss. Carbon brushes require larger brush holders because of lower current density.

### 1.26.2 E.M.F commutation:

To neutralize sparking caused by reactance voltage in this method an emf is produced which acts in opposite direction to that of reactance voltage, so that the reactance voltage is completely eliminated. The neutralization of emf may be done in two ways (i) by giving brush a forward lead sufficient enough to bring the short circuited coil under the influence of next pole of opposite polarity or (ii) by using

interpoles or composites. The second method is commonly employed.

### 1.26.3 Interpoles or Composites

These are small poles fixed to the yoke and placed in between the main poles as shown in figure 1.28. They are wound with few turns of heavy gauge copper wire and are connected in series with the armature so that they carry full armature current. Their polarity in case of generator is that of the main pole ahead in the direction of rotation. Their polarity in case of motor is that of the main pole behind in the direction of rotation.

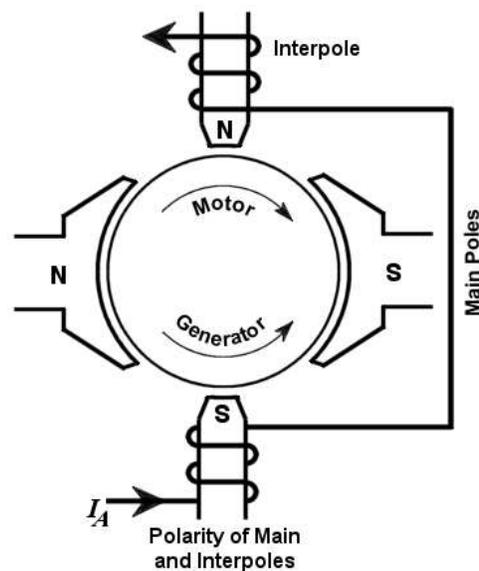


Fig. 1.28 Inter-poles of DC machines

The function of interpoles is (i) to induce an emf which is equal and opposite to that of the reactance voltage. Interpoles neutralize the cross magnetizing effect of armature reaction.

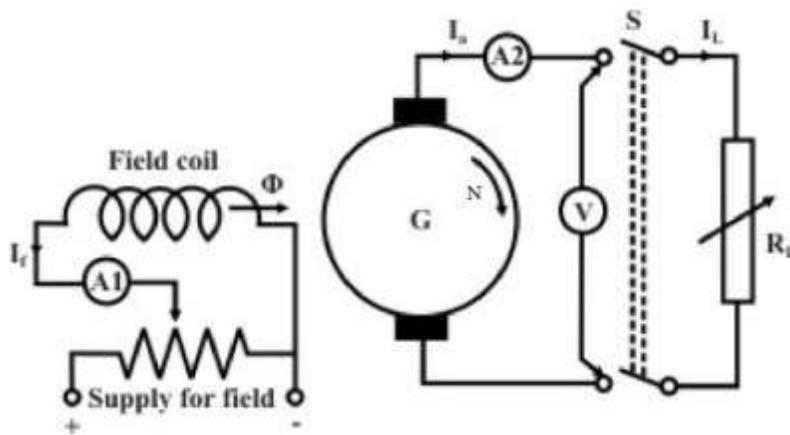
### 1.27 DC Machines Characteristics

There main characteristics of dc machines are

- i. No-load characteristics or Open Circuit Characteristics
- ii. Load characteristics

### 1.27.1 No-load characteristics or Open Circuit Characteristics of separately excited generator

In this type of generator field winding is excited from a separate source, hence field current is independent of armature terminal voltage as shown on figure 1.29. The generator is driven by a prime mover at rated speed, say  $N$  rpm. With switch  $S$  in opened condition, field is excited via a potential divider connection from a separate d.c source and field current is gradually increased. The field current will establish the flux per pole  $\phi$ . The voltmeter  $V$  connected across the armature terminals of the machine will record the generated emf ( $E_g = \frac{P\phi ZN}{60A} = k\phi N$ ). As field current is increased,  $E_g$  will increase.  $E_g$  versus  $I_f$  plot at speed  $N_1, N_2, N_3$  is shown in figure below. Where  $N_1 > N_2 > N_3$ .



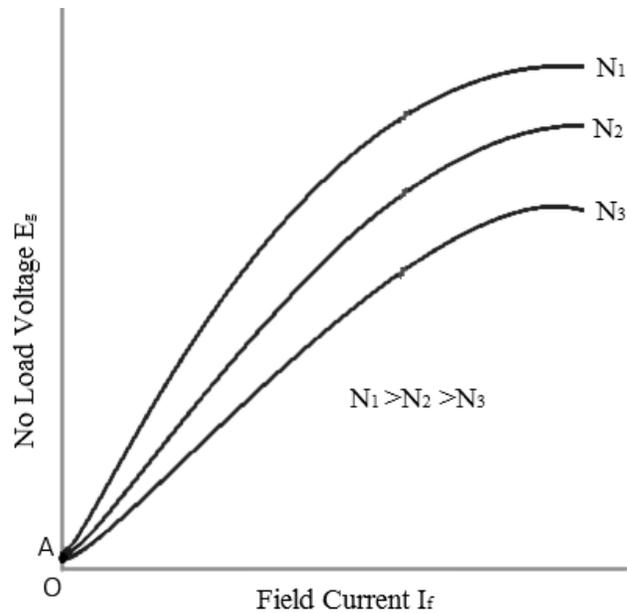


Fig. 1.29 Open circuit characteristics of DC separately excited generator

### 1.27.2 Load characteristic of separately excited generator

Load characteristic is the characteristics in between terminal voltage  $V_T$  with load current  $I_L$  of a generator with constant speed and constant field current. For  $I_L = 0$ ,  $V_T = E_g$  should be the first point on the load characteristic. With increase of load current the terminal voltage will drop due to armature resistance and reaction drop. In the figure below the rated load current shown by point A. Hence the load characteristic will be drooping in nature as shown in figure 1.30.

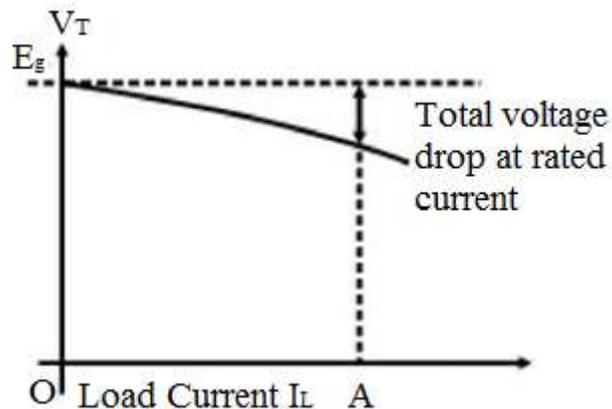
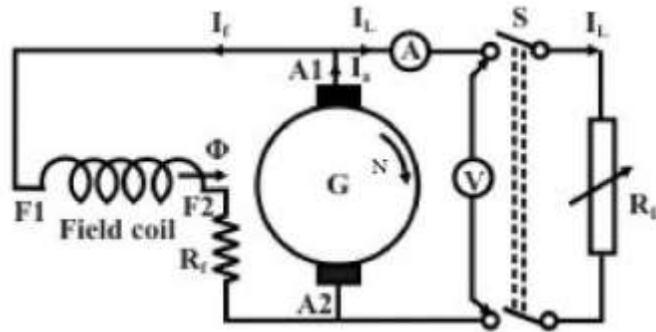


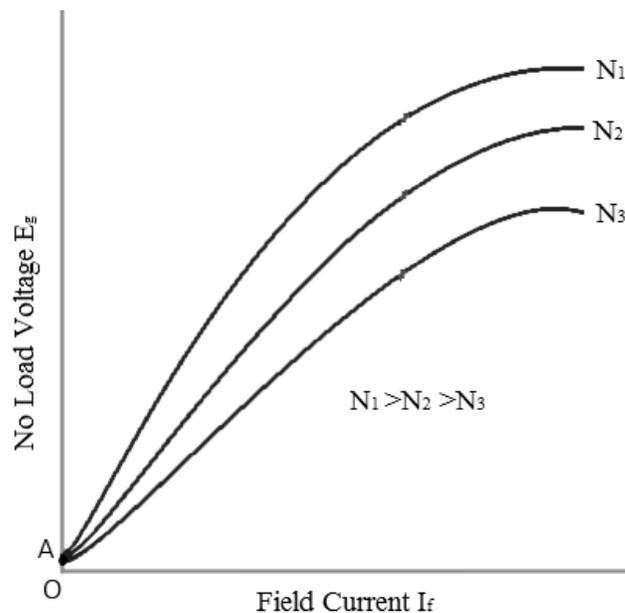
Fig 1.30 Load characteristics of DC separately excited generator

### 1.27.3 Characteristics of a shunt generator



**Fig 1.31 Connection diagram to obtain no-load and load characteristic**

To obtain OCC of DC shunt generator the above circuit will be used with switch S kept open in fig. 1.31. As this machine is self-excited thus there is no need to use separate dc source for producing field current. The voltage build up process is described earlier. The open nature of the OCC will be similar to separately excited machine.



**Fig. 1.32 Open circuit characteristics of DC shunt generator**

#### 1.27.4 Load characteristic of shunt generator

With switch S in open condition in fig. 1.31, the generator is practically under no load condition as field current is pretty small. The voltmeter reading will be  $E_g$  as shown in figures below. In other words,  $V_T = E_g$ ,  $I_L = 0$  is the first point in the load characteristic. To load the machine S is closed and the load resistances decreased so that it delivers load current  $I_L$ . Unlike separately excited motor, here  $I_L \neq I_a$ . In

fact, for shunt generator,  $I_a = I_L - I_f$ . So increase of  $I_L$  will mean increase of  $I_a$  as well. The drop in the terminal voltage will be caused by the usual  $I_a r_a$  drop, brush voltage drop and armature reaction effect. Apart from these, in shunt generator, as terminal voltage decreases, field current hence  $\phi$  also decreases causing additional drop in terminal voltage. Remember in shunt generator, field current is decided by the terminal voltage by virtue of its parallel connection with the armature. Figure 1.33 gives the plot of terminal voltage versus load current which is called the load characteristic.

As the load resistance is decreased (load current increased), the terminal voltage drops until point B is reached. If load resistance is further decreased, the load current increases momentarily. This momentary increase in load current produces more armature reaction thus causing a reduction in the terminal voltage and field current. The net reduction in terminal voltage is so large that the load current decreases and the characteristic turns back. In case the machine is short circuited, the curve terminates at point H. Here OH is the load current due to the voltage generated by residual flux.

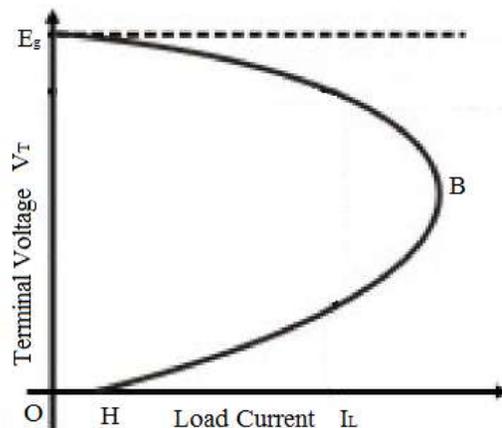
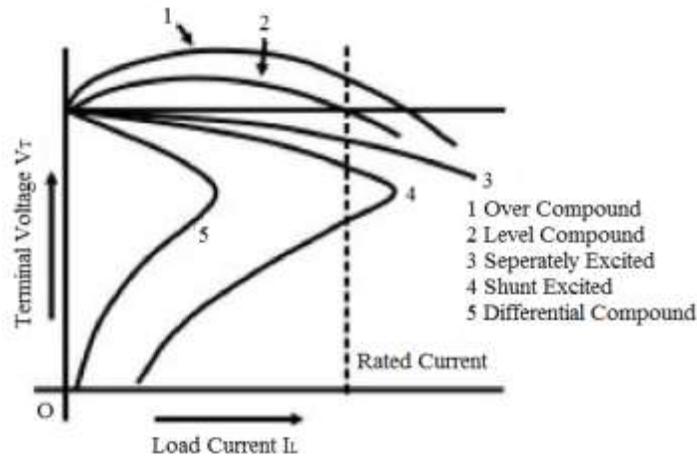


Fig. 1.33 Load characteristics of DC shunt generator

### 1.27.5 Compound generator

As introduced earlier, compound machines have both series and shunt field coils. Series field coil may be connected in such a way that the mmf produced by it aids the shunt field mmf-then the machine is said to be cumulative compound machine, otherwise if the series field mmf acts in opposition with the shunt field mmf – then the machine is said to be differential compound machine.



**Fig. 1.34 Load characteristics of DC compound generator**

In a compound generator, series field coil current is load dependent. Therefore, for a cumulatively compound generator, with the increase of load, flux per pole increases. This in turn increases the generated emf and terminal voltage. Unlike a shunt motor, depending on the strength of the series field mmf, terminal voltage at full load current may be same or more than the no load voltage. When the terminal voltage at rated current is same that at no load condition, then it is called a level compound generator. If however, terminal voltage at rated current is more than the voltage at no load, it is called an over compound generator. The load characteristic of a cumulative compound generator will naturally be above the load characteristic of a shunt generator as depicted in figure 1.34. At load current higher than the rated current, terminal voltage starts decreasing due to saturation, armature reaction effect and more drop in armature and series field resistances.

## **1.28 Parallel Operation of DC Generator**

### **1.28.1 Advantages of DC generator operating in parallel**

In a dc power plant, power is usually supplied from several generators of small ratings connected in parallel instead of from one large generator. This is due to the following reasons:

#### ***a. Continuity of service:***

If a single large generator is used in the power plant, then in case of its breakdown, the whole plant will be shut down. However, if power is supplied from a number of small units operating in parallel, then in case of failure of one unit, the continuity of supply can be maintained by other healthy units.

**b. Efficiency:**

Generators run most efficiently when loaded to their rated capacity. Therefore, when load demand on power plant decreases, one or more generators can be shut down and the remaining units can be efficiently loaded.

**c. Maintenance and repair:**

Generators generally require routine-maintenance and repair. Therefore, if generators are operated in parallel, the routine or emergency operations can be performed by isolating the affected generator while load is being supplied by other units. This leads to both safety and economy.

**d. Increasing plant capacity:**

In the modern world of increasing population, the use of electricity is continuously increasing. When added capacity is required, the new unit can be simply paralleled with the old units.

**e. Non-availability of single large unit:**

In many situations, a single unit of desired large capacity may not be available. In that case a number of smaller units can be operated in parallel to meet the load requirement. Generally a single large unit is more expensive.

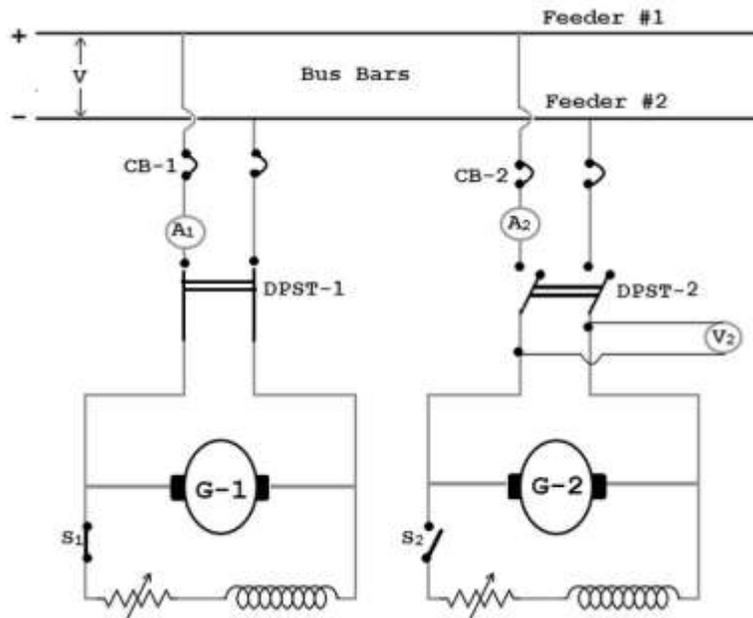
**1.28.2 Connecting Shunt Generators in Parallel:**

The generators in a power plant are connected in parallel through bus-bars. The bus-bars are heavy thick copper bars and they act as +ve and -ve terminals. The positive terminals of the generators are connected to the +ve side of bus-bars and negative terminals to the negative side of bus-bars. Fig. 1.35 shown shunt generator 1 connected to the bus-bars and supplying load. When the load on the power plant increases beyond the capacity of this generator, the second shunt generator 2 is connected in parallel with the first to meet the increased load demand.

***The procedure for paralleling generator 2 with generator 1 is as under:***

- i. The prime mover of generator 2 is brought up to the rated speed. Now switch S2 in the field circuit of the generator 2 is closed.
- ii. Next circuit breaker CB-2 is closed and the excitation of generator 2 is adjusted till it generates voltage equal to the bus-bars voltage. This is indicated by voltmeter V2.

- iii. Now the generator 2 is ready to be paralleled with generator 1. The main switch DPST2 is closed, thus putting generator 2 in parallel with generator 1. Note that generator 2 is not supplying any load because its generated emf is equal to bus-bars voltage. The generator is said to be “floating” (i.e. not supplying any load) on the bus-bars.



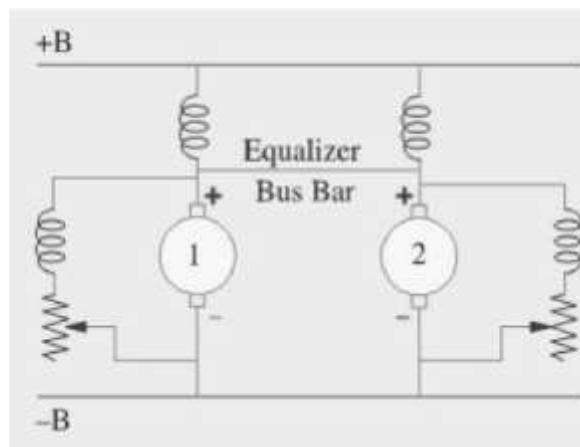
**Fig. 1.35 Schematic diagram of DC generator connected in parallel**

- iv. If generator 2 is to deliver any current, then its generated voltage  $E_g$  should be greater than the bus-bars voltage  $V_T$ . In that case, current supplied by it is  $I = (E_g - V_T)/R_a$  where  $R_a$  is the resistance of the armature circuit. By increasing the field current (and hence induced emf  $E_g$ ), the generator 2 can be made to supply proper amount of load.
- v. The load may be shifted from one shunt generator to another merely by adjusting the field excitation. Thus if generator 1 is to be shut down, the whole load can be shifted onto generator 2 provided it has the capacity to supply that load. In that case, reduce the current supplied by generator 1 to zero (This will be indicated by ammeter A1) open C.B.-1 and then open the main switch DPST1.

### **1.29 Equalizer Bar:**

**Compound Generators in Parallel:** Under-compounded generators also operate satisfactorily in

parallel but over compounded generators will not operate satisfactorily unless their series fields are paralleled. This is achieved by connecting two negative brushes together as shown in Fig. 1.36. The conductor used to connect these brushes is generally called equalizer bar. Suppose that an attempt is made to operate the two generators in parallel without an equalizer bar. If, for any reason, the current supplied by generator 1 increases slightly, the current in its series field will increase and raise the generated voltage. This will cause generator 1 to take more load. Since total load supplied to the system is constant, the current in generator 2 must decrease and as a result its series field is weakened. Since this effect is cumulative, the generator 1 will take the entire load and drive generator 2 as a motor. After machine 2 changes from a generator to a motor, the current in the shunt field will remain in the same direction, but the current in the armature and series field will reverse. Thus the magnetizing action, of the series field opposes that of the shunt field. As the current taken by the machine 2 increases, the demagnetizing action of series field becomes greater and the resultant field becomes weaker. The resultant field will finally become zero and at that time machine 2 will be short circuited machine 1, opening the breaker of either or both machines.



**Fig. 1.36 Connection of equalizer bar in parallel connection of DC compound generator**

When the equalizer bar is used, a stabilizing action exists and neither machine tends to take all the load. To consider this, suppose that current delivered by generator 1 increases. The increased current will not only pass through the series field of generator 1 but also through the equalizer bar and series field of generator 2. Therefore, the voltage of both the machines increases and the generator 2 will take a part

of the load.

### **1.30 Load Sharing:**

The load sharing between shunt generators in parallel can be easily regulated because of their drooping characteristics. The load may be shifted from one generator to another merely by adjusting the field excitation. Let us discuss the load sharing of two generators which have unequal no-load voltages. Let  $E_1, E_2$  = no-load voltages of the two generators  $R_1, R_2$  = their armature resistances

$V_T$  = common terminal voltage (Bus-bars voltage). Then

$$I_1 = \frac{E_1 - V_T}{R_1} \quad \text{and} \quad I_2 = \frac{E_2 - V_T}{R_2}$$

Thus current output of the generators depends upon the values of  $E_1$  and  $E_2$ . These values may be changed by field rheostats. The common terminal voltage (or bus-bars voltage) will depend upon (i) the emfs of individual generators and (ii) the total load current supplied. It is generally desired to keep the busbars voltage constant. This can be achieved by adjusting the field excitations of the generators operating in parallel.



## Module II

### [TRANSFORMER]

#### TOPICS

**Transformers:** Single phase transformer, Constructional details, Core, windings, Insulation, principle of operation, emf equation, magnetising current and core losses, no load and on load operation, Phasor diagram, equivalent circuit, losses and efficiency, condition for maximum efficiency, voltage regulation, approximate expression for voltage regulation, open circuit and short circuit tests, Sumpner's test, Inrush of switching currents, harmonics in single phase transformers, magnetizing current wave form, Parallel operation of transformers.

[Topics are arranged as per above sequence]

# Transformers

## 2.1 Introduction

The transformer is a device that transfers electrical energy from one electrical circuit to another electrical circuit. The two circuits may be operating at different voltage levels but always work at the same frequency. Basically transformer is an electro-magnetic energy conversion device. It is commonly used in electrical power system and distribution systems. It can change the magnitude of alternating voltage or current from one value to another. This useful property of transformer is mainly responsible for the widespread use of alternating currents rather than direct currents i.e., electric power is generated, transmitted and distributed in the form of alternating current. Transformers have no moving parts, rugged and durable in construction, thus requiring very little attention. They also have a very high efficiency as high as 99%.

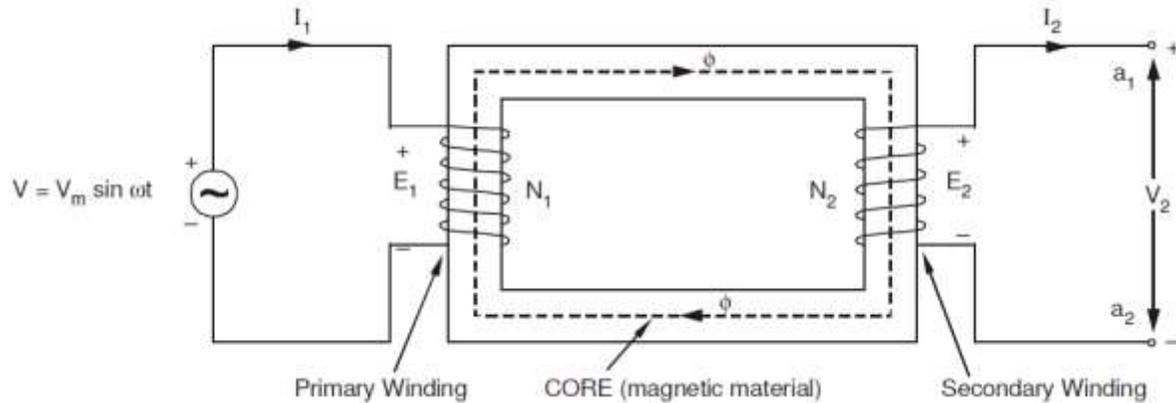
## 2.2. Single Phase Transformer

A transformer is a static device of equipment used either for raising or lowering the voltage of an a.c. supply with a corresponding decrease or increase in current. It essentially consists of two windings, the primary and secondary, wound on a common laminated magnetic core as shown in Fig 1. The winding connected to the a.c. source is called primary winding (or primary) and the one connected to load is called secondary winding (or secondary). The alternating voltage  $V_1$  whose magnitude is to be changed is applied to the primary.

Depending upon the number of turns of the primary ( $N_1$ ) and secondary ( $N_2$ ), an alternating e.m.f.  $E_2$  is induced in the secondary. This induced e.m.f.  $E_2$  in the secondary causes a secondary current  $I_2$ . Consequently, terminal voltage  $V_2$  will appear across the load.

If  $V_2 > V_1$ , it is called a step up-transformer.

If  $V_2 < V_1$ , it is called a step-down transformer.



**Fig. 2.1 Schematic diagram of single phase transformer**

## **2.3 Constructional Details**

Depending upon the manner in which the primary and secondary windings are placed on the core, and the shape of the core, there are two types of transformers, called (a) core type, and (b) shell type.

### **2.3.1 Core-type and Shell-type Construction**

In core type transformers, the windings are placed in the form of concentric cylindrical coils placed around the vertical limbs of the core. The low-voltage (LV) as well as the high-voltage (HV) winding are made in two halves, and placed on the two limbs of core. The LV winding is placed next to the core for economy in insulation cost. Figure 2.1(a) shows the cross-section of the arrangement. In the shell type transformer, the primary and secondary windings are wound over the central limb of a three-limb core as shown in Figure 2.1(b). The HV and LV windings are split into a number of sections, and the sections are interleaved or sandwiched i.e. the sections of the HV and LV windings are placed alternately.

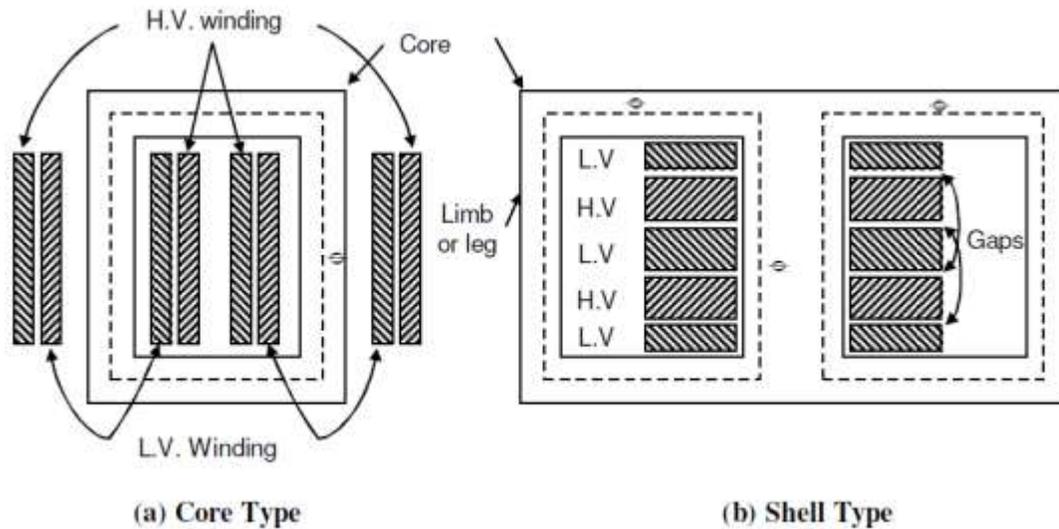


Fig: 2.1 Core type & shell type transformer

### Core

The core is built-up of thin steel laminations insulated from each other. This helps in reducing the eddy current losses in the core, and also helps in construction of the transformer. The steel used for core is of high silicon content, sometimes heat treated to produce a high permeability and low hysteresis loss. The material commonly used for core is CRGO (Cold Rolled Grain Oriented) steel. Conductor material used for windings is mostly copper. However, for small distribution transformer aluminium is also sometimes used. The conductors, core and whole windings are insulated using various insulating materials depending upon the voltage.

### Insulating Oil

In oil-immersed transformer, the iron core together with windings is immersed in insulating oil. The insulating oil provides better insulation, protects insulation from moisture and transfers the heat produced in core and windings to the atmosphere.

The transformer oil should possess the following qualities:

- (a) High dielectric strength,
- (b) Low viscosity and high purity,
- (c) High flash point, and

(d) Free from sludge.

Transformer oil is generally a mineral oil obtained by fractional distillation of crude oil.

### **Tank and Conservator**

The transformer tank contains core wound with windings and the insulating oil. In large transformers small expansion tank is also connected with main tank is known as conservator. Conservator provides space when insulating oil expands due to heating. The transformer tank is provided with tubes on the outside, to permits circulation of oil, which aides in cooling. Some additional devices like breather and Buchholz relay are connected with main tank. Buchholz relay is placed between main tank and conservator. It protect the transformer under extreme heating of transformer winding. Breather protects the insulating oil from moisture when the cool transformer sucks air inside. The silica gel filled breather absorbs moisture when air enters the tank. Some other necessary parts are connected with main tank like, Bushings, Cable Boxes, Temperature gauge, Oil gauge, Tappings, etc.

