CE 15008
Fluid
Mechanics

LECTURE NOTES

Module-III

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COURSE CONTENT

CE 15008:

FLUID MECHANICS (3-1-0)

CR-04

Module – III **(12 Hours)**

Fluid dynamics: Basic equations: Equation of continuity; One-dimensional Euler's equation of motion and its integration to obtain Bernoulli's equation and momentum equation.

Flow through pipes: Laminar and turbulent flow in pipes; Hydraulic mean radius; Concept of losses; Darcy-Weisbach equation; Moody's (Stanton) diagram; Flow in sudden expansion and contraction; Minor losses in fittings; Branched pipes in parallel and series, Transmission of power; Water hammer in pipes (Sudden closure condition).

Lecture Notes

Module 3

FLUID DYNAMICS

CONTINUITY EQUATION

The continuity equation of fluid mechanics expresses the notion that mass cannot be created nor destroyed or that mass is conserved. It relates the flow field variables at a point of the flow in terms of the fluid density and the fluid velocity vector, and is given by:

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \bar{V}) = 0 \qquad (1)$$

We consider the vector identity resembling the chain rule of differentiation:

$$\nabla.(\rho \bar{V}) \equiv \rho \nabla.\bar{V} + \bar{V}.\nabla \rho \qquad (2)$$

where the divergence operator is noted to act on a vector quantity, and the gradient operator acts on a scalar quantity.

This allows us to rewrite the continuity equation as:

$$\frac{D}{Dt} = \underbrace{\frac{\partial}{\partial t}}_{\text{Local Derivative}} + \underbrace{(\bar{V}.\nabla)}_{\text{Convective Derivative}} \qquad (4)$$

where the partial time derivative is called the local derivative and the dot product term is called the convective derivative.

In terms of the substantial derivative the continuity equation can be expressed as::

$$\frac{D\rho}{Dt} + \rho \nabla.\bar{V} = 0 \qquad (5)$$

MOMENTUM CONSERVATION OR EQUATION OF MOTION

Newton's second law is frequently written in terms of an acceleration and a force vectors as:

$$\bar{F} = m\bar{a} \qquad (6)$$

A more general form describes the force vector as the rate of change of the momentum vector as:

$$\bar{F} = \frac{d}{dt}(m\bar{V}) \qquad (7)$$

Its general form is written in term of volume integrals and a surface integral over an arbitrary control volume v as:

$$\iiint_v \frac{\partial(\rho\bar{V})}{\partial t}dv + \iint_S (\rho\bar{V}\cdot d\bar{S})\bar{V} = -\iiint_v \nabla p\, dv + \iiint_v \rho\bar{f}\, dv + \iiint_v \bar{F}_{viscous}\, dv \qquad (8)$$

where the velocity vector is:

$$\bar{V} = u\ddot{x} + v\ddot{y} + w\ddot{z} \qquad (9)$$

The cartesian coordinates x, y and z components of the continuity equation are:

$$\iiint_v \frac{\partial(\rho u)}{\partial t}dv + \iint_S (\rho\bar{V}\cdot d\bar{S})u = -\iiint_v \frac{\partial p}{\partial x}dv + \iiint_v \rho f_x dv + \iiint_v (F_x)_{viscous}dv$$

$$\iiint_v \frac{\partial(\rho v)}{\partial t}dv + \iint_S (\rho\bar{V}\cdot d\bar{S})v = -\iiint_v \frac{\partial p}{\partial y}dv + \iiint_v \rho f_y dv + \iiint_v (F_x)_{viscous}dv \qquad (10)$$

$$\iiint_v \frac{\partial(\rho w)}{\partial t}dv + \iint_S (\rho\bar{V}\cdot d\bar{S})w = -\iiint_v \frac{\partial p}{\partial z}dv + \iiint_v \rho f_z dv + \iiint_v (F_z)_{viscous}dv$$

(23)

EULER'S EQUATION

Multiplying the flow equations respectively by dx, dy and dz, we get:

$$u\frac{\partial u}{\partial x}dx + v\frac{\partial u}{\partial y}dx + w\frac{\partial u}{\partial z}dx = -\frac{1}{\rho}\frac{\partial p}{\partial x}dx$$
$$u\frac{\partial v}{\partial x}dy + v\frac{\partial v}{\partial y}dy + w\frac{\partial v}{\partial z}dy = -\frac{1}{\rho}\frac{\partial p}{\partial y}dy \qquad (24)$$
$$u\frac{\partial w}{\partial x}dz + v\frac{\partial w}{\partial y}dz + w\frac{\partial w}{\partial z}dz = -\frac{1}{\rho}\frac{\partial p}{\partial z}dz$$

Using the streamline differential equations, we can write:

$$u\frac{\partial u}{\partial x}dx + u\frac{\partial u}{\partial y}dy + w\frac{\partial u}{\partial z}dz = -\frac{1}{\rho}\frac{\partial p}{\partial x}dx$$
$$u\frac{\partial v}{\partial x}dx + v\frac{\partial v}{\partial y}dy + w\frac{\partial v}{\partial z}dz = -\frac{1}{\rho}\frac{\partial p}{\partial y}dy \qquad (25)$$
$$u\frac{\partial w}{\partial x}dx + v\frac{\partial w}{\partial y}dy + w\frac{\partial w}{\partial z}dz = -\frac{1}{\rho}\frac{\partial p}{\partial z}dz$$

The differentials of functions u = u(x,y,z), v = v(x,y,z), w =w (x,y,z) are:

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz$$
$$dv = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy + \frac{\partial v}{\partial z}dz \qquad (26)$$
$$dw = \frac{\partial w}{\partial x}dx + \frac{\partial w}{\partial y}dy + \frac{\partial w}{\partial z}dz$$

This allows us to write:

$$udu = -\frac{1}{\rho}\frac{\partial p}{\partial x}dx$$
$$vdv = -\frac{1}{\rho}\frac{\partial p}{\partial y}dy \qquad (27)$$
$$wdw = -\frac{1}{\rho}\frac{\partial p}{\partial z}dz$$

Through integration we can write:

$$\frac{1}{2}d(u^2) = -\frac{1}{\rho}\frac{\partial p}{\partial x}dx$$
$$\frac{1}{2}d(v^2) = -\frac{1}{\rho}\frac{\partial p}{\partial y}dy \qquad (28)$$
$$\frac{1}{2}d(w^2) = -\frac{1}{\rho}\frac{\partial p}{\partial z}dz$$

Adding the three last equations we get:

$$\frac{1}{2}d(u^2+v^2+w^2) = -\frac{1}{\rho}\left(\frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial y}dy + \frac{\partial p}{\partial z}dz\right)$$
$$\frac{1}{2}d(V^2) = -\frac{1}{\rho}dp \qquad (29)$$

From the last equation we can write a simple form of Euler's equation as:

$$dp = -\rho V dV \qquad (30)$$

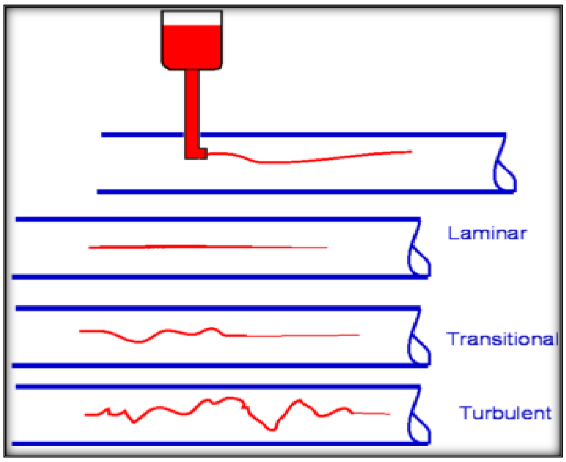
Euler's equation applies to an inviscid flow with no body forces. It relates the change in velocity along a streamline dV to the change in pressure dp along the same streamline.

FLOW THROUGH PIPES

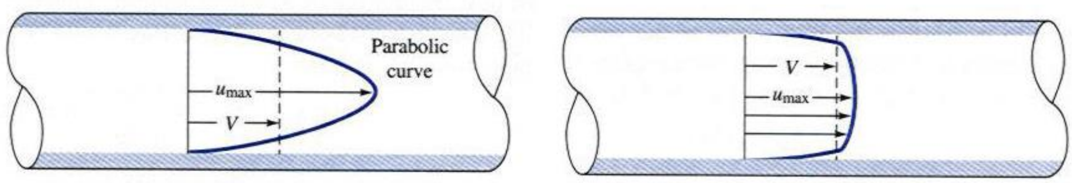
LAMINAR AND TURBULENT FLOW

When a fluid is flowing through a closed channel such as a pipe or between two flat plates, either of two types of flow may occur depending on the velocity and viscosity of the fluid: laminar flow or turbulent flow.

A flow can be Laminar, Turbulent or Transitional in nature. This classification of flows is brought out vividly by the experiment conducted by Osborne Reynolds (1842 - 1912). Into a flow through a glass tube he injected a dye to observe the nature of flow. When the speeds were small the flow seemed to follow a straight line path (with a slight blurring due to dye diffusion). As the flow speed was increased the dye fluctuates and one observes intermittent bursts. As the flow speed is further increased the dye is blurred and seems to fill the entire pipe. These are what we call Laminar, Transitional and Turbulent Flows.



In laminar flow the fluid particles move along smooth, regular paths or laminas gliding over adjacent layers. The turbulent flow is characterized by random and erratic movement of fluid particles resulting in the formation of eddies.



Laminar flow Turbulent flow

HYDRAULIC MEAN RADIUS

The hydraulic radius of a section is not a directly measurable characteristic, but it is used frequently during calculations. It is defined as the area divided by the wetted perimeter, and therefore has units of length. The hydraulic radius can often be related directly to the geometric properties of the channel. For example, the hydraulic radius of a full circular pipe (such as a pressure pipe) can be directly computed as:

$$R = \frac{A}{P}$$

Where R = hydraulic radius (m, ft.) A = cross-sectional area (m^2, ft.) Pw = wetted perimeter (m, ft.) D = pipe diameter (m, ft.)

CONCEPT OF LOSSES

It is often necessary to determine the head loss that occur in a pipe flow so that the energy equation, can be used in the analysis of pipe flow problems. The overall head loss for the pipe system consists of the head loss due to viscous effects in the straight pipes, termed the major loss and denoted h_L-major. The head loss in various pipe components, termed the minor loss and denoted h_L-minor.

Major Losses The head loss, h_L-major is given as;

$$h_L = \frac{fLV^2}{D\,2g}$$

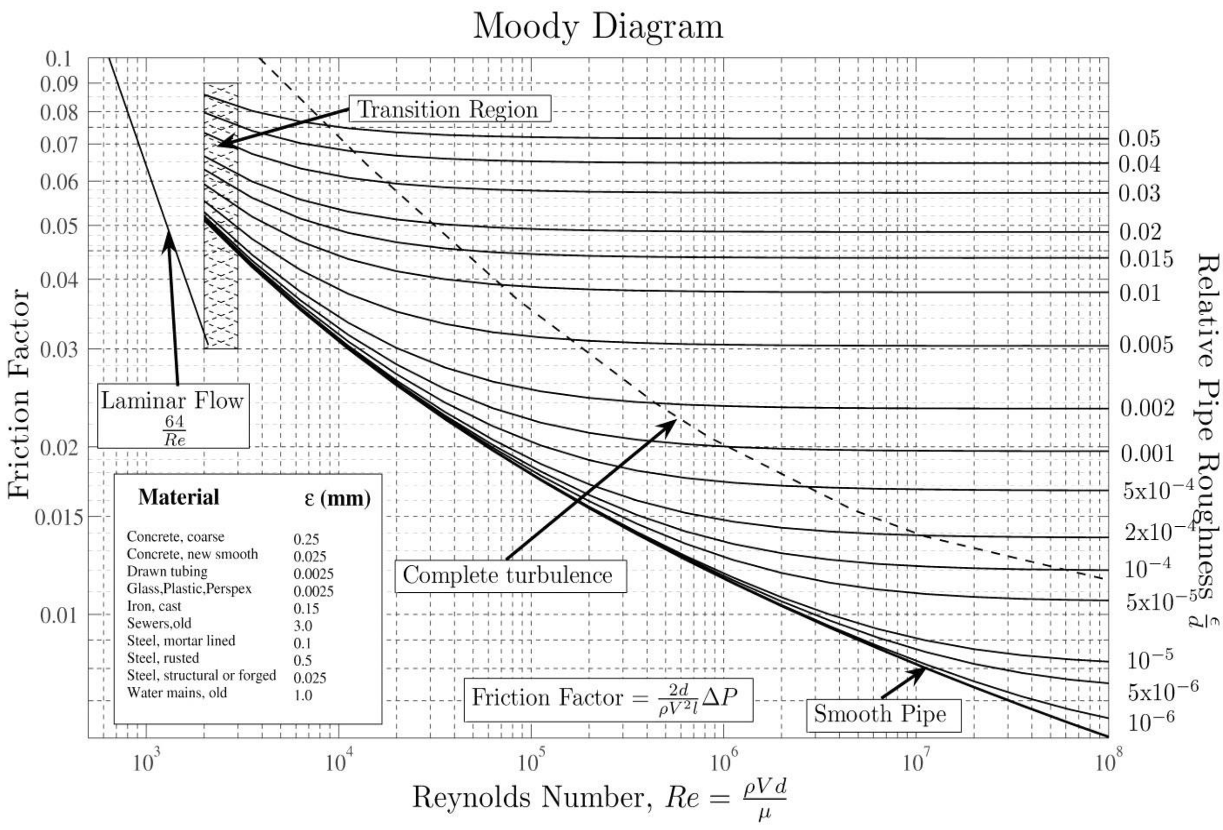
where f is friction factor. Above mention equation is called the Darcy-Weisbach equation. It is valid for any fully developed, steady, incompressible pipe flow, whether the pipe is horizontal or on hill.

Friction factor for laminar flow is; f $=\frac{64}{R_e}$

Friction factor for turbulent flow is based on Moody chart.

MOODY'S DIAGRAM

The Moody diagram is a plot of the Darcy friction factor as a function of Reynolds number and relative roughness. The Moody diagram shows both the laminar and turbulent regimes as well as a transition zone between laminar and turbulent flow. The determination of the Darcy friction factor using the Moody diagram requires several pieces of information. First, the Reynolds number based on the diameter of the pipe, kinematic viscosity, and average velocity must be known. Second, the relative roughness must be determined. The horizontal axis of a Moody diagram contains the Reynolds number and the vertical axis is a plot of the friction factor. The Reynolds number is first located on the horizontal axis. Next, the Reynolds number is followed vertically to the desired relative roughness value. The friction factor is then read off of the vertical axis. If the flow is laminar, the friction factor does not depend on the relative roughness and a single straight line is used to determine the friction factor.



Under normal conditions the flow in pipes remain in laminar state upto a Reynolds no value of 2000 and disturbances tending to cause turbulence are damped by viscous action. The region within the Reynolds no range of 2000 to 4000 is known as critical zone in which flow may be laminar or turbulent. In transition zone the surface roughness and viscous action both influence the pipe resistance. Outside this region the friction factor is governed by the relative roughness alone.

FLOW IN SUDDEN EXPANSION AND CONTRACTION

- If the cross-section of a pipe with fluid flowing through it, is abruptly enlarged at certain place, fluid emerging from the smaller pipe is unable to follow the abrupt deviation of the boundary. The streamline takes a typical diverging pattern. This creates pockets of turbulent eddies in the corners resulting in the dissipation of mechanical energy into intermolecular energy. The fluid flows against an adverse pressure gradient. The upstream pressure p_1 at section a-b is lower than the downstream pressure p_2 at section e-f since the upstream velocity V_1 is higher than the downstream velocity V_2 as a consequence of continuity. The fluid particles near the wall due to their low kinetic energy cannot overcome the adverse pressure hill in the direction of flow and hence follow up the reverse path under the favorable pressure gradient (from p_2 to p_1). This creates a zone of re-circulating flow with turbulent eddies near the wall of the larger tube at the abrupt change of cross-section, resulting in a loss of total mechanical energy. For high values of Reynolds number, usually found in practice, the velocity in the smaller pipe may be assumed sensibly uniform over the cross section. Due to the vigorous mixing caused by the turbulence, the velocity becomes again uniform at a far downstream section e-f from the enlargement (approximately 8 times the larger diameter).

A control volume abcdefgh is considered for which the momentum theorem can be written as

$$p_1A_1 + p'(A_2 - A_1) - p_2A_2 = \rho Q(V_2 - V_1)$$

Where A_1, A_2 are the cross-sectional areas of the smaller and larger parts of the pipe respectively, Q is the volumetric flow rate and p' is the mean pressure of the eddying fluid over the annular face, gd. It is known from experimental evidence, the $p' = p_1$.

Hence the Eq. becomes

$$(p_2 - p_1)A_2 = \rho Q(V_1 - V_2)$$

From the equation of continuity

$$Q = V_2A_2$$

$$p_2 - p_1 = \rho V_2(V_1 - V_2)$$

Applying Bernoulli's equation between sections ab and ef in consideration of the flow to be incompressible and the axis of the pipe to be horizontal, we can write

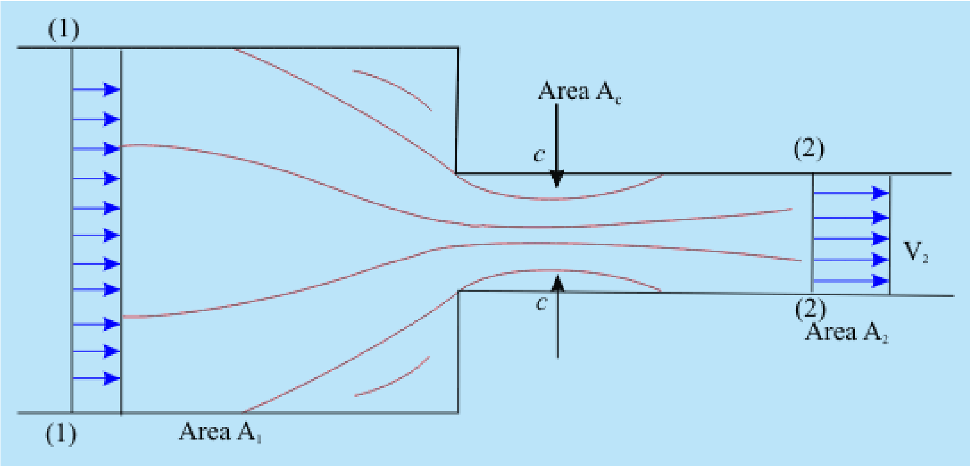
$$\frac{p_1}{\rho}+\frac{V_1^2}{2}=\frac{p_2}{\rho}+\frac{V_2^2}{2}+gh_L$$

$$\frac{p_2-p_1}{\rho}=\frac{V_1^2-V_2^2}{2}-gh_L$$

where h_L is the loss of head

$$h_L=\frac{(V_1-V_2)^2}{2g}= \quad \frac{V_1^2}{2g}\left[(1\text{-}(\frac{A_1}{A_2})\right]^2$$

- An abrupt contraction is geometrically the reverse of an abrupt enlargement (Fig.). Here also the streamlines cannot follow the abrupt change of geometry and hence gradually converge from an upstream section of the larger tube. However, immediately downstream of the junction of area contraction, the cross-sectional area of the stream tube becomes the minimum and less than that of the smaller pipe. This section of the stream tube is known as vena contracta, after which the stream widens again to fill the pipe. The velocity of flow in the converging part of the stream tube from Sec. 1-1 to Sec. c-c (vena contracta) increases due to continuity and the pressure decreases in the direction of flow accordingly in compliance with the Bernoulli's theorem. In an accelerating flow,under a favourable pressure gradient, losses due to separation cannot take place. But in the decelerating part of the flow from Sec. c-c to Sec. 2-2, where the stream tube expands to fill the pipe, losses take place in the similar fashion as occur in case of a sudden geometrical enlargement. Hence eddies are formed between the vena contracta c-c and the downstream Sec. 2-2. The flow pattern after the vena contracta is similar to that after an abrupt enlargement, and the loss of head is thus confined between Sec. c-c to Sec. 2-2. Therefore, we can say that the losses due to contraction are not for the contraction itself, but due to the expansion followed by the contraction.



The loss of head in this case can be written as

$$h_L = \frac{V_2^2}{2g}\left[\left(\frac{A_2}{A_c}\right)-1\right]^2 = \frac{V_2^2}{2g}\left[\left(\frac{1}{C_c}\right)-1\right]^2$$

MINOR LOSSES IN FITTINGS

The loss of energy caused by commercial pipe fittings, such as valves, elbows, bends etc. occur because of their rough and irregular interior surfaces which produce excessive large scale turbulence. These components interrupt the smooth flow of the fluid and cause additional losses because of the flow separation and mixing they induce. In a typical system with long pipes, these losses are minor compared to the total head loss in the pipes (the major losses) and are called minor losses. The loss of energy in pipe fittings is generally expressed as

$$h_L = k_L \frac{V^2}{2g}$$

Where, V= Mean velocity in pipe

k_L= Loss coefficient

WATER HAMMER IN PIPES

Water and most fluids are comparatively incompressible and heavy. When they flow down a pipe, depending on the diameter and length, there is a weight of fluid in motion. If a valve at the end of the pipe is suddenly closed, the momentum of the fluid is changed and this will give rise to forces on the valve and within the pipe. This is called Water Hammer and can, depending upon the magnitude of the force, be very damaging.

Water hammer (or, more generally, fluid hammer) is a pressure surge or wave caused when a fluid (usually a liquid but sometimes also a gas) in motion is forced to stop or change direction suddenly (momentum change). A water hammer commonly occurs when a valve closes suddenly at an end of a pipeline system, and a pressure wave propagates in the pipe. It is also called hydraulic shock. This pressure wave can cause major problems, from noise and vibration to pipe collapse.

When a pipe is suddenly closed at the outlet (downstream), the mass of water before the closure is still moving, thereby building up high pressure and a resulting shock wave. In domestic plumbing this is experienced as a loud banging, resembling a hammering noise. Water hammer can cause pipelines to break if the pressure is high enough. Air traps or stand pipes (open at the top) are sometimes added as dampers to water systems to absorb the potentially damaging forces caused by the moving water.

On the other hand, when an upstream valve in a pipe closes, water downstream of the valve attempts to continue flowing, creating a vacuum that may cause the pipe to collapse or implode. This problem can be particularly acute if the pipe is on a downhill slope. To prevent this, air and vacuum relief valves, or air vents, are installed just downstream of the valve to allow air to enter the line for preventing this vacuum from occurring.

References

Text Books:
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Reference Books:
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2. Engineering Fluid Mechanics by K.L. Kumar, S. Chand & Co.
3. Fluid Mechanics by V.L. Streeter, MGH