Lecture Notes On Analogue Communication Techniques(Module 1 & 2)

<u>Topics Covered:</u>

1. **Spectral Analysis of Signals**
2. **Amplitude Modulation Techniques**
3. **Angle Modulation**
4. **Mathematical Representation of Noise**
5. **Noise in AM System**
6. **Noise in FM system**

Module-I (12 Hours)

Spectral Analysis: Fourier Series: The Sampling Function, The Response of a linear System, Normalized Power in a Fourier expansion, Impulse Response, Power Spectral Density, Effect of Transfer Function on Power Spectral Density, The Fourier Transform, Physical Appreciation of the Fourier Transform, Transform of some useful functions, Scaling, Time-shifting and Frequency shifting properties, Convolution, Parseval's Theorem, Correlation between waveforms, Auto-and cross correlation, Expansion in Orthogonal Functions, Correspondence between signals and Vectors, Distinguishability of Signals.

Module-II (14 Hours)

Amplitude Modulation Systems: A Method of frequency translation, Recovery of base band Signal, Amplitude Modulation, Spectrum of AM Signal, The Balanced Modulator, The Square law Demodulator, DSB-SC, SSB-SC and VSB, Their Methods of Generation and Demodulation, Carrier Acquisition, Phase-locked Loop (PLL),Frequency Division Multiplexing. Frequency Modulation Systems: Concept of Instantaneous Frequency, Generalized concept of Angle Modulation, Frequency modulation, Frequency Deviation, Spectrum of FM Signal with Sinusoidal Modulation, Bandwidth of FM Signal Narrowband and wideband FM, Bandwidth required for a Gaussian Modulated WBFM Signal, Generation of FM Signal, FM Demodulator, PLL, Pre-emphasis and De-emphasis Filters.

Module-III (12 Hours)

Mathematical Representation of Noise: Sources and Types of Noise, Frequency Domain Representation of Noise, Power Spectral Density, Spectral Components of Noise, Response of a Narrow band filter to noise, Effect of a Filter on the Power spectral density of noise, Superposition of Noise, Mixing involving noise, Linear Filtering, Noise Bandwidth, and Quadrature Components of noise. Noise in AM Systems: The AM Receiver, Super heterodyne Principle, Calculation of Signal Power and Noise Power in SSB-SC, DSB-SC and DSB, Figure of Merit ,Square law Demodulation, The Envelope Demodulation, Threshold

Module-IV (8 Hours)

Noise in FM System: Mathematical Representation of the operation of the limiter, Discriminator, Calculation of output SNR, comparison of FM and AM, SNR improvement using pre-emphasis, Multiplexing, Threshold infrequency modulation, The Phase locked Loop.

Text Books:

1. Principles of Communication Systems by Taub & Schilling, 2nd Edition. Tata Mc Graw Hill. Selected portion from Chapter1, 3, 4, 8, 9 & 10

2. Communication Systems by Siman Haykin,4th Edition, John Wiley and Sons Inc.

References Books:

1. Modern digital and analog communication system, by B. P. Lathi, 3rd Edition, Oxford University Press.

2. Digital and analog communication systems, by L.W.Couch, 6th Edition, Pearson Education, Pvt. Ltd.

Spectral Analysis of Signals

A signal under study in a communication system is generally expressed as a function of time or as a function of frequency. When the signal is expressed as a function of time, it gives us an idea of how that instantaneous amplitude of the signal is varying with respect to time. Whereas when the same signal is expressed as function of frequency, it gives us an insight of what are the contributions of different frequencies that compose up that particular signal. Basically a signal can be expressed both in time domain and the frequency domain. There are various mathematical tools that aid us to get the frequency domain expression of a signal from the time domain expression and vice-versa. *FourierSeries* is used when the signal in study is a periodic one, whereas *Fourier Transform* may be used for both periodic as well as non-periodic signals.