### **2.4 Principle of Operation**

When an alternating voltage  $V_1$  is applied to the primary, an alternating flux  $\phi$  is set up in the core. This alternating flux links both the windings and induces e.m.f.s  $E_1$  and  $E_2$  in them according to Faraday's laws of electromagnetic induction. The e.m.f.  $E_1$  is termed as primary e.m.f. and e.m.f.  $E_2$  is termed as secondary e.m.f.

$$\begin{aligned} \text{Clearly, } E_1 &= -N_1 \frac{d\phi}{dt} \\ \text{and } E_2 &= -N_2 \frac{d\phi}{dt} \\ \therefore \frac{E_2}{E_1} &= \frac{N_2}{N_1} \end{aligned}$$

Note that magnitudes of  $E_2$  and  $E_1$  depend upon the number of turns on the secondary and primary respectively.

If  $N_2 > N_1$ , then  $E_2 > E_1$  (or  $V_2 > v_1$ ) and we get a step-up transformer. If  $N_2 < N_1$ , then  $E_2 < E_1$  (or  $V_2 < V_1$ ) and we get a step-down transformer.

If load is connected across the secondary winding, the secondary e.m.f.  $E_2$  will cause a current  $I_2$  to flow through the load. Thus, a transformer enables us to transfer a.c. power from one circuit to another with a change in voltage level.

**The following points may be noted carefully:**

- (a) The transformer action is based on the laws of electromagnetic induction.
- (b) There is no electrical connection between the primary and secondary.
- (c) The a.c. power is transferred from primary to secondary through magnetic flux.
- (d) There is no change in frequency i.e., output power has the same frequency as the input power.
- (e) The losses that occur in a transformer are:
  - (a) *core losses*—eddy current and hysteresis losses
  - (b) *copper losses*—in the resistance of the windings

In practice, these losses are very small so that output power is nearly equal to the input primary power. In other words, a transformer has very high efficiency.

### **2.4.1 E.M.F. Equation of a Transformer**

Consider that an alternating voltage  $V_1$  of frequency  $f$  is applied to the primary as shown in Fig.2.3. The sinusoidal flux  $\phi$  produced by the primary can be represented as:

$$\phi = \phi_m \sin \omega t$$

When the primary winding is excited by an alternating voltage  $V_1$ , it is circulating alternating current, producing an alternating flux  $\phi$ .

$\phi$  - Flux

$\phi_m$  - maximum value of flux

$N_1$  - Number of primary turns

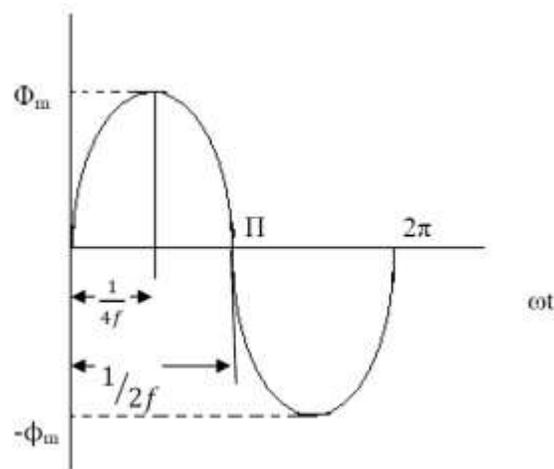
$N_2$  - Number of secondary turns

$f$  - Frequency of the supply voltage

$E_1$  - R.M.S. value of the primary induced e.m.f

$E_2$  - R.M.S. value of the secondary induced e.m.f

The instantaneous e.m.f.  $e_1$  induced in the primary is -



**Fig. 2.3**

From Faraday's law of electromagnetic induction -

$$\text{Average e.m.f per turns} = \frac{d\Phi}{dt}$$

$d\Phi$  = change in flux

$dt$  = time required for change in flux

The flux increases from zero value to maximum value  $\phi_m$  in  $1/4f$  of the time period that is in  $1/4f$  seconds.

The change of flux that takes place in  $1/4f$  seconds =  $\phi_m - 0 = \phi_m$  webers

$$\frac{d\phi}{dt} = \frac{dt}{1/4f} = 4f\phi_m \text{ wb/sec.}$$

Since flux  $\phi$  varies sinusoidally, the R.m.s value of the induced e.m.f is obtained by multiplying the average value with the form factor

$$\text{Form factor of a sinwave} = \frac{\text{R.m.s value}}{\text{Average value}} = 1.11$$

R.M.S Value of e.m.f induced in one turns =  $4\phi_m f \times 1.11$  Volts.

$$= 4.44\phi_m f \text{ Volts.}$$

R.M.S Value of e.m.f induced in primary winding =  $4.44\phi_m f N_1$  Volts.

R.M.S Value of e.m.f induced in secondary winding =  $4.44\phi_m f N_2$  Volts.

The expression of  $E_1$  and  $E_2$  are called e.m.f equation of a transformer

$$\begin{aligned} V_1 = E_1 &= 4.44\phi_m f N_1 \text{ Volts.} \\ V_2 = E_2 &= 4.44\phi_m f N_2 \text{ Volts.} \end{aligned}$$

### 2.4.2 Voltage Ratio

Voltage transformation ratio is the ratio of e.m.f induced in the secondary winding to the e.m.f induced in the primary winding.

$$\frac{E_2}{E_1} = \frac{4.44\phi_m f N_2}{4.44\phi_m f N_1}$$

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = K$$

This ratio of secondary induced e.m.f to primary induced e.m.f is known as voltage transformation ratio

$$E_2 = KE_1 \quad \text{where } K = \frac{N_2}{N_1}$$

1. If  $N_2 > N_1$  i.e.  $K > 1$  we get  $E_2 > E_1$  then the transformer is called step up transformer.
2. If  $N_2 < N_1$  i.e.  $K < 1$  we get  $E_2 < E_1$  then the transformer is called step down transformer.
3. If  $N_2 = N_1$  i.e.  $K = 1$  we get  $E_2 = E_1$  then the transformer is called isolation transformer or 1:1

transformer.

### 2.4.3 Current Ratio

Current ratio is the ratio of current flow through the primary winding ( $I_1$ ) to the current flowing through the secondary winding ( $I_2$ ). In an ideal transformer -

Apparent input power = Apparent output power.

$$V_1 I_1 = V_2 I_2$$

$$\frac{I_1}{I_2} = \frac{V_2}{V_1} = \frac{N_2}{N_1} = K$$

### Volt-Ampere Rating

i) The transformer rating is specified as the products of voltage and current (VA rating).

ii) On both sides, primary and secondary VA rating remains same. This rating is generally expressed in KVA (Kilo Volts Amperes rating).

$$\frac{V_1}{V_2} = \frac{I_2}{I_1} = K$$

$$V_1 I_1 = V_2 I_2$$

$$\text{KVA Rating of a transformer} = \frac{V_1 I_1}{1000} = \frac{V_2 I_2}{1000} \quad (1000 \text{ is to convert KVA to VA})$$

$V_1$  and  $V_2$  are the  $V_t$  of primary and secondary by using KVA rating we can calculate  $I_1$  and  $I_2$  Full load current and it is safe maximum current.

$$I_1 \text{ Full load current} = \frac{\text{KVA Rating} \times 1000}{V_1}$$

$$I_2 \text{ Full load current} = \frac{\text{KVA Rating} \times 1000}{V_2}$$

### 2.4.4 Transformer on No-load

a) Ideal transformer

b) Practical transformer

#### *a) Ideal Transformer*

An ideal transformer is one that has

(i) No winding resistance

(ii) No leakage flux i.e., the same flux links both the windings

(iii) No iron losses (i.e., eddy current and hysteresis losses) in the core

Although ideal transformer cannot be physically realized, yet its study provides a very powerful tool in the analysis of a practical transformer. In fact, practical transformers have properties that approach very close to an ideal transformer.

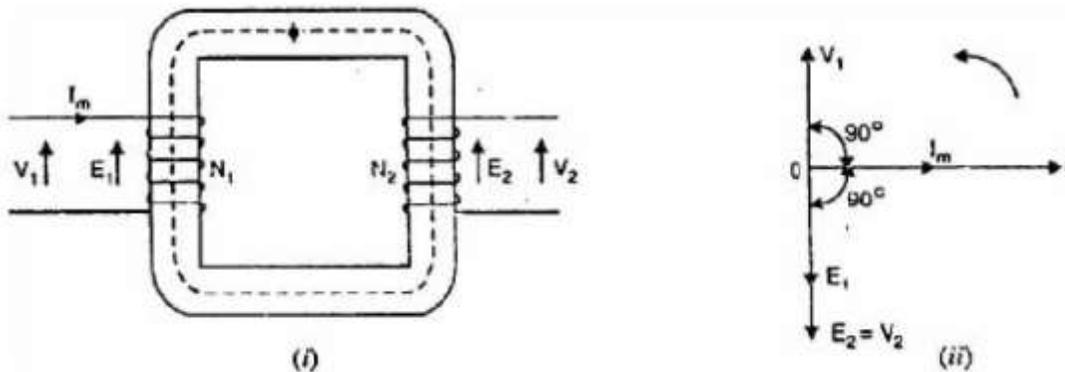


Fig: 2.4

Consider an ideal transformer on no load i.e., secondary is open-circuited as shown in *Fig.2.4 (i)*. under such conditions, the primary is simply a coil of pure inductance. When an alternating voltage  $V_1$  is applied to the primary, it draws a small magnetizing current  $I_m$  which lags behind the applied voltage by  $90^\circ$ . This alternating current  $I_m$  produces an alternating flux  $\phi$  which is proportional to and in phase with it. The alternating flux  $\phi$  links both the windings and induces e.m.f.  $E_1$  in the primary and e.m.f.  $E_2$  in the secondary. The primary e.m.f.  $E_1$  is, at every instant, equal to and in opposition to  $V_1$  (Lenz's law). Both e.m.f.s  $E_1$  and  $E_2$  lag behind flux  $\phi$  by  $90^\circ$ . However, their magnitudes depend upon the number of primary and secondary turns. *Fig. 2.4 (ii)* shows the phasor diagram of an ideal transformer on no load. Since flux  $\phi$  is common to both the windings, it has been taken as the reference phasor. The primary e.m.f.  $E_1$  and secondary e.m.f.  $E_2$  lag behind the flux  $\phi$  by  $90^\circ$ . Note that  $E_1$  and  $E_2$  are in phase. But  $E_1$  is equal to  $V_1$  and  $180^\circ$  out of phase with it.

$$\frac{E_2}{E_1} = \frac{V_2}{V_1} = K$$

### 2.4.5 Phasor Diagram

- i)  $\Phi$  (flux) is reference
- ii)  $I_m$  produce  $\phi$  and it is in phase with  $\phi$ ,  $V_1$  Leads  $I_m$  by  $90^\circ$
- iii)  $E_1$  and  $E_2$  are in phase and both opposing supply voltage  $V_1$ , winding is purely inductive

So current has to lag voltage by  $90^\circ$ .

iv) The power input to the transformer

$$P = V_1 I_1 \cos(90^\circ) \dots\dots\dots (\cos 90^\circ = 0)$$

$$P = 0 \text{ (ideal transformer)}$$

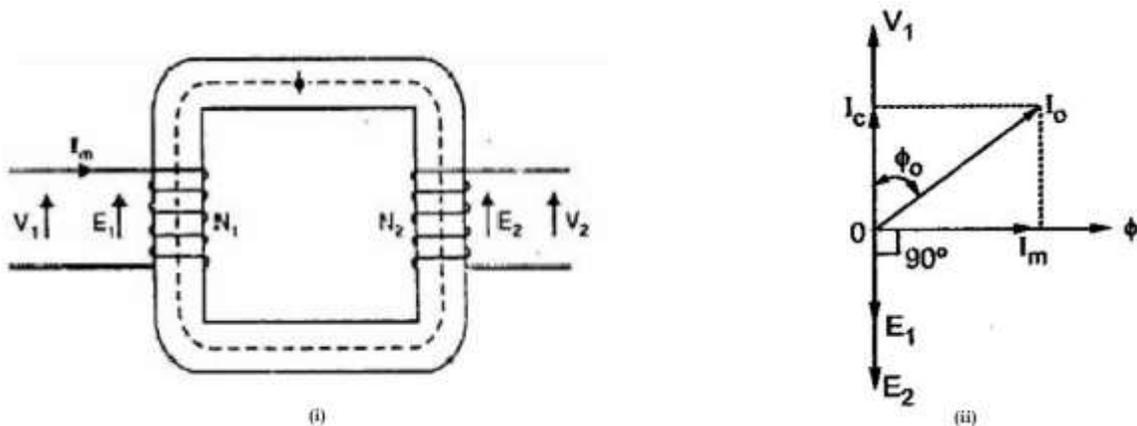
**b)i) Practical Transformer on no load**

A practical transformer differs from the ideal transformer in many respects. The practical transformer has (i) iron losses (ii) winding resistances and (iii) Magnetic leakage

**(i) Iron losses.** Since the iron core is subjected to alternating flux, there occurs eddy current and hysteresis loss in it. These two losses together are known as iron losses or core losses. The iron losses depend upon the supply frequency, maximum flux density in the core, volume of the core etc. It may be noted that magnitude of iron losses is quite small in a practical transformer.

**(ii) Winding resistances.** Since the windings consist of copper conductors, it immediately follows that both primary and secondary will have winding resistance. The primary resistance  $R_1$  and secondary resistance  $R_2$  act in series with the respective windings as shown in Fig. When current flows through the windings, there will be power loss as well as a loss in voltage due to IR drop. This will affect the power factor and  $E_1$  will be less than  $V_1$  while  $V_2$  will be less than  $E_2$ .

Consider a practical transformer on no load i.e., secondary on open-circuit as Shown in Fig 2.5.



**Fig: 2.5 Phasor diagram of transformer at no load**

Here the primary will draw a small current  $I_0$  to supply -

(i) the iron losses and

(ii) a very small amount of copper loss in the primary.

Hence the primary no load current  $I_0$  is not  $90^\circ$  behind the applied voltage  $V_1$  but lags it by an angle  $\phi_0 < 90^\circ$  as shown in the phasor diagram.

No load input power,  $W_0 = V_1 I_0 \cos \phi_0$

As seen from the phasor diagram in Fig.2.5 (ii), the no-load primary current  $I_0$

(i) The component  $I_c$  in phase with the applied voltage  $V_1$ . This is known as active or working or iron loss component and supplies the iron loss and a very small primary copper loss.

$$I_c = I_0 \cos \phi_0$$

The component  $I_m$  lagging behind  $V_1$  by  $90^\circ$  and is known as magnetizing component. It is this component which produces the mutual flux  $\phi$  in the core.

$$I_m = I_0 \sin \phi_0$$

Clearly,  $I_0$  is phasor sum of  $I_m$  and  $I_c$ ,

$$I_0 = \sqrt{I_m^2 + I_c^2}$$

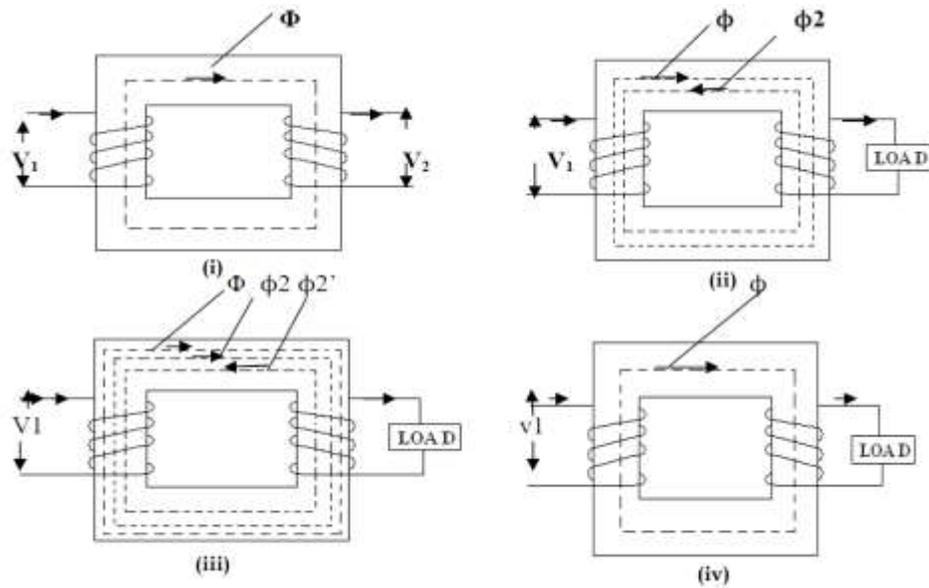
$$\text{No load P.F., } \cos \phi_0 = \frac{I_c}{I_0}$$

The no load primary copper loss (i.e.  $I_0^2 R_1$ ) is very small and may be neglected.

Therefore, the no load primary input power is practically equal to the iron loss in the transformer i.e.,

No load input power,  $W_0 = V_1 I_0 \cos \phi_0 = P_i = \text{Iron loss}$

### ***b) ii) Practical Transformer on Load***



**Fig: 2.6**

At no load, there is no current in the secondary so that  $V_2 = E_2$ . On the primary side, the drops in  $R_1$  and  $X_1$ , due to  $I_0$  are also very small because of the smallness of  $I_0$ . Hence, we can say that at no load,  $V_1 = E_1$ .

i) When transformer is loaded, the secondary current  $I_2$  flows through the secondary winding.

ii) Already  $I_m$  magnetizing current flow in the primary winding fig. 2.6(i).

iii) The magnitude and phase of  $I_2$  with respect to  $V_2$  is determined by the characteristics of the load.

a)  $I_2$  in phase with  $V_2$  (resistive load)

b)  $I_2$  lags with  $V_2$  (Inductive load)

c)  $I_2$  leads with  $V_2$  (capacitive load)

iv) Flow of secondary current  $I_2$  produce new Flux  $\phi_2$  fig.2.6 (ii)

v)  $\Phi$  is main flux which is produced by the primary to maintain the transformer as constant magnetising component.

vi)  $\Phi_2$  opposes the main flux  $\phi$ , the total flux in the core reduced. It is called demagnetising Ampere-turns due to this  $E_1$  reduced.

vii) To maintain the  $\phi$  constant primary winding draws more current ( $I_2'$ ) from the supply (load component of primary) and produce  $\phi_2'$  flux which is oppose  $\phi_2$  (but in same direction as  $\phi$ ), to maintain flux constant in the core fig.2.6 (iii).

viii) The load component current  $I_2'$  always neutralizes the changes in the load.

ix) Whatever the load conditions, the net flux passing through the core is approximately the same as at no-load. An important deduction is that due to the constancy of core flux at all loads, the core loss is also practically the same under all load conditions fig.2.6 (iv).

$$\Phi_2 = \phi_2' \quad N_2 I_2 = N_1 I_2' \quad I_2' = \frac{N_2}{N_1} X I_2 = K I_2$$

### 2.4.6 Phasor Diagram

- i) Take ( $\phi$ ) flux as reference for all load
- ii) The no load  $I_0$  which lags by an angle  $\phi_0$ .  $I_0 = \sqrt{I_c^2 + I_m^2}$ .
- iii) The load component  $I_2'$ , which is in anti-phase with  $I_2$  and phase of  $I_2$  is decided by the load.
- iv) Primary current  $I_1$  is vector sum of  $I_0$  and  $I_2'$

$$\vec{I}_1 = \vec{I}_0 + \vec{I}_2'$$

$$I_1 = \sqrt{I_0^2 + I_2'^2}$$

- a) If load is Inductive,  $I_2$  lags  $E_2$  by  $\phi_2$ , shown in phasor diagram fig 2.7 (a).
- b) If load is resistive,  $I_2$  in phase with  $E_2$  shown in phasor diagram fig. 2.7 (b).
- c) If load is capacitive load,  $I_2$  leads  $E_2$  by  $\phi_2$  shown in phasor diagram fig. 2.7 (c).

For easy understanding at this stage here we assumed  $E_2$  is equal to  $V_2$  neglecting various drops.

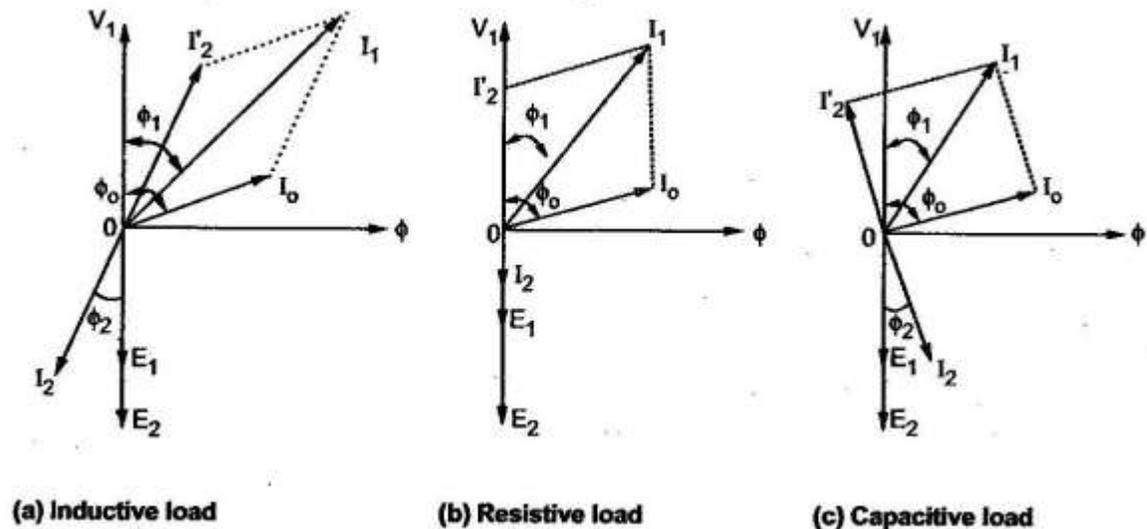


Fig: 2.7.a

$$I_1 \cong I_2'$$

Balancing the ampere – turns

$$N_1 I_2' = N_1 I_1 + N_2 I_2$$

$$\frac{I_1}{I_2} = \frac{N_2}{N_1} = K$$

Now we going to construct complete phasor diagram of a transformer (shown in Fig: 2.7.b)

### Effect of Winding Resistance

In practical transformer it process its own winding resistance causes power loss and also the voltage drop.

$R_1$  – primary winding resistance in ohms.

$R_2$  – secondary winding resistance in ohms.

The current flow in primary winding make voltage drop across it is denoted as  $I_1 R_1$  here supply voltage  $V_1$  has to supply this drop primary induced e.m.f  $E_1$  is the vector difference between  $V_1$  and  $I_1 R_1$ .

$$\vec{E}_1 = \vec{V}_1 - \vec{I}_1 R_1$$

Similarly the induced e.m.f in secondary  $E_2$ , The flow of current in secondary winding makes voltage drop across it and it is denoted as  $I_2 R_2$  here  $E_2$  has to supply this drop.

The vector difference between  $E_2$  and  $I_2R_2$

$$\vec{V}_2 = \vec{E}_2 - \vec{I}_2R_2 \quad (\text{Assuming as purely resistive drop here.})$$

### Equivalent Resistance

- 1) It would now be shown that the resistances of the two windings can be transferred to any one of the two winding.
- 2) The advantage of concentrating both the resistances in one winding is that it makes calculations very simple and easy because one has then to work in one winding only.
- 3) Transfer to any one side either primary or secondary without affecting the performance of the transformer.

The total copper loss due to both the resistances.

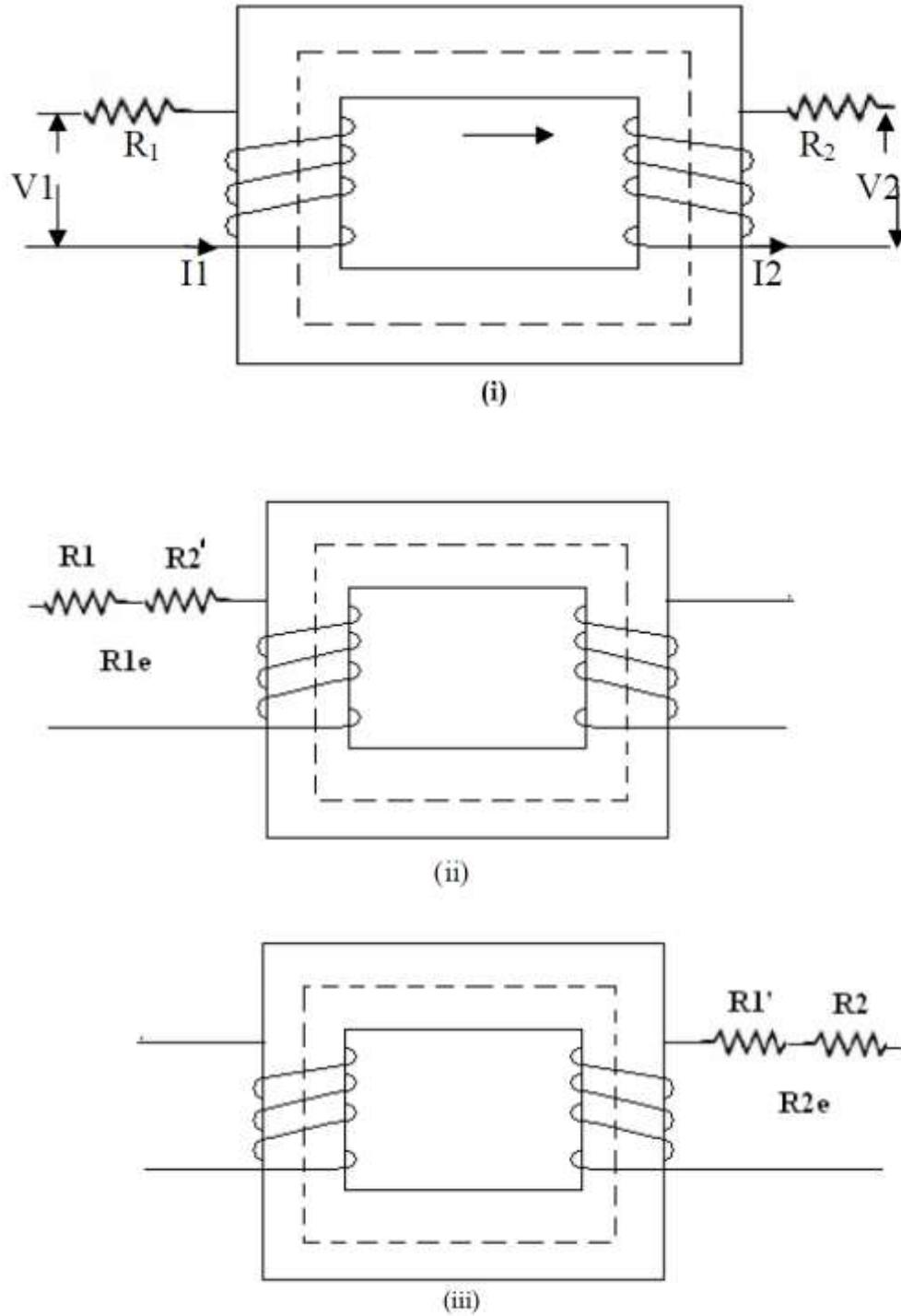
$$\begin{aligned} \text{Total copper loss} &= I_1^2R_1 + I_2^2R_2 \\ &= I_1^2\left[R_1 + \frac{I_2^2}{I_1^2}\right] \\ &= I_1^2\left[R_1 + \frac{1}{K} R_2\right] \end{aligned}$$

$\frac{R_2}{K^2}$  is the resistance value of  $R_2$  shifted to primary side and denoted as  $R_2'$ .  
 $R_2'$  is the equivalent resistance of secondary referred to primary

$$R_2' = \frac{R_2}{K^2}$$

Equivalent resistance of transformer referred to primary fig (ii)

$$R_{1e} = R_1 + R_2' = R_1 + \frac{R_2}{K^2}$$

**Fig:2.8**

Similarly it is possible to refer the equivalent resistance to secondary winding.

$$\text{Total copper loss} = I_1^2 R_1 + I_2^2 R_2 = I_2^2 \left[ \frac{I_1^2}{I_2^2} R_1 + R_2 \right]$$

$$= I_2^2 [K^2 R_1 + R_2]$$

$K^2 R_1$  is primary resistance referred to secondary denoted as  $R_1'$ .

$$R_1' = K^2 R_1$$

Equivalent resistance of transformer referred to secondary, denoted as  $R_{2e}$

$$R_{2e} = R_2 + R_1' = R_2 + K^2 R_1$$

$$\text{Total copper loss} = I_2^2 R_{2e}$$

*Note:*

*Note:*

i) When a resistance is to be transferred from the primary to secondary, it must be multiplied by  $K^2$ , it must be divided by  $K^2$  while transferred from the secondary to primary.

High voltage side  $\longrightarrow$  low current side  $\longrightarrow$  high resistance side

Low voltage side  $\longrightarrow$  high current side  $\longrightarrow$  low resistance side

### Effect of Leakage Reactance

i) It has been assumed that all the flux linked with primary winding also links the secondary winding.

But, in practice, it is impossible to realize this condition.

ii) However, primary current would produce flux  $\phi$  which would not link the secondary winding.

Similarly, current would produce some flux  $\phi$  that would not link the primary winding.

iii) The flux  $\phi_{L1}$  complete its magnetic circuit by passing through air rather than around the core, as shown in fig.2.9. This flux is known as primary leakage flux and is proportional to the primary ampere – turns alone because the secondary turns do not links the magnetic circuit of  $\phi_{L1}$ . It induces an e.m.f  $e_{L1}$  in primary but not in secondary.

iv) The flux  $\phi_{L2}$  complete its magnetic circuit by passing through air rather than around the core, as shown in fig. This flux is known as secondary leakage flux and is proportional to the secondary ampere – turns alone because the primary turns do not links the magnetic circuit of  $\phi_{L2}$ . It induces an e.m.f  $e_{L2}$  in secondary but not in primary.

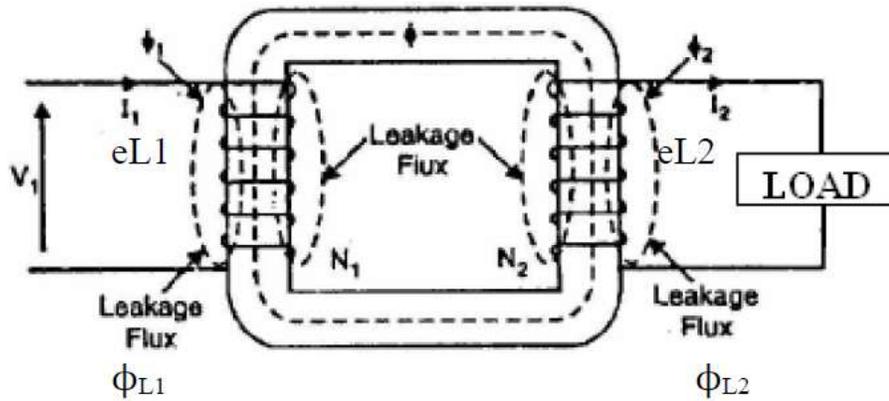


Fig: 2.9

$\phi_{L1}$  – primary leakage flux

$\phi_{L2}$  – secondary leakage flux

$e_{L1}$  – self induced e.m.f (primary)

$e_{L2}$  – self induced e.m.f (secondary)

### Equivalent Leakage Reactance

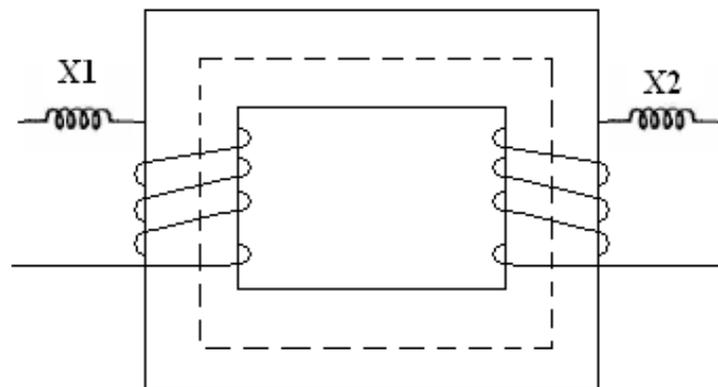


Fig: 2.10

Similarly to the resistance, the leakage reactance also can be transferred from primary to secondary. The relation through  $K^2$  remains same for the transfer of reactance as it is studied earlier for the resistance

$X_1$  – leakage reactance of primary.

$X_2$  - leakage reactance of secondary.

Then the total leakage reactance referred to primary is  $X_{1e}$  given by

$$X_{1e} = X_1 + X_2'$$

$$X_2' = \frac{X_2}{K^2}$$

The total leakage reactance referred to secondary is  $X_{2e}$  given by

$$X_{2e} = X_2 + X_1'$$

$$X_1' = K^2 X_1$$

$X_{1e} = X_1 + X_2'$ $X_{2e} = X_2 + X_1'$
---

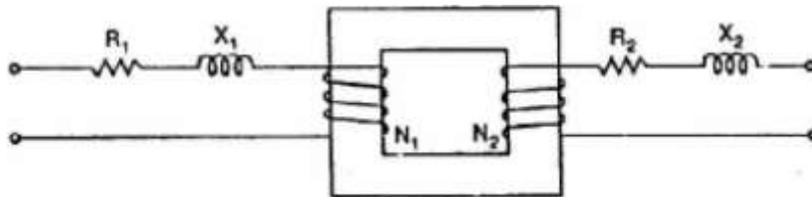
### Equivalent Impedance

The transformer winding has both resistance and reactance ( $R_1, R_2, X_1, X_2$ ). Thus we can say that the total impedance of primary winding is  $Z_1$  which is,

$$Z_1 = R_1 + jX_1 \text{ ohms}$$

On secondary winding,

$$Z_2 = R_2 + jX_2 \text{ ohms}$$

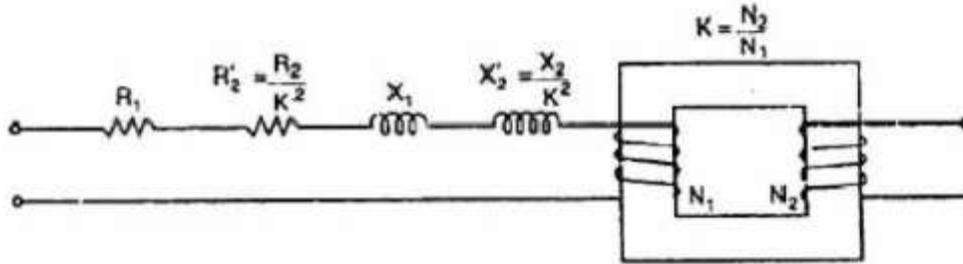


Individual magnitude of  $Z_1$  and  $Z_2$  are

$$Z_1 = \sqrt{R_1^2 + X_1^2}$$

$$Z_2 = \sqrt{R_2^2 + X_2^2}$$

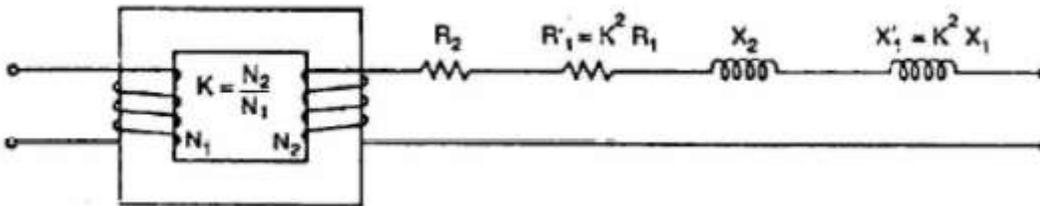
Similar to resistance and reactance, the impedance also can be referred to any one side,



$Z_{1e}$  = total equivalent impedance referred to primary

$$Z_{1e} = R_{1e} + jX_{1e} = Z_1 + Z_2' = Z_1 + \frac{Z_2}{K^2}$$

$Z_{2e}$  = total equivalent impedance referred to secondary.



$$Z_{2e} = R_{2e} + jX_{2e} = Z_2 + Z_1' = Z_2 + K^2 Z_1$$

The magnitudes of  $Z_{1e}$  and  $Z_{2e}$

$$Z_1 = \sqrt{R_1^2 + X_1^2}$$

$$Z_2 = \sqrt{R_2^2 + X_2^2}$$

It can be noted that

$$Z_{2e} = K^2 Z_{1e} \text{ and } Z_{1e} = \frac{Z_{2e}}{K^2}$$

### 2.4.7 Complete Phasor Diagram of a Transformer (for Inductive Load or Lagging pf)

We now restrict ourselves to the more commonly occurring load i.e. inductive along with resistance,

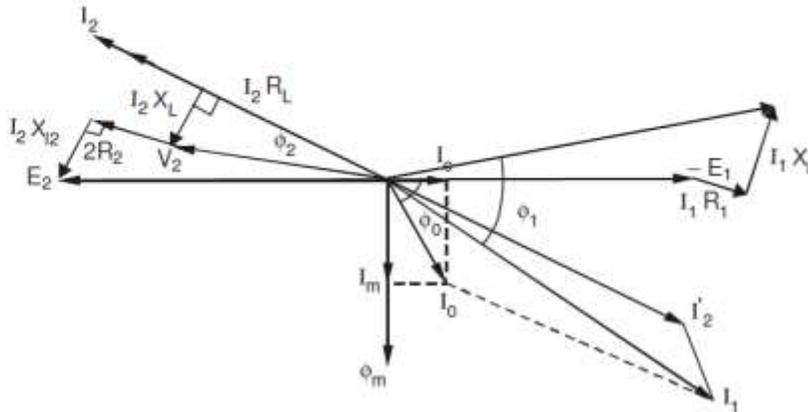
which has a lagging power factor.

For drawing this diagram, we must remember that

$$\bar{V}_2 = \bar{E}_2 - \bar{I}_2 (R_2 + j X_{L2})$$

and

$$\bar{V}_1 = -\bar{E}_1 + \bar{I}_1 (R_1 + j X_{L1})$$



### 2.5 Equivalent Circuit of Transformer

No load equivalent circuit

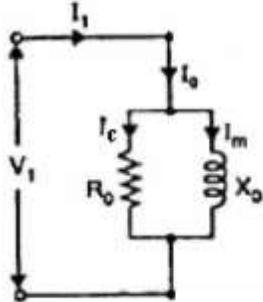


Fig:11

$$I_m = I_0 \sin \phi_0 = \text{magnetizing component}$$

$$I_c = I_0 \cos \phi_0 = \text{Active component}$$

$$R_o = \frac{V_1}{I_c}, \quad X_o = \frac{V_1}{I_m}$$

- i)  $I_m$  produces the flux and is assumed to flow through reactance  $X_o$  called no load reactance while  $I_c$  is active component representing core losses hence is assumed to flow through the resistance  $R_o$
- ii) Equivalent resistance is shown in fig.2.12.
- iii) When the load is connected to the transformer then secondary current  $I_2$  flows causes voltage drop across  $R_2$  and  $X_2$ . Due to  $I_2$ , primary draws an additional current.

$$I_2' = \frac{I_2}{K}$$

$I_1$  is the phasor addition of  $I_0$  and  $I_2'$ . This  $I_1$  causes the voltage drop across primary resistance  $R_1$  and reactance  $X_1$ .

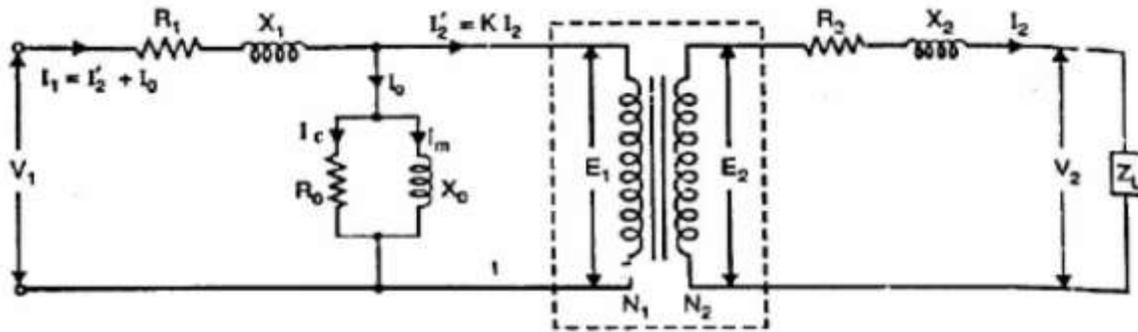


Fig: 2.12

To simplify the circuit the winding is not taken in equivalent circuit while transfer to one side.

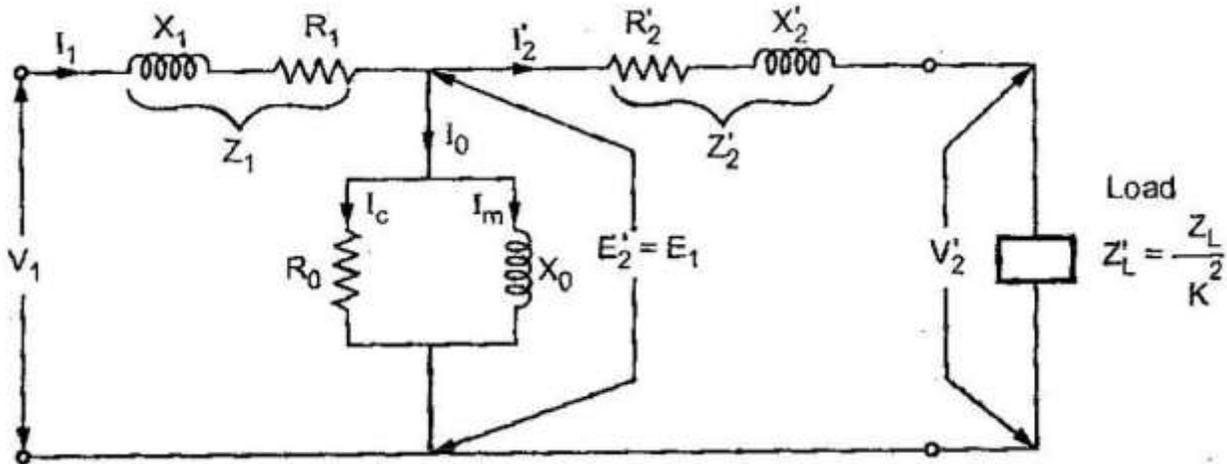


Fig: 2.13

### 2.5.1 Exact equivalent circuit referred to primary

Transferring secondary parameter to primary -

$$R_2' = \frac{R_2}{K^2}, X_2' = \frac{X_2}{K^2}, Z_2' = \frac{Z_2}{K^2}, E_2' = \frac{E_2}{K}, I_2' = KI_2, K = \frac{N_2}{N_1}$$

High voltage winding  $\Rightarrow$  low current  $\Rightarrow$  high impedance

Low voltage winding  $\Rightarrow$  high current  $\Rightarrow$  low impedance

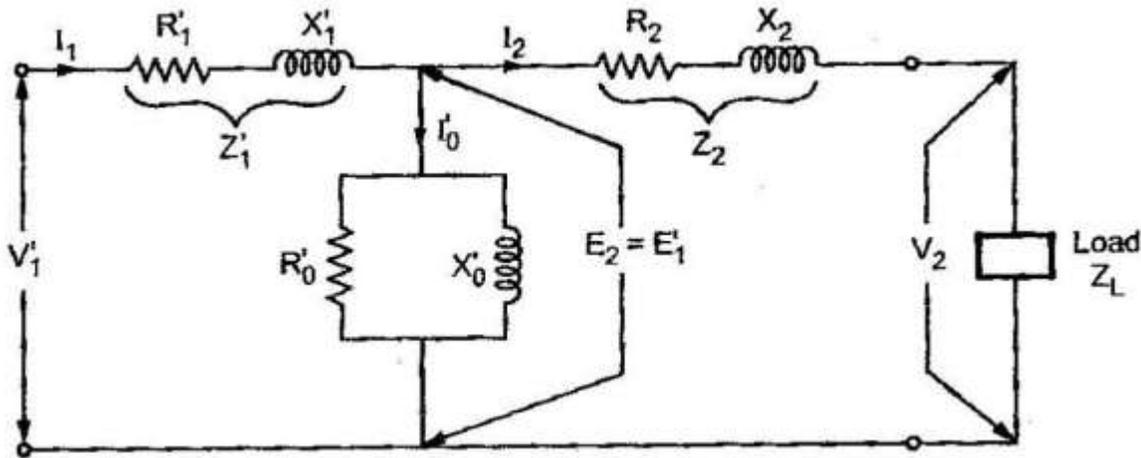


Fig: 2.14

### 2.5.2 Exact equivalent circuit referred to secondary

$$R_1' = R_1 K^2, X_1' = K^2 X_1, E_1' = K E_1$$

$$Z_1' = K^2 Z_1, I_1' = \frac{I_1}{K}, I_0 = \frac{I_0}{K}$$

Now as long as no load branch i.e. exciting branch is in between  $Z_1$  and  $Z_2'$ , the impedances cannot be combined. So further simplification of the circuit can be done. Such circuit is called approximate equivalent circuit.

### 2.5.3 Approximate Equivalent Circuit

- i) To get approximate equivalent circuit, shift the no load branch containing  $R_0$  and  $X_0$  to the left of  $R_1$  and  $X_1$ .
- ii) By doing this we are creating an error that the drop across  $R_1$  and  $X_1$  to  $I_0$  is neglected due to this circuit because simpler.
- iii) This equivalent circuit is called approximate equivalent circuit Fig: 2.15 & Fig: 2.16.

In this circuit new  $R_1$  and  $R_2'$  can be combined to get equivalent circuit referred to primary  $R_{1e}$ , similarly

$X_1$  and  $X_2'$  can be combined to get  $X_{1e}$ .

$$R_{1e} = R_1 + R_2' = R_1 + \frac{R_2}{K^2}$$

$$X_{1e} = X_1 + X_2' = X_1 + \frac{X_2}{K^2}$$

$$Z_{1e} = R_{1e} + jX_{1e}, \quad R_0 = \frac{V_1}{I_c}, \quad \text{and } X_0 = \frac{V_1}{I_m}$$

$$I_c = I_0 \cos\phi_0, \quad \text{and } I_m = I_0 \sin\phi_0$$

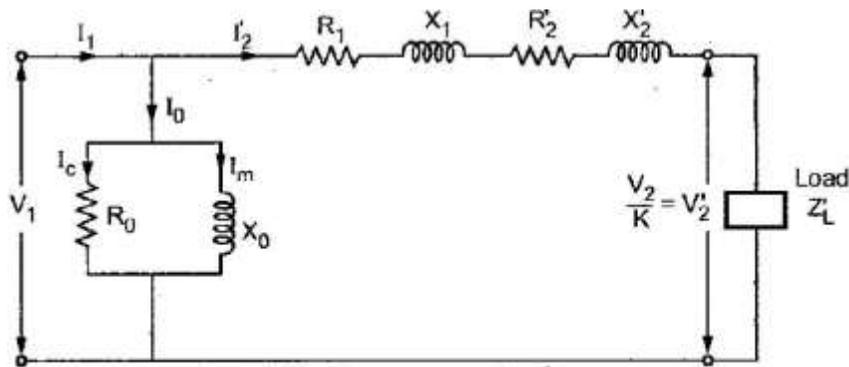


Fig:2.15 Approximate equivalent circuit referred to primary

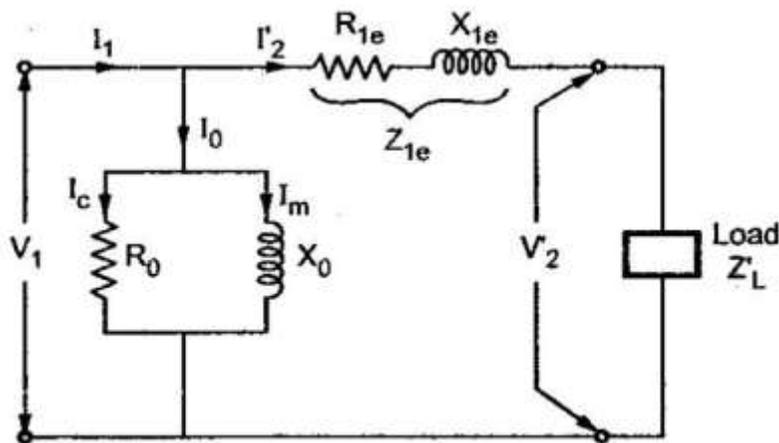


Fig:2.16 Simplified equivalent circuit

## 2.6 Approximate Voltage Drop in a Transformer

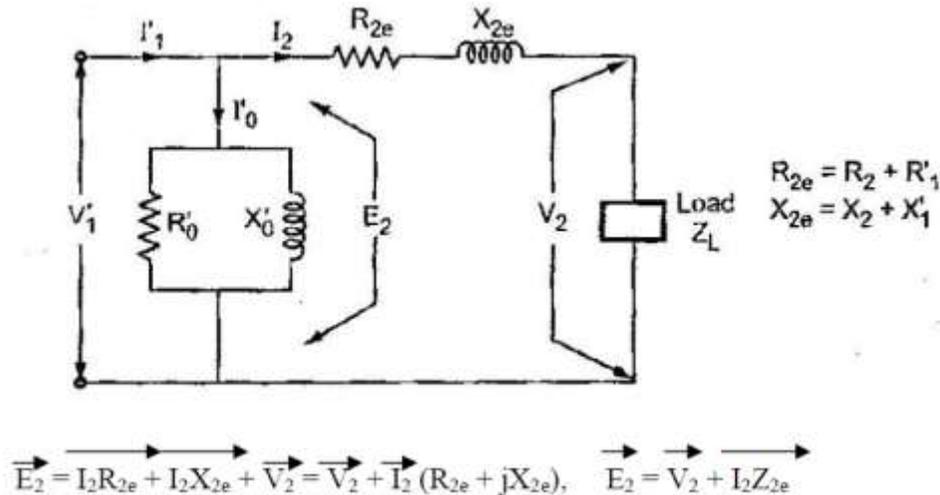


Fig. 2.17

Primary parameter is referred to secondary there are no voltage drop in primary. When there is no load,  $I_2 = 0$  and we get no load terminal voltage drop in

$V_{20} = E_2 =$  no load terminal voltage

$V_2 =$  terminal voltage on load

### 2.6.1 For Lagging P.F.

i) The current  $I_2$  lags  $V_2$  by angle  $\phi_2$

ii) Take  $V_2$  as reference

iii)  $I_2 R_{2e}$  is in phase with  $I_2$  while  $I_2 X_{2e}$  leads  $I_2$  by  $90^\circ$

iv) Draw the circle with O as centre and OC as radius cutting extended OA at M. as OA

$= V_2$  and now  $OM = E_2$ .

v) The total voltage drop is  $AM = I_2 Z_{2e}$ .

vi) The angle  $\alpha$  is practically very small and in practice M&N are very close to each other. Due to this the approximate voltage drop is equal to AN instead of AM

AN – approximate voltage drop

To find AN by adding AD & DN

$$AD = AB \cos\phi = I_2 R_{2e} \cos\phi$$

$$DN = BL \sin\phi = I_2 X_{2e} \sin\phi$$

$$AN = AD + DN = I_2 R_{2e} \cos\phi + I_2 X_{2e} \sin\phi$$

Assuming:  $\phi_2 = \phi_1 = \phi$

Approximate voltage drop =  $I_2 R_{2e} \cos\phi + I_2 X_{2e} \sin\phi$  (referred to secondary)

Similarly: Approximate voltage drop =  $I_1 R_{1e} \cos\phi + I_1 X_{1e} \sin\phi$  (referred to primary)

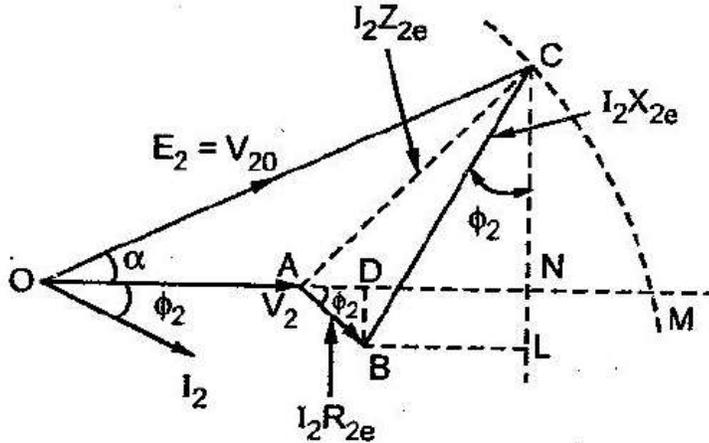


Fig:2.18

### **2.6.2 For Leading P.F Loading**

$I_2$  leads  $V_2$  by angle  $\phi_2$

Approximate voltage drop =  $I_2 R_{2e} \cos\phi - I_2 X_{2e} \sin\phi$  (referred to secondary)

Similarly: Approximate voltage drop =  $I_1 R_{1e} \cos\phi - I_1 X_{1e} \sin\phi$  (referred to primary)

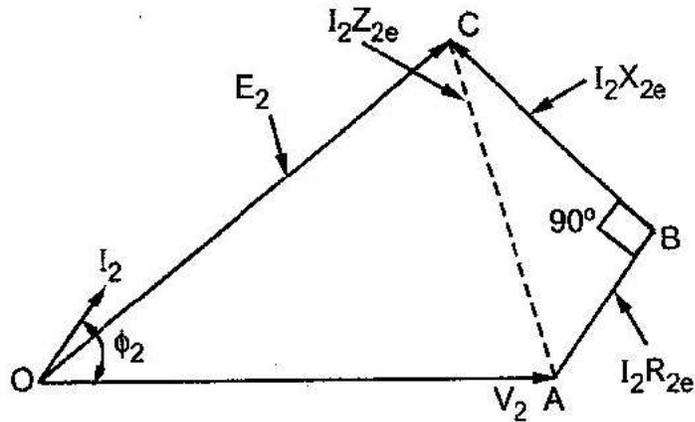


Fig: 2.19

### 2.6.3 For Unity P.F. Loading

Approximate voltage drop =  $I_2 R_{2e}$  (referred to secondary)

Similarly: Approximate voltage drop =  $I_1 R_{1e}$  (referred to primary)

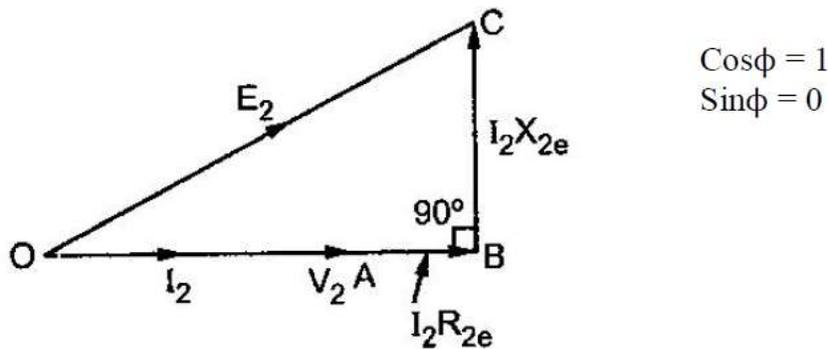


Fig: 2.20

Approximate voltage drop =  $E_2 - V_2$

$$= I_2 R_{2e} \cos \phi \pm I_2 X_{2e} \sin \phi \quad (\text{referred to secondary})$$

$$= I_1 R_{1e} \cos \phi \pm I_1 X_{1e} \sin \phi \quad (\text{referred to primary})$$

### 2.7 Losses in a Transformer

The power losses in a transformer are of two types, namely;

1. Core or Iron losses

2. Copper losses

These losses appear in the form of heat and produce (i) an increase in Temperature and (ii) a drop in efficiency.

### **2.7.1 Core or Iron losses ( $P_i$ )**

These consist of hysteresis and eddy current losses and occur in the transformer core due to the alternating flux. These can be determined by open-circuit test.

$$\text{Hysteresis loss} = k_h f B_m^{1.6} \text{ watts /m}^3$$

$K_h$  - hysteresis constant depend on material

f - Frequency

$B_m$  - maximum flux density

$$\text{Eddy current loss} = K_e f^2 B_m^2 t^2 \text{ watts /m}^3$$

$K_e$  - eddy current constant

t - Thickness of the core

Both hysteresis and eddy current losses depend upon

(i) Maximum flux density  $B_m$  in the core

(ii) Supply frequency f. Since transformers are connected to constant-frequency, constant voltage supply, both f and  $B_m$  are constant. Hence, core or iron losses are practically the same at all loads.

$$\text{Iron or Core losses, } P_i = \text{Hysteresis loss} + \text{Eddy current loss} = \text{Constant losses (} P_i \text{)}$$

The hysteresis loss can be minimized by using steel of high silicon content .Whereas eddy current loss can be reduced by using core of thin laminations.

### **Copper losses ( $P_{cu}$ )**

These losses occur in both the primary and secondary windings due to their ohmic resistance. These

can be determined by short-circuit test. The copper loss depends on the magnitude of the current flowing through the windings.

$$\text{Total copper loss} = I_1^2 R_1 + I_2^2 R_2 = I_1^2 (R_1 + R_2') = I_2^2 (R_2 + R_1')$$

$$\text{Total loss} = \text{iron loss} + \text{copper loss} = P_i + P_{cu}$$

## 2.8 Efficiency of a Transformer

Like any other electrical machine, the efficiency of a transformer is defined as the ratio of output power (in watts or kW) to input power (watts or kW) i.e.

$$\text{Power output} = \text{power input} - \text{Total losses}$$

$$\text{Power input} = \text{power output} + \text{Total losses}$$

$$= \text{power output} + P_i + P_{cu}$$

$$\text{Efficiency} = \frac{\text{power output}}{\text{power input}}$$

$$\text{Efficiency} = \frac{\text{power output}}{\text{power input} + P_i + P_{cu}}$$

Power output =  $V_2 I_2 \cos \phi$ ,  $\cos \phi$  = load power factor

Transformer supplies full load of current  $I_2$  and with terminal voltage  $V_2$

$P_{cu}$  = copper losses on full load =  $I_2^2 R_{2e}$

$$\text{Efficiency} = \frac{V_2 I_2 \cos \phi}{V_2 I_2 \cos \phi + P_i + I_2^2 R_{2e}}$$

$V_2 I_2$  = VA rating of a transformer

$$\text{Efficiency} = \frac{(\text{VA rating}) \times \cos \phi}{(\text{VA rating}) \times \cos \phi + P_i + I_2^2 R_{2e}}$$

$$\% \text{ Efficiency} = \frac{(\text{VA rating}) \times \cos \phi}{(\text{VA rating}) \times \cos \phi + P_i + I_2^2 R_{2e}} \times 100$$

This is full load efficiency and  $I_2$  = full load current.

We can now find the full-load efficiency of the transformer at any p.f. without actually loading the transformer.

$$\text{Full load Efficiency} = \frac{(\text{Full load VA rating}) \times \cos\phi}{(\text{Full load VA rating}) \times \cos\phi + P_i + I_2^2 R_{2e}}$$

Also for any load equal to  $n$  x full-load,

$$\text{Corresponding total losses} = P_i + n^2 P_{Cu}$$

$$n = \text{fractional by which load is less than full load} = \frac{\text{actual load}}{\text{full load}}$$

$$n = \frac{\text{half load}}{\text{full load}} = \frac{(\frac{1}{2})}{1} = 0.5$$

$$\text{Corresponding (n) \% Efficiency} = \frac{n(\text{VA rating}) \times \cos\phi}{n(\text{VA rating}) \times \cos\phi + P_i + n^2 P_{Cu}} \times 100$$

### 2.8.1 Condition for Maximum Efficiency

Voltage and frequency supply to the transformer is constant the efficiency varies with the load. As load increases, the efficiency increases. At a certain load current, it loaded further the efficiency start decreases as shown in fig. 2.21.

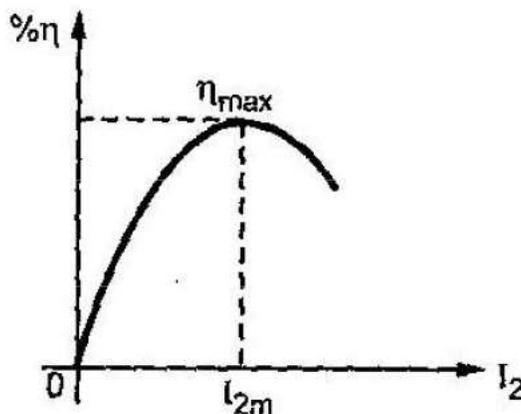


Fig: 2.21

The load current at which the efficiency attains maximum value is denoted as  $I_{2m}$  and maximum efficiency is denoted as  $\eta_{max}$ , now we find -

- (a) condition for maximum efficiency
- (b) load current at which  $\eta_{\max}$  occurs
- (c) KVA supplied at maximum efficiency

Considering primary side,

$$\text{Load output} = V_1 I_1 \cos\phi_1$$

$$\text{Copper loss} = I_1^2 R_{1e} \quad \text{or} \quad I_2^2 R_{2e}$$

$$\text{Iron loss} = \text{hysteresis} + \text{eddy current loss} = P_i$$

$$\begin{aligned} \text{Efficiency} &= \frac{V_1 I_1 \cos\phi_1 - \text{losses}}{V_1 I_1 \cos\phi_1} = \frac{V_1 I_1 \cos\phi_1 - I_1^2 R_{1e} + P_i}{V_1 I_1 \cos\phi_1} \\ &= 1 - \frac{I_1 R_{1e}}{V_1 I_1 \cos\phi_1} = \frac{P_i}{V_1 I_1 \cos\phi_1} \end{aligned}$$

Differentiating both sides with respect to  $I_2$ , we get

$$\frac{d\eta}{dI_2} = 0 - \frac{R_{1e}}{V_1 \cos\phi_1} = \frac{P_i}{V_1 I_1^2 \cos\phi_1}$$

For  $\eta$  to be maximum,  $\frac{d\eta}{dI_2} = 0$ . Hence, the above equation becomes

$$\frac{R_{1e}}{V_1 \cos\phi_1} = \frac{P_i}{V_1 I_1^2 \cos\phi_1} \quad \text{OR} \quad P_i = I_1^2 R_{1e}$$

$$P_{cu} \text{ loss} = P_i \text{ iron loss}$$

The output current which will make  $P_{cu}$  loss equal to the iron loss. By proper design, it is possible to make the maximum efficiency occur at any desired load.

### 2.8.2 Load current $I_{2m}$ at maximum efficiency

For  $\eta_{\max}$   $I_2^2 R_{2e} = P_i$  but  $I_2 = I_{2m}$

$$I_{2m}^2 R_{2e} = P_i \quad I_{2m} = \sqrt{\frac{P_i}{R_{2e}}}$$

This is the load current at  $\eta_{\max}$ .  
 $(I_2)_{F.L}$  = full load current

$$\frac{I_{2m}}{(I_2)_{F.L}} = \frac{1}{(I_2)_{F.L}} \sqrt{\frac{P_i}{R_{2e}}}$$

$$\frac{I_{2m}}{(I_2)_{F.L}} = \sqrt{\frac{P_i}{[(I_2)_{F.L}]^2 R_{2e}}} = \sqrt{\frac{P_i}{[P_{cu}]_{F.L}}}$$

$$I_{2m} = (I_2)_{F.L.} \sqrt{\frac{P_i}{[P_{cu}]_{F.L}}}$$

This is the load current at  $\eta_{\max}$  in terms of full load current

### **2.8.3 KVA Supplied at Maximum Efficiency**

For constant  $V_2$  the KVA supplied is the function of load current.

$$\text{KVA at } \eta_{\max} = I_{2m} V_2 = V_2(I_2)_{\text{F.L.}} \times \sqrt{\frac{P_i}{[P_{cu}]_{\text{F.L.}}}}$$

$$\text{KVA at } \eta_{\max} = (\text{KVA rating}) \times \sqrt{\frac{P_i}{[P_{cu}]_{\text{F.L.}}}}$$

Substituting condition for  $\eta_{\max}$  in the expression of efficiency, we can write expression for  $\eta_{\max}$  as ,

$$\text{as } P_{cu} = P_i$$

$$\% \eta_{\max} = \frac{V_2 I_{2m} \cos \phi}{V_2 I_{2m} \cos \phi + 2P_i} \times 100$$

$$\% \eta_{\max} = \frac{\text{KVA for } \eta_{\max} \cos \phi}{\text{KVA for } \eta_{\max} \cos \phi + 2P_i}$$

#### **12.8.4 All Day Efficiency (Energy Efficiency)**

In electrical power system, we are interested to find out the all-day efficiency of any transformer because the load at transformer is varying in the different time duration of the day. So all day efficiency is defined as the ratio of total energy output of transformer to the total energy input in 24 hours.

$$\text{All day efficiency} = \frac{\text{kWh output during a day}}{\text{kWh input during the day}}$$

Here, kWh is kilowatt hour.

#### **12.9 Testing of Transformer**

The testing of transformer means to determine efficiency and regulation of a transformer at any load and at any power factor condition.

There are two methods

i) Direct loading test

ii) Indirect loading test

*a. Open circuit test*

*b. Short circuit test*

### i) Load test on transformer

This method is also called as direct loading test on transformer because the load is directly connected to the transformer. We required various meters to measure the input and output reading while change the load from zero to full load. Fig. 2.22 shows the connection of transformer for direct load test. The primary is connected through the variac to change the input voltage as we required. Connect the meters as shown in the figure below.

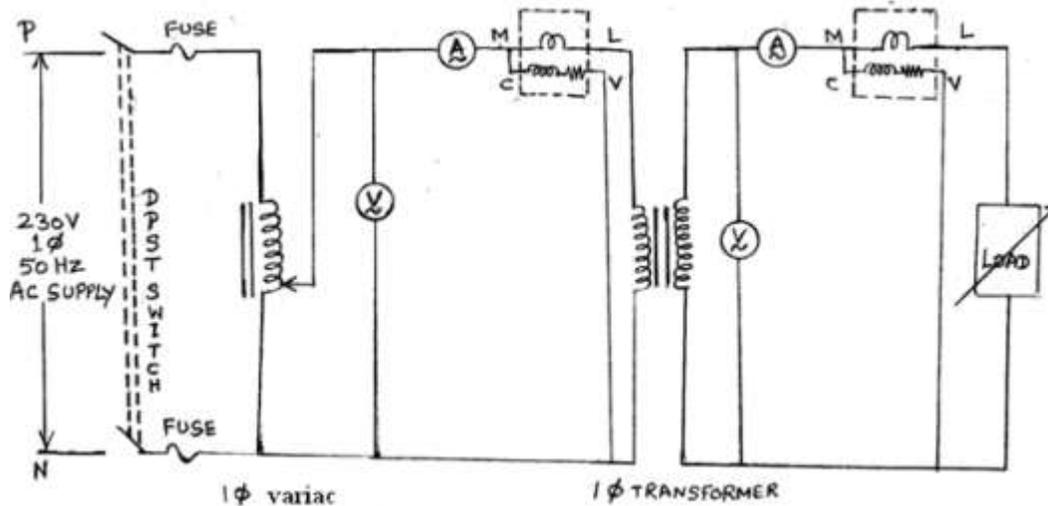


Fig: 2.22

The load is varied from no load to full load in desired steps. All the time, keep primary voltage  $V_1$  constant at its rated value with help of variac and tabulated the reading. The first reading is to be noted on no load for which  $I_2 = 0$  A and  $W_2 = 0$  W.

### Calculation

From the observed reading

$W_1$  = input power to the transformer

$W_2$  = output power delivered to the load

$$\% \eta = \frac{W_2}{W_1} \times 100$$

The first reading is no load so  $V_2 = E_2$

The regulation can be obtained as

$$\% R = \frac{E_2 - V_2}{V_2} \times 100$$

The graph of  $\% \eta$  and  $\% R$  on each load against load current  $I_L$  is plotted as shown in fig. 2.23.

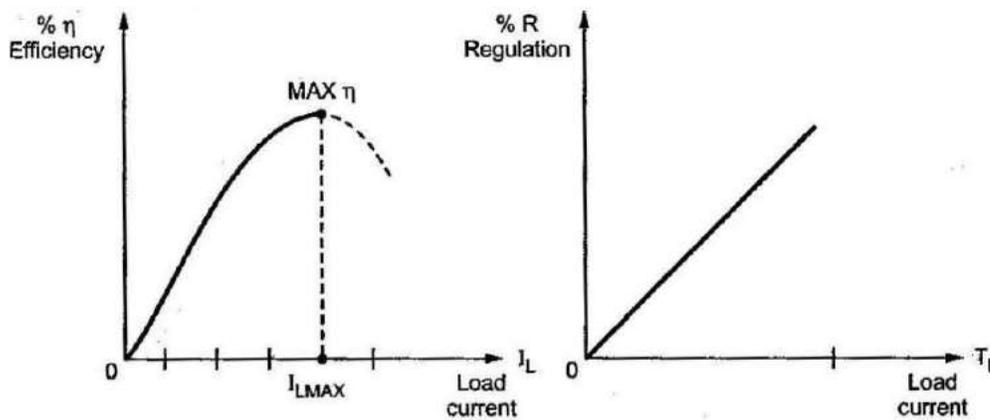


Fig: 2.23

**Advantages:**

- 1) This test enables us to determine the efficiency of the transformer accurately at any load.
- 2) The results are accurate as load is directly used.

**Disadvantages:**

- 1) There are large power losses during the test.
- 2) Load not avail in lab while test conduct for large transformer.

**ii) a. Open-Circuit or No-Load Test**

This test is conducted to determine the iron losses (or core losses) and parameters  $R_0$  and  $X_0$  of the transformer. In this test, the rated voltage is applied to the primary (usually low-voltage winding) while

the secondary is left open circuited. The applied primary voltage  $V_1$  is measured by the voltmeter, the no load current  $I_0$  by ammeter and no-load input power  $W_0$  by wattmeter as shown in Fig.2.24.a. As the normal rated voltage is applied to the primary, therefore, normal iron losses will occur in the transformer core. Hence wattmeter will record the iron losses and small copper loss in the primary. Since no-load current  $I_0$  is very small (usually 2-10 % of rated current). Cu losses in the primary under no-load condition are negligible as compared with iron losses. Hence, wattmeter reading practically gives the iron losses in the transformer. It is reminded that iron losses are the same at all loads.

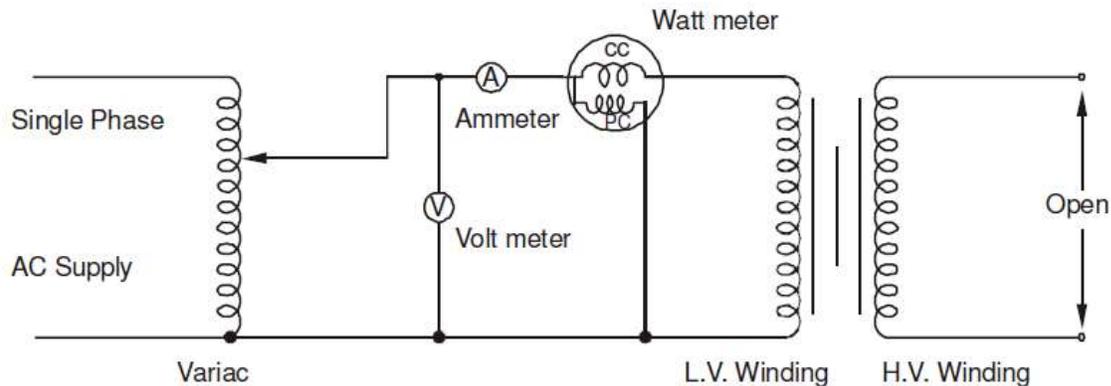


Fig: 2.24.a

$$\begin{aligned} \text{Iron losses, } P_i &= \text{Wattmeter reading} = W_0 \\ \text{No load current} &= \text{Ammeter reading} = I_0 \\ \text{Applied voltage} &= \text{Voltmeter reading} = V_1 \\ \text{Input power, } W_0 &= V_1 I_0 \cos \phi_0 \\ \text{No - load p.f., } \cos \phi &= \frac{W_0}{V_0 I_0} = \text{no load power factor} \end{aligned}$$

$$\begin{aligned} I_m &= I_0 \sin \phi_0 = \text{magnetizing component} \\ I_c &= I_0 \cos \phi_0 = \text{Active component} \end{aligned}$$

$$R_o = \frac{V_o}{I_c} \Omega, \quad X_o = \frac{V_o}{I_m} \Omega$$

Under no load conditions the PF is very low (near to 0) in lagging region. By using the above data we can draw the equivalent parameter shown in Figure 2.24.b.

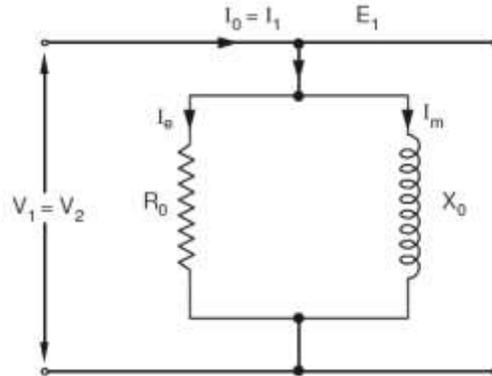


Fig: 2.24.b

Thus open-circuit test enables us to determine iron losses and parameters  $R_0$  and  $X_0$  of the transformer.

### ii) b. Short-Circuit or Impedance Test

This test is conducted to determine  $R_{1e}$  (or  $R_{2e}$ ),  $X_{1e}$  (or  $X_{2e}$ ) and full-load copper losses of the transformer. In this test, the secondary (usually low-voltage winding) is short-circuited by a thick conductor and variable low voltage is applied to the primary as shown in Fig.2.25. The low input voltage is gradually raised till at voltage  $V_{sc}$ , full-load current  $I_1$  flows in the primary. Then  $I_2$  in the secondary also has full-load value since  $I_1/I_2 = N_2/N_1$ . Under such conditions, the copper loss in the windings is the same as that on full load. There is no output from the transformer under short-circuit conditions. Therefore, input power is all loss and this loss is almost entirely copper loss. It is because iron loss in the core is negligibly small since the voltage  $V_{sc}$  is very small. Hence, the wattmeter will practically register the full load copper losses in the transformer windings.

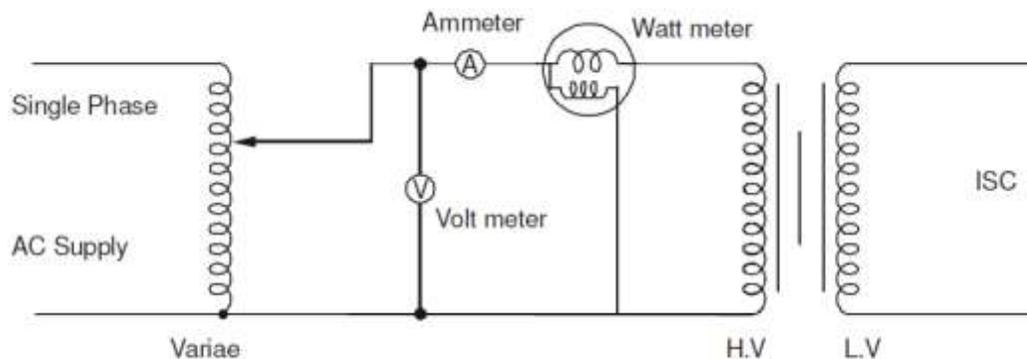


Fig: 2.25.a

Full load Cu loss, PC = Wattmeter reading =  $W_{sc}$

Applied voltage = Voltmeter reading =  $V_{sc}$

F.L. primary current = Ammeter reading =  $I_1$

$$P_{cu} = I_1^2 R_1 + I_1^2 R_2' = I_1^2 R_{1e}, \quad R_{1e} = \frac{P_{cu}}{I_1^2}$$

Where  $R_{1e}$  is the total resistance of transformer referred to primary.

Total impedance referred to primary,  $Z_{1e} = \sqrt{Z_{1e}^2 - R_{1e}^2}$ ,

short-circuit P.F,  $\cos \Phi = \frac{P_{cu}}{V_{sc} I_1}$  Thus short-circuit test gives full-load Cu loss,  $R_{1e}$  and  $X_{1e}$ .

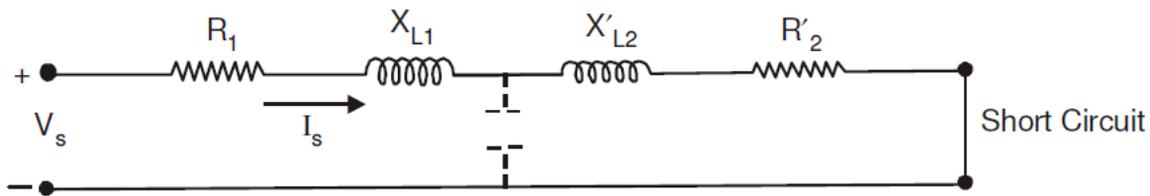


Fig: 2.25.b

From fig: 2.25.b we can calculate,

$$\text{equivalent resistance } R_{eq} = \frac{W_s}{I_s^2} = R_1 + R'_2$$

$$\text{and equivalent impedance } Z_{eq} = \frac{V_s}{I_s}$$

So we calculate equivalent reactance

$$X_{eq} = \sqrt{Z_{eq}^2 - R_{eq}^2} = X_{L1} + X'_{L2}$$

These  $R_{eq}$  and  $X_{eq}$  are equivalent resistance and reactance of both windings referred in HV side. These are known as equivalent circuit resistance and reactance.

## 2.8 Voltage Regulation of Transformer

Under no load conditions, the voltage at the secondary terminals is  $E_2$  and

$$E_2 \approx V_1 \cdot \frac{N_2}{N_1}$$

(This approximation neglects the drop  $R_1$  and  $X_{L1}$  due to small no load current). As load is applied to the transformer, the load current or the secondary current increases. Correspondingly, the primary current

$I_1$  also increases. Due to these currents, there is a voltage drop in the primary and secondary leakage reactances, and as a consequence the voltage across the output terminals or the load terminals changes. In quantitative terms this change in terminal voltage is called Voltage Regulation.

Voltage regulation of a transformer is defined as the drop in the magnitude of load voltage (or secondary terminal voltage) when load current changes from zero to full load value. This is expressed as a fraction of secondary rated voltage.

$$\text{Regulation} = \frac{\text{Secondary terminal voltage at no load} - \text{Secondary terminal voltage at any load}}{\text{Secondary rated voltage}}$$

The secondary rated voltage of a transformer is equal to the secondary terminal voltage at no load (i.e.  $E_2$ ), this is as per IS.

Voltage regulation is generally expressed as a percentage.

$$\text{Percent voltage regulation (\% VR)} = \frac{E_2 - V_2}{E_2} \times 100.$$

Note that  $E_2$ ,  $V_2$  are magnitudes, and not phasor or complex quantities. Also note that voltage regulation depends not only on load current, but also on its power factor. Using approximate equivalent circuit referred to primary or secondary, we can obtain the voltage regulation. From approximate equivalent circuit referred to the secondary side and phasor diagram for the circuit.

$$E_2 = V_2 + I_2 r_{eq} \cos \phi_2 \pm I_2 x_{eq} \sin \phi_2$$

where  $r_{eq} = r_2 + r_1'$  (referred to secondary)  $x_e = x_2 + x_1'$  (+ sign applies lagging power factor load and – sign applies to leading pf load).

$$\text{So } \frac{E_2 - V_2}{E_2} = \frac{I_2 r_{eq} \cos \phi_2 \pm I_2 x_{eq} \sin \phi_2}{E_2}$$

$$\frac{E_2 - V_2}{E_2} = \frac{I_2 r_{eq}}{E_2} \cos \phi_2 \pm \frac{I_2 x_{eq}}{E_2} \sin \phi_2$$

% Voltage regulation = (% resistive drop)  $\cos \phi_2 \pm$  (% reactive drop)  $\sin \phi_2$ .

Ideally voltage regulation should be zero.

## 2.9 Auto-transformers

The transformers we have considered so far are two-winding transformers in which the electrical circuit connected to the primary is electrically isolated from that connected to the secondary. An auto-transformer does not provide such isolation, but has economy of cost combined with increased efficiency. Fig.2.26 illustrates the auto-transformer which consists of a coil of  $N_A$  turns between terminals 1 and 2, with a third terminal 3 provided after  $N_B$  turns. If we neglect coil resistances and leakage fluxes, the flux linkages of the coil between 1 and 2 equals  $N_A \phi_m$  while the portion of coil between 3 and 2 has a flux linkage  $N_B \phi_m$ . If the induced voltages are designated as  $E_A$  and  $E_B$ , just as in a two winding transformer,

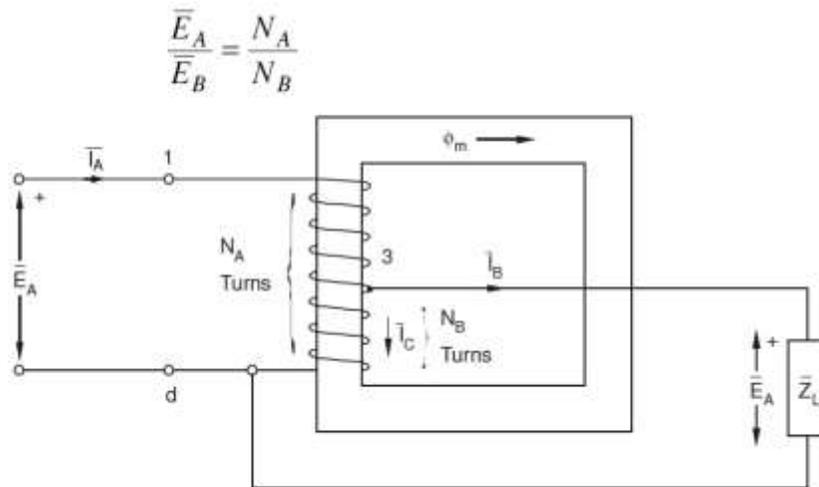


Fig: 2.26

Neglecting the magnetizing ampere-turns needed by the core for producing flux, as in an ideal transformer, the current  $I_A$  flows through only  $(N_A - N_B)$  turns. If the load current is  $I_B$ , as shown by

Kirchhoff's current law, the current  $I_C$  flowing from terminal 3 to terminal 2 is  $(I_A - I_B)$ . This current flows through  $N_B$  turns. So, the requirement of a net value of zero ampere-turns across the core demands that

$$(N_A - N_B) \bar{I}_A + (\bar{I}_A - \bar{I}_B) N_B = 0$$

or 
$$N_A \bar{I}_A - N_B \bar{I}_B = 0$$

Hence, just as in a two-winding transformer,

$$\frac{\bar{I}_A}{\bar{I}_B} = \frac{N_B}{N_A}$$

Consequently, as far as voltage, current converting properties are concerned, the autotransformer of Figure: 26 behaves just like a two-winding transformer. However, in the autotransformer we don't need two separate coils, each designed to carry full load values of current.

## **2.10 Parallel Operation of Transformers**

It is economical to install numbers of smaller rated transformers in parallel than installing a bigger rated electrical power transformers. This has mainly the following advantages,

To maximize electrical power system efficiency: Generally electrical power transformer gives the maximum efficiency at full load. If we run numbers of transformers in parallel, we can switch on only those transformers which will give the total demand by running nearer to its full load rating for that time. When load increases, we can switch none by one other transformer connected in parallel to fulfil the total demand. In this way we can run the system with maximum efficiency.

To maximize electrical power system availability: If numbers of transformers run in parallel, we can shut down any one of them for maintenance purpose. Other parallel transformers in system will serve the load without total interruption of power.

To maximize power system reliability: if any one of the transformers run in parallel, is tripped due to fault of other parallel transformers is the system will share the load, hence power supply may not be interrupted if the shared loads do not make other transformers over loaded.

To maximize electrical power system flexibility: There is always a chance of increasing or decreasing future demand of power system. If it is predicted that power demand will be increased in future, there must be a provision of connecting transformers in system in parallel to fulfil the extra demand because, it is not economical from business point of view to install a bigger rated single transformer by forecasting the increased future demand as it is unnecessary investment of money. Again if future demand is decreased, transformers running in parallel can be removed from system to balance the capital investment and its return.

### **2.10.1 Conditions for Parallel Operation of Transformers**

When two or more transformers run in parallel, they must satisfy the following conditions for satisfactory performance. These are the conditions for parallel operation of transformers.

- *Same voltage ratio of transformer.*
- *Same percentage impedance.*
- *Same polarity.*
- *Same phase sequence.*
- *Same Voltage Ratio*

#### **Same voltage ratio of transformer.**

If two transformers of different voltage ratio are connected in parallel with same primary supply voltage, there will be a difference in secondary voltages. Now say the secondary of these transformers are connected to same bus, there will be a circulating current between secondaries and therefore between primaries also. As the internal impedance of transformer is small, a small voltage difference may cause sufficiently high circulating current causing unnecessary extra  $I^2R$  loss.

#### **Same Percentage Impedance**

The current shared by two transformers running in parallel should be proportional to their MVA ratings. Again, current carried by these transformers are inversely proportional to their internal impedance. From these two statements it can be said that, impedance of transformers running in parallel are inversely proportional to their MVA ratings. In other words, percentage impedance or per unit values of impedance should be identical for all the transformers that run in parallel.

### **Same Polarity**

Polarity of all transformers that run in parallel, should be the same otherwise huge circulating current that flows in the transformer but no load will be fed from these transformers. Polarity of transformer means the instantaneous direction of induced emf in secondary. If the instantaneous directions of induced secondary emf in two transformers are opposite to each other when same input power is fed to both of the transformers, the transformers are said to be in opposite polarity. If the instantaneous directions of induced secondary e.m.f in two transformers are same when same input power is fed to the both of the transformers, the transformers are said to be in same polarity.

### **Same Phase Sequence**

The phase sequence or the order in which the phases reach their maximum positive voltage, must be identical for two parallel transformers. Otherwise, during the cycle, each pair of phases will be short circuited.

The above said conditions must be strictly followed for parallel operation of transformers but totally identical percentage impedance of two different transformers is difficult to achieve practically, that is why the transformers run in parallel may not have exactly same percentage impedance but the values would be as nearer as possible.

### **2.11 Why Transformer Rating in kVA?**

An important factor in the design and operation of electrical machines is the relation between the life of the insulation and operating temperature of the machine. Therefore, temperature rise resulting from the losses is a determining factor in the rating of a machine. We know that copper loss in a transformer depends on current and iron loss depends on voltage. Therefore, the total loss in a transformer depends on the volt-ampere product only and not on the phase angle between voltage and current i.e., it is independent of load power factor. For this reason, the rating of a transformer is in kVA and not kW.

## Acknowledgement

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However apart from this lecture note students/readers are strongly recommended to follow the below mentioned books in the references and above all confer with the faculty for thorough knowledge of this authoritative subject of electrical engineering.

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**Best of Luck to All the Students**