CE 15023

FLUID DYNAMICS

LECTURE NOTES

MODULE-I

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Course Contents

MODULE 1

Dimensional Analysis: Introduction, Dimensional homogeneity, Methods of Dimensional Analysis, Model investigation, Similitude, Types of similarity, Model Laws, types of Models, Dimensionless numbers, Application of dynamic similarity to specific models.

Lecture Note 1

Dimensional Analysis

1.1 Introduction

In engineering and science, dimensional analysis is the analysis of the relationships between different physical quantities by identifying their base quantities (such as length, mass, time, and electric charge) and units of measure (such as miles vs. kilometers, or pounds vs. kilograms vs. grams) and tracking these dimensions as calculations or comparisons are performed. Converting from one dimensional unit to another is often somewhat complex. Dimensional analysis, or more specifically the factor-label method, also known as the unit-factor method, is a widely used technique for such conversions using the rules of algebra.

The concept of physical dimension was introduced by Joseph Fourier in 1822. Physical quantities that are of the same kind (also called commensurable) have the same dimension (length, time, mass) and can be directly compared to each other, even if they are originally expressed in differing units of measure (such as inches and meters, or pounds and Newton's). If physical quantities have different dimensions (such as length vs. mass), they cannot be expressed in terms of similar units and cannot be compared in quantity (also called incommensurable). For example, asking whether a kilogram is greater than, equal to, or less than an hour is meaningless.

Any physically meaningful equation (and likewise any inequality and in equation) will have the same dimensions on its left and right sides, a property known as dimensional homogeneity. Checking for dimensional homogeneity is a common application of dimensional analysis, serving as a plausibility check on derived equations and computations. It also serves as a guide and constraint in deriving equations that may describe a physical system in the absence of a more rigorous derivation.

Many practical flow problems of different nature can be solved by using equations and analytical procedures, as discussed in the previous modules. However, solutions of some real flow problems depend heavily on experimental data and the refinements in the analysis are made, based on the measurements. Sometimes, the experimental work in the laboratory is not only time-consuming, but also expensive. So, the dimensional analysis is an important tool that helps in correlating analytical results with experimental data for such unknown flow problems. Also, some dimensionless parameters and scaling laws can be framed in order to predict the prototype behavior from the measurements on the model. The important terms used in this module may be defined as below;

Dimensional Analysis: The systematic procedure of identifying the variables in a physical phenomena and correlating them to form a set of dimensionless group is known as dimensional analysis.

Dimensional Homogeneity: If an equation truly expresses a proper relationship among variables in a physical process, then it will be dimensionally homogeneous. The equations are correct for any system of units and consequently each group of terms in the equation must have the same dimensional representation. This is also known as the law of dimensional homogeneity.

Dimensional variables: These are the quantities, which actually vary during a given case and can be plotted against each other.

Dimensional constants: These are normally held constant during a given run. But, they may vary from case to case.

Pure constants: They have no dimensions, but, while performing the mathematical manipulation, they can arise.

Let us explain these terms from the following examples: -

Displacement of a free falling body is given as,

$$S = S_O + V_O t + \frac{1}{2} g t^2$$

where, V_0 is the initial velocity,

g is the acceleration due to gravity,

t is the time,

S and S_O are the final and initial distances, respectively. Each term in this equation has the dimension of length [L] and hence it is dimensionally homogeneous.

Here, S and t are the dimensional variables,

g, S_O, and V_O are the dimensional constants and 1/2 arises due to mathematical manipulation and is the pure constant.

- Bernoulli's equation for incompressible flow is written as,

$$\frac{p}{\rho} + \frac{1}{2} V^2 + gz = C$$

Here, p is the pressure, V is the velocity, z is the distance, ρ is the density and g is the acceleration due to gravity. In this case, the dimensional variables are pV z , and , the dimensional constants are g C , and ρ and 1/2 is the pure constant. Each term in this equation including the constant has dimension of $[L^2T^{-2}]$and hence it is dimensionally homogeneous.

Buckingham pi Theorem

The dimensional analysis for the experimental data of unknown flow problems leads to some non-dimensional parameters. These dimensionless products are frequently referred as pi terms. Based on the concept of dimensional homogeneity, these dimensionless parameters may be grouped and expressed in functional forms. This idea was explored by the famous scientist Edgar Buckingham (1867-1940) and the theorem is named accordingly.