Fourier Series

Let the signal x(t) be a periodic signal with period T_0. The Fourier series of a signal can be obtained, if the following conditions known as the Dirichlet conditions are satisfied:

1. x(t) must be a single valued function of 't'.
2. x(t) is absolutely integrable over its domain, i.e.

$$\int_{-\infty}^{\infty} |x(t)| dt = 0$$

3. The number of maxima and minima of x(t) must be finite in its domain.
4. The number of discontinuities of x(t) must be finite in its domain.

A periodic function of time, say *x(t)* having a fundamental period T_0 can be represented as an infinite sum of sinusoidal waveforms, the summation being called as the *Fourier series* expansion of the signal.

$$x(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{2\pi n t}{T_0}\right) + \sum_{n=1}^{\infty} B_n \sin\left(\frac{2\pi n t}{T_0}\right)$$

Where A_0 is the average value of *v(t)* given by:

$$A_0 = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t)\, dt$$

And the coefficients A_n and B_n are given by

$$A_n = \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} x(t) \cos\left(\frac{2\pi n t}{T_0}\right) dt$$

$$B_n = \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} x(t) \sin\left(\frac{2\pi n t}{T_0}\right) dt$$

Alternate form of Fourier Series is

$$\mathrm{x(t)} = C_0 + \sum_{n=1}^{\infty} C_n \cos\left(\frac{2\pi nt}{T_0} - \phi_n\right)$$
$$C_0 = A_0$$
$$C_n = \sqrt{A_n^{\,2} + B_n^{\,2}}$$
$$\phi_n = \tan^{-1}\frac{B_n}{A_n}$$

The Fourier series hence expresses a periodic signal as an infinite summation of harmonics of fundamental frequency $f_0 = \dfrac{1}{T_0}$. The coefficients C_n are called spectral amplitudes i.e. C_n is the amplitude of the spectral component $C_n \cos\left(\dfrac{2\pi nt}{T_0} - \phi_n\right)$ at frequency nf_0. This form gives one sided spectral representation of a signal as shown in1st plot of Figure 1.

Exponential Form of Fourier Series

This form of Fourier series expansion can be expressed as :

$$\mathrm{x(t)} = \sum_{n=-\infty}^{\infty} V_n e^{j2\pi nt/T_0}$$
$$V_n = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \mathrm{x(t)} e^{j2\pi nt/T_0}\, dt$$

The spectral coefficients $V_n$and V_{-n} have the property that they are complex conjugates of each other $V_n = V_{-n}^{\;*}$. This form gives two sided spectral representation of a signal as shown in 2nd plot of Figure-1. The coefficients V_n can be related to C_n as :

$$V_0 = C_0$$
$$V_n = \frac{C_n}{2} e^{-j\phi_n}$$

The V_n*'s* are the spectral amplitude of spectral components $V_n e^{j2\pi ntf_0}$.

C_n

0 f_0 $2f_0$ $3f_0$ frequency→

V_n

$-3f_0$ $-2f_0$ $-f_0$ 0 f_0 $2f_0$ $3f_0$ frequency→

Figure 1 One sided and corresponding two sided spectral amplitude plot

The Sampling Function

The sampling function denoted as *Sa(x)* is defined as:

$$Sa(x) = \frac{\mathrm{Sin}(x)}{x}$$

And a similar function *Sinc(x)* is defined as :

$$Sinc(x) = \frac{Sin(\pi x)}{\pi x}$$

The *Sa(x)* is symmetrical about *x=0*, and is maximum at this point *Sa(x)=1*. It oscillates with an amplitude that decreases with increasing *x*. It crosses zero at equal intervals on *x* at every $x = \pm n\pi$, where *n* is an non-zero integer.

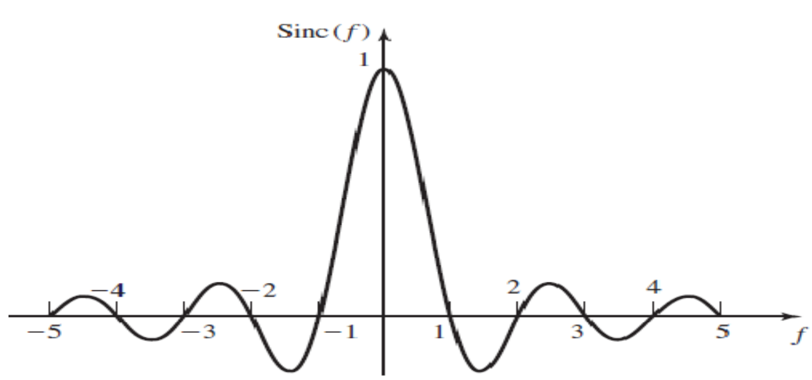


Figure 2 Plot of Sinc(f)

Fourier Transform

The Fourier transform is the extension of the Fourier series to the general class of signals (periodic and nonperiodic). Here, as in Fourier series, the signals are expressed in terms of complex exponentials of various frequencies, but these frequencies are not discrete. Hence, in this case, the signal has a continuous spectrum as opposed to a discrete spectrum. Fourier Transform of a signal *x(t)* can be expressed as:

$$F[\mathrm{x(t)}] = \mathrm{X(f)} = \int_{-\infty}^{\infty} x(\mathrm{t})\,\mathrm{e}^{-j2\pi ft}\,dt$$

$x(\mathrm{t}) \Leftrightarrow \mathrm{X(f)}$ represents a Fourier Transform pair

The time-domain signal *x(t)* can be obtained from its frequency domain signal *X(f)* by Fourier inverse defined as:

$$x(\mathrm{t}) = \mathrm{F}^{-1}\left[X(\mathrm{f})\right] = \int_{-\infty}^{\infty} X(\mathrm{f})e^{j2\pi ft}df$$

When frequency is defined in terms of angular frequency ω ,then Fourier transform relation can be expressed as:

$$F[\mathrm{x(t)}] = \mathrm{X}(\omega) = \int_{-\infty}^{\infty} x(\mathrm{t})\,\mathrm{e}^{-j\omega t}\,dt$$

and

$$x(\mathrm{t}) = \mathrm{F}^{-1}\left[X(\omega)\right] = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$$

Properties of Fourier Transform

Let there be signals x(t) and y(t) ,with their Fourier transform pairs:

$$x(\mathrm{t}) \Leftrightarrow \mathrm{X(f)}$$
$$\mathrm{y(t)} \Leftrightarrow \mathrm{Y(f)}\ \text{then,}$$

1. **Linearity Property**
   $ax(\mathrm{t}) + \mathrm{by(t)} \Leftrightarrow \mathrm{aX(f)} + \mathrm{bY(f)}$, where *a* and *b* are the constants

2. **Duality Property**
   $X(\mathrm{t}) \Leftrightarrow x(-f)\ \text{or}$
   $\mathrm{X(t)} \Leftrightarrow 2\pi\mathrm{X}(-\omega)$

3. **Time Shift Property**
   $x(\mathrm{t}-\mathrm{t}_0) \Leftrightarrow \mathrm{e}^{-j2\pi f t_0}X(\mathrm{f})$

4. **Time Scaling Property**

$$x(\mathrm{at}) \Leftrightarrow \frac{1}{|a|} X\left(\frac{f}{a}\right)$$

5. **Convolution Property:** If convolution operation between two signals is defined as:

$$x(\mathrm{t}) \otimes \mathrm{y}(\mathrm{t}) = \int_{-\infty}^{\infty} x\left(\tau\right) x\left(t-\tau\right) d\tau \text{ , then}$$

$$x(\mathrm{t}) \otimes \mathrm{y}(\mathrm{t}) \Leftrightarrow \mathrm{X(f)\,Y(f)}$$

6. **Modulation Property**

$$e^{j2\pi f_0 t} x(\mathrm{t}) \Leftrightarrow \mathrm{X(f-f_0)}$$

7. **Parseval's Property**

$$\int_{-\infty}^{\infty} x(\mathrm{t})\,\mathrm{y}^*(\mathrm{t})\,\mathrm{dt} = \int_{-\infty}^{\infty} X(\mathrm{f})\,\mathrm{Y}^*(\mathrm{f}) df$$

8. **Autocorrelation Property:** If the time autocorrelation of signal *x(t)* is expressed as:

$$R_x(\tau) = \int_{-\infty}^{\infty} x(\mathrm{t})\,\mathrm{x}*(\mathrm{t}-\tau) dt \text{ ,then}$$

$$R_x(\tau) \Leftrightarrow \left|X(\mathrm{f})\right|^2$$

9. **Differentiation Property:**

$$\frac{d}{dt} x(\mathrm{t}) \Leftrightarrow j2\pi f X(\mathrm{f})$$

Response of a linear system

The reason what makes Trigonometric Fourier Series expansion so important is the unique characteristic of the sinusoidal waveform that such a signal always represent a particular frequency. When any linear system is excited by a sinusoidal signal, the response also is a sinusoidal signal of same frequency. In other words, a sinusoidal waveform preserves its wave-shape throughout a linear system. Hence the response-excitation relationship for a linear system can be characterised by, how the response amplitude is related to the excitation amplitude (amplitude ratio) and how the response phase is related to the excitation phase (phase difference) for a particular frequency. Let the input to a linear system be :

$$v_i\left(t,\omega_n\right) = V_n e^{j\omega_n t}$$

Then the filter output is related to this input by the *Transfer Function* (characteristic of the Linear Filter): $H\left(\omega_n\right) = \left|H\left(\omega_n\right)\right| e^{-j\theta\left(\omega_n\right)}$, such that the filter output is given as

$$v_o\left(t,\omega_n\right) = V_n \left|H\left(\omega_n\right)\right| e^{j\left(\omega_n t - j\theta\left(\omega_n\right)\right)}$$

Normalised Power

While discussing communication systems, rather than the absolute power we are interested in another quantity called Normalised Mean Power. It is an average power normalised across a 1 ohm resistor, averaged over a single time-period for a periodic signal. In general irrespective of the fact, whether it is a periodic or non-periodic signal, average normalised power of a signal *v(t)* is expressed as :

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{\frac{-T}{2}}^{\frac{T}{2}} v^2(t) dt$$

Energy of signal

For a continuous-time signal, the energy of the signal is expressed as:

$$E = \int_{-\infty}^{\infty} x^2(\mathrm{t}) dt$$

A signal is called an ***Energy Signal*** if

$$0 < E < \infty$$
$$P = 0$$

A signal is called ***Power Signa***l if

$$0 < P < \infty$$
$$E = \infty$$

Normalised Power of a Fourier Expansion

If a periodic signal can be expressed as a Fourier Series expansion as:

$$v(t) = C_0 + C_1 \cos(2\pi f_0 t) + C_2 \cos(4\pi f_0 t) + \ldots$$

Then, its normalised average power is given by :

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{\frac{-T}{2}}^{\frac{T}{2}} v^2(t) dt$$

Integral of the cross-product terms become zero, since the integral of a product of orthogonal signals over period is zero. Hence the power expression becomes:

$$P = C_0^{\,2} + \frac{C_1^2}{2} + \frac{C_2^{\,2}}{2} + \ldots$$

By generalisation, normalised average power expression for entire Fourier Series becomes:

$$P = C_0^2 + \sum_{n=1}^{\infty} \frac{C_n^2}{2} + ...$$

In terms of trigonometric Fourier coefficients A_n's, B_n's, the power expression can be written as:

$$P = A_0^2 + \sum_{n=1}^{\infty} A_n^2 + \sum_{n=1}^{\infty} B_n^2$$

In terms of complex exponential Fourier series coefficients V_n's, the power expressions becomes:

$$P = \sum_{n=-\infty}^{\infty} V_n V_n^*$$

Energy Spectral Density(*ESD*)

It can be proved that energy ***E*** of a signal *x(t)* is given by :

$$E = \int_{-\infty}^{\infty} x^2(\mathrm{t})dt = \int_{-\infty}^{\infty} |X(\mathrm{f})|^2 df \rightarrow \textit{Parseval's Theorem for energy signals}$$

So, $E = \int_{-\infty}^{\infty} \psi(\mathrm{f})df$, where $\psi(\mathrm{f}) = |X(\mathrm{f})|^2 \rightarrow$ *Energy Spectral Density*

The above expression says that $\psi(\mathrm{f})$ integrated over all of the frequencies, gives the total energy of the signal. Hence *Energy Spectral Density* (***ESD***) quantifies the energy contribution from every frequency component in the signal, and is a function of frequency.

Power Spectral Density(*PSD*)

It can be proved that the average normalised power ***P*** of a signal *x(t),*such that $x_\tau(\mathrm{t})$ is a truncated and periodically repeated version of *x(t)* such that $x_\tau(\mathrm{t}) = \begin{cases} x(\mathrm{t}); \frac{-\tau}{2} < t < \frac{\tau}{2} \\ 0; elsewhere \end{cases}$ is given by :

$$P = \lim_{\tau \to \infty} \frac{1}{\tau} \int_{\frac{-\tau}{2}}^{\frac{\tau}{2}} x^2(t)dt = \lim_{\tau \to \infty} \frac{1}{\tau} \int_{\frac{-\tau}{2}}^{\frac{\tau}{2}} |X_\tau(t)|^2 dt \rightarrow \textit{Parseval's Theorem for power signals}$$

So, $P = \int_{-\infty}^{\infty} \mathrm{S}(\mathrm{f})df$, where $S(\mathrm{f}) = \lim_{\tau \to \infty} \frac{|X_\tau(\mathrm{f})|^2}{\tau} \rightarrow$ *Power Spectral Density*

The above expression says that *S(f)*integrated over all of the frequencies, gives the total *normalised power* of the signal. Hence *Power Spectral Density* (***PSD***) quantifies the power contribution from every frequency component in the signal, and is a function of frequency.

Expansion in Orthogonal Functions

Let there be a set of functions $g_1(x), g_2(x), g_3(x), \ldots, g_n(x)$, defined over the interval $x_1 < x < x_2$ and such that any two functions of the set have a special relation:

$$\int_{x_1}^{x_2} g_i(x)\, g_j(x)\, dx = 0 \ \cdot$$

The set of functions showing the above property are said to be an *orthogonal set of functions* in the interval $x_1 < x < x_2$. We can then write a function $f(x)$ in the same interval $x_1 < x < x_2$, as a linear sum of such $g_n(x)$'s as:

$f(x) = C_1 g_1(x) + C_2\, g_2(x) + C_3\, g_3(x) + \ldots + C_n\, g_n(x)$, where C_n's are the numerical coefficients

The numerical value of any coefficient C_n can be found out as:

$$C_n = \frac{\int_{x_1}^{x_2} f(x)\, g_n(x) dx}{\int_{x_1}^{x_2} g_n^{\,2}(x)\, dx}$$

In a special case when the functions $g_n(x)$ in the set are chosen such that $\int_{x_1}^{x_2} g_n^{\,2}(x)\, dx = 1$, then such a set is called as a set of *orthonormal functions*, that is the functions are orthogonal to each other and each one is a normalised function too.

<u>Amplitude Modulation Systems</u>

In communication systems, we often need to design and analyse systems in which many independent message can be transmitted simultaneously through the same physical transmission channel. It is possible with a technique called *frequency division multiplexing*, in which each message is translated in frequency to occupy a different range of spectrum. This involves an auxiliary signal called *carrier* which determines the amount of frequency translation. It requires modulation, in which either the amplitude, frequency or phase of the carrier signal is varied as according to the instantaneous value of the message signal. The resulting signal then is called a modulated signal. When the amplitude of the carrier is changed as according to the instantaneous value of the message/baseband signal, it results in *Amplitude Modulation*. The systems implementing such modulation are called as Amplitude modulation systems.

Frequency Translation

Frequency translation involves translating the signal from one region in frequency to another region. A signal band-limited in frequency lying in the frequencies from f_1 to f_2, after frequency translation can be translated to a new range of frequencies from $f_1^{'}$ to $f_2^{'}$. The information in the original message signal at baseband frequencies can be recovered back even from the frequency-translated signal. The advantagesof frequency translation are as follows:

1. <u>Frequency Multiplexing</u>: In a case when there are more than one sources which produce band-limited signals that lie in the same frequency band. Such signals if transmitted as such simultaneously through a transmission channel, they will interfere with each other and cannot be recovered back at the intended receiver. But if each signal is translated in frequency such that they encompass different ranges of frequencies, not interfering with other signal spectrums, then each signal can be separated back at the receiver with the use of proper filters. The output of filters then can be suitably processed to get back the original message signal.
2. <u>Practicability of antenna</u>: In a wireless medium, antennas are used to radiate and to receive the signals. The antenna operates effectively, only when the dimension of the antenna is of the order of magnitude of the wavelength of the signal concerned. At baseband low frequencies, wavelength is large and so is the dimension of antenna required is impracticable. By frequency translation, the signal can be shifted in frequency to higher range of frequencies. Hence the corresponding wavelength is small to the extent that the dimension of antenna required is quite small and practical.
3. <u>Narrow banding</u>: For a band-limited signal, an antenna dimension suitable for use at one end of the frequency range may fall too short or too large for use at another end of the frequency range. This happens when the ratio of the highest to lowest frequency contained in the signal is large (wideband signal). This ratio can be reduced to close around one by translating the signal to a higher frequency range, the resulting signal being called as a narrow-banded signal. Narrowband signal works effectively well with the same antenna dimension for both the higher end frequency as well as lower end frequency of the band-limited signal.
4. <u>Common Processing</u>: In order to process different signals occupying different spectral ranges but similar in general character, it may always be necessary to adjust the frequency range of operation of the apparatus. But this may be avoided, by keeping the frequency range of operation of the apparatus constant, and instead every time the signal of interest beingtranslated down to the operating frequency range of the apparatus.

Amplitude Modulation Types:

1. Double-sideband with carrier (DSB+C)
2. Double-sideband suppressed carrier (DSB-SC)
3. Single-sideband suppressed carrier (SSB-SC)
4. Vestigial sideband (VSB)

Double-sideband with carrier (DSB+C)

Let there be a sinusoidal carrier signal $c(t) = A Cos(2\pi f_c t)$, of frequency f_c . With the concept of amplitude modulation, the instantaneous amplitude of the carrier signal will be modulated (changed) proportionally according to the instantaneous amplitude of the baseband or modulating signal *x(t)*. So the expression for the Amplitude Modulated (AM) wave becomes:

$$s(t) = \left[A + x(t)\right] Cos(2\pi f_c t) = E(t) Cos(2\pi f_c t)$$

$$E(t) = A + x(t)$$

The time varying amplitude *E(t)* of the AM wave is called as the envelope of the AM wave. The envelope of the AM wave has the same shape as the message signal or baseband signal.

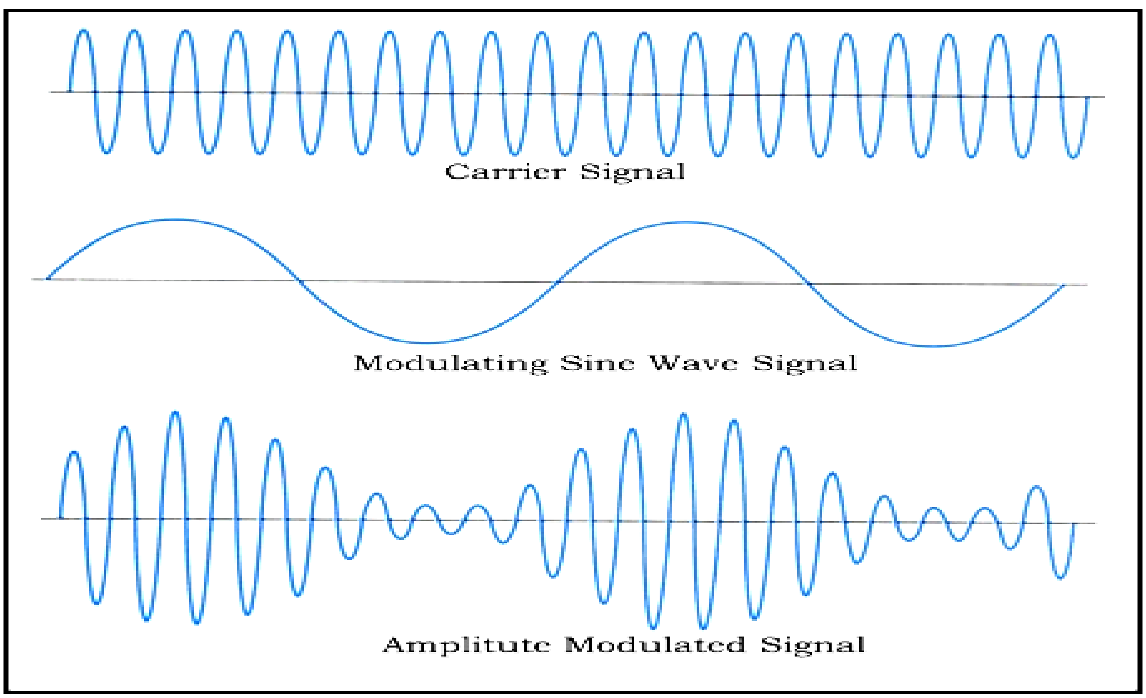


Figure 3 Amplitude modulation time-domain plot

Modulation Index (m_a): It is defined as the measure of extent of amplitude variation about unmodulated maximum carrier amplitude. It is also called as depth of modulation, degree of modulation or modulation factor.

$$m_a = \frac{|x(t)|_{max}}{A}$$

On the basis of modulation index, AM signal can be from any of these cases:

I. $m_a > 1$: Here the maximum amplitude of baseband signal exceeds maximum carrier amplitude, $|x(t)|_{max} > A$. In this case, the baseband signal is not preserved in the AM envelope, hence baseband signal recovered from the envelope will be distorted.

II. $m_a \leq 1$: Here the maximum amplitude of baseband signal is less than carrier amplitude $|x(t)|_{max} \leq A$. The baseband signal is preserved in the AM envelope.

Spectrum of Double-sideband with carrier (DSB+C)

Let *x(t)* be a bandlimited baseband signal with maximum frequency content f_m. Let this signal modulate a carrier $c(t) = ACos(2\pi f_c t)$ .Then the expression for AM wave in time-domain is given by:

$$\begin{aligned} s(t) &= [A + x(t)]Cos(2\pi f_c t) \\ &= ACos(2\pi f_c t) + x(t)Cos(2\pi f_c t) \end{aligned}$$

Taking the Fourier transform of the two terms in the above expression will give us the spectrum of the DSB+C AM signal.

$$ACos(2\pi f_c t) \Leftrightarrow \frac{1}{2}[\delta(f+f_c) + \delta(f-f_c)]$$

$$x(t)Cos(2\pi f_c t) \Leftrightarrow \frac{1}{2}[X(f+f_c) + X(f-f_c)]$$

So, first transform pair points out two impulses at $f = \pm f_c$, showing the presence of carrier signal in the modulated waveform. Along with that, the second transform pair shows that the AM signal spectrum contains the spectrum of original baseband signal shifted in frequency in both negative and positive direction by amount f_c . The portion of AM spectrum lying from f_c to $f_c + f_m$ in positive frequency and from $-f_c$ to $-f_c - f_m$ in negative frequency represent the *Upper Sideband(USB).* The portion of AM spectrum lying from $f_c - f_m$ to f_c in positive frequency and from $-f_c + f_m$ to $-f_c$ in negative frequency represent the *Lower Sideband(LSB).* Total AM signal spectrum spans a frequency from $f_c - f_m$ to $f_c + f_m$, hence has a bandwidth of $2f_m$.

Power Content in AM Wave

By the general expression of AM wave:

$$s(t) = ACos(2\pi f_c t) + x(t)Cos(2\pi f_c t)$$

Hence, total average normalised power of an AM wave comprises of the carrier power corresponding to first term and sideband power corresponding to second term of the above expression.

$$P_{total} = P_{carrier} + P_{sideband}$$

$$P_{carrier} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} A^2 Cos^2(2\pi\, \mathrm{f}_c \mathrm{t})dt = \frac{A^2}{2}$$

$$P_{sideband} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(\mathrm{t}) Cos^2(2\pi\, \mathrm{f}_c \mathrm{t})dt = \frac{1}{2}\overline{x^2(\mathrm{t})}$$

In the case of single-tone modulating signal where $x(\mathrm{t}) = \mathrm{V}_m Cos(2\pi \mathrm{f}_m \mathrm{t})$:

$$P_{carrier} = \frac{A^2}{2}$$

$$P_{sideband} = \frac{V_m^{\ 2}}{4}$$

$$P_{total} = P_{carrier} + P_{sideband} = \frac{A^2}{2} + \frac{V_m^{\ 2}}{4}$$

$$\Rightarrow P_{total} = P_{carrier}\left[1 + \frac{m_a^{\ 2}}{2}\right]$$

Where, m_a is the modulation index given as $m_a = \frac{V_m}{A}$.

Net Modulation Index for Multi-tone Modulation: If modulating signal is a multitone signal expressed in the form:

$$x(\mathrm{t}) = \mathrm{V}_1 Cos(2\pi \mathrm{f}_1 \mathrm{t}) + \mathrm{V}_2 Cos(2\pi \mathrm{f}_2 \mathrm{t}) + \mathrm{V}_3 Cos(2\pi \mathrm{f}_3 \mathrm{t}) + ... + \mathrm{V}_n Cos(2\pi \mathrm{f}_n \mathrm{t})$$

Then, $P_{total} = P_{carrier}\left[1 + \frac{m_1^{\ 2}}{2} + \frac{m_2^{\ 2}}{2} + \frac{m_3^{\ 2}}{2} ... \frac{m_n^{\ 2}}{2}\right]$

Where $m_1 = \frac{V_1}{A}, m_2 = \frac{V_2}{A}, m_3 = \frac{V_2}{A}, ..., m_n = \frac{V_n}{A}$

Generation of DSB+C AM by Square Law Modulation

Square law diode modulation makes use of non-linear current-voltage characteristics of diode. This method is suited for low voltage levels as the current-voltage characteristic of diode is highly non-linear in the low voltage region. So the diode is biased to operate in this non-linear region for this application. A DC battery V_c is connected across the diode to get such a operating point on the characteristic. When the carrier and modulating signal are applied at the input of diode, different frequency terms appear at the output of the diode. These when applied across a tuned circuit tuned to carrier frequency and a narrow bandwidth just to allow the two pass-bands, the output has the carrier and the sidebands only which is essentially the DSB+C AM signal.