Buckingham pi theorem, states that if an equation involving k variables is dimensionally homogeneous, then it can be reduced to a relationship among (k−r) independent dimensionless products, where r is the minimum number of reference dimensions required to describe the variable. For a physical system, involving k variables, the functional relation of variables can be written mathematically as,

$y = f(x_1,x_2,\ldots\ldots x_k)$ …………………………………………….(1)

In Eq. (1), it should be ensured that the dimensions of the variables on the left side of the equation are equal to the dimensions of any term on the right side of equation. Now, it is possible to rearrange the above equation into a set of dimensionless products (pi terms), so that

$\pi_1 = \varphi(\pi_{2,}\pi_3, \ldots \ldots . \pi_{k-r})$………………………………………….(2)

Here, $\varphi(\pi_{2,}\pi_3, \ldots \ldots . \pi_{k-r})$is a function of $\pi_{2,}$ through π_{k-r} . The required number of pi terms is less than the number of original reference variables by r . These reference dimensions are usually the basic dimensions ML T , and (Mass, Length and Time)

Determination of pi Terms

Several methods can be used to form dimensionless products or pi terms that arise in dimensional analysis. But, there is a systematic procedure called method of repeating variables that allows in deciding the dimensionless and independent pi terms. For a given problem, following distinct steps are followed. Step I: List out all the variables that are involved in the problem. The 'variable' is any quantity including dimensional and non-dimensional constants in a physical situation under investigation. Typically, these variables are those that are necessary to describe the "geometry" of the system (diameter, length etc.), to define fluid properties (density, viscosity etc.) and to indicate the external effects influencing the system (force, pressure etc.). All the variables must be independent in nature so as to minimize the number of variables required to describe the complete system. Step II: Express each variable in terms of basic dimensions. Typically, for fluid mechanics problems, the basic dimensions will be either ML T , and or FL T , and .

Dimensionally, these two sets are related through Newton's second law (F =ma) so that

$F = MLT^{-2}$ = e.g. $\rho = ML^{-3}$ or $\rho = FL^{-4}T^{2}$ = It should be noted that these basic dimensions should not be mixed.

Step III: Decide the required number of pi terms. It can be determined by using Buckingham pi theorem which indicates that the number of pi terms is equal to (k −r), where k is the number of variables in the problem (determined from Step I) and r is the number of reference dimensions required to describe these variables (determined from Step II).

Step IV: Amongst the original list of variables, select those variables that can be combined to form pi terms. These are called as repeating variables. The required number of repeating variables is equal to the number of reference dimensions. Each repeating variable must be dimensionally independent of the others, i.e. they cannot be combined themselves to form any dimensionless product. Since there is a possibility of repeating variables to appear in more than one pi term, so dependent variables should not be chosen as one of the repeating variable.

Step V: Essentially, the pi terms are formed by multiplying one of the non-repeating variables by the product of the repeating variables each raised to an exponent that will make the combination dimensionless. It usually takes the form of $x_i x_1^a x_2^b x_3^c$ where the exponents a,b, and c are determined so that the combination is dimensionless.

Step VI: Repeat the 'Step V' for each of the remaining non-repeating variables. The resulting set of pi terms will correspond to the required number obtained from Step III.

Step VII: After obtaining the required number of pi terms, make sure that all the pi terms are dimensionless. It can be checked by simply substituting the basic dimension (M, L , and T) of the variables into the pi terms.

Step VIII: Typically, the final form of relationship among the pi terms can be written in the form of Eq. (.1) where, Π1 would contain the dependent variable in the numerator. The actual functional relationship among pi terms is determined from experiment.

Illustration of Pi Theorem

Let us consider the following example to illustrate the procedure of determining the various steps in the pi theorem.

Example (Pressure drop in a pipe flow)

Consider a steady flow of an incompressible Newtonian fluid through a long, smooth walled, horizontal circular pipe. It is required to measure the pressure drop per unit length of the pipe and find the number of non-dimensional parameters involved in the problem. Also, it is desired to know the functional relation among these dimensionless parameters.

Step I: Let us express all the pertinent variables involved in the experimentation of pressure drop per unit length (Δp_l) of the pipe, in the following form;

$\Delta p_l = f(D, \rho, \mu, V)$ …… ………………………. ……………. ………… …………….(3)

where, D is the pipe diameter, ρ is the fluid density, µ is the viscosity of the fluid and V is the mean velocity at which the fluid is flowing through the pipe.

Step II: Next step is to express all the variables in terms of basic dimensions i.e. M,L and T . It then follows that

$$\Delta p_l = ML^{-2}T^{-2};$$

$$D = L;$$

Step III: Apply *Buckingham theorem* to decide the number of *pi terms* required. There are five variables (including the dependent variable Δp_l) and three reference dimensions. Since, $k = 5$ and $r = 3$, only *two pi terms* are required for this problem.

Step IV: The repeating variables to form pi terms, need to be selected from the list ,ρ, µ and V . It is to be noted that the dependent variable should not be used as one of the repeating variable. Since, there are three reference dimensions involved, so we need to select three repeating variable. These repeating variables should be dimensionally independent, i.e. dimensionless product cannot be formed from this set. In this case, D, ρ and V may be chosen as the repeating variables.

Step V: Now, first *pi term* is formed between the dependent variable and the repeating variables. It is written as,

$$\pi = \Delta P_I D^a V^b \rho^c$$

Since, this combination need to be dimensionless, it follows that

$$(ML^{-2}T^{-2})(L)^a (LT^{-1})^b (ML^{-3})^c = M_0 L_0 T_0$$

The exponents a , b and c must be determined by equating the exponents for each of the terms M , L and T i.e.

For M : $1 + c = 0$

For L : $-2 + a + b - 3c = 0$

For T : $-2 - b = 0$

The solution of this algebraic equations gives $a = 1$; $b = -2$; $c = -1$. Therefore,

$$\pi_1 = \frac{\Delta P_1 D}{\rho V^2}$$

The process is repeated for remaining non-repeating variables with other additional variable (μ) so that,

$$\Pi_2 = \mu . D^d . V^e . \rho^f$$

Since, this combination need to be dimensionless, it follows that

$$(ML^{-1}T^{-1})(L)^d (LT^{-1})^e (ML^{-3})^f = M^0 L^0 T^0$$

Equating the exponents

For M : $1 + f = 0$

For L : $-1 + d + e - 3f = 0$

For T : $-1 - e = 0$

The solution of this algebraic equation gives $d = -1$; $e = -1$; $f = -1$. Therefore

$$\Pi_2 = \frac{\mu}{\rho V D}$$

Step VI: Now, the correct numbers of *pi terms* are formed as determined in "Step III". In order to make sure about the dimensionality of *pi terms*, they are written as,

$$\Pi_1 = \frac{\Delta p\, D}{\rho V^2} = \frac{(ML^{-2}T^{-2})(L)}{(ML^{-3})(LT^{-1})^2} = M^0 L^0 T^0$$

$$\Pi_2 = \frac{\mu}{\rho V D} = \frac{(ML^{-1}T^{-1})(L)}{(ML^{-3})(LT^{-1})(L)} = M^0 L^0 T^0$$

Step VII: Finally, the result of dimensional analysis is expressed among the *pi terms*

$$\frac{D\,\Delta p}{\rho V^2} = \varphi\frac{\mu}{\rho V D} = \varphi\frac{1}{\text{Re}}$$

It may be noted here that Re is the Reynolds number.

Remarks

If the difference in the number of variables for a given problem and number of reference dimensions is equal to unity, then only one Pi term is required to describe the phenomena. Here, the functional relationship for the one Pi term is a constant quantity and it is determined from the experiment

$$\Pi_1 = \text{Constant} \qquad (6.1.15)$$

The problems involving two *Pi* terms can be described such that

$$\Pi_1 = \varphi(\Pi_2) \qquad (6.1.16)$$

Here, the functional relationship among the variables can then be determined by varying Π_2 and measuring the corresponding values of Π_1.

References

Text Book:

1. Fluid Mechanics by A.K. Jain, Khanna Publishers

Reference Book:

1. Fluid Mechanics and Hydraulic Machines, Modi & Seth, Standard Pulishers

2. Introduction to Fluid Mechanics and Fluid Machines, S.K. Som & G. Biswas,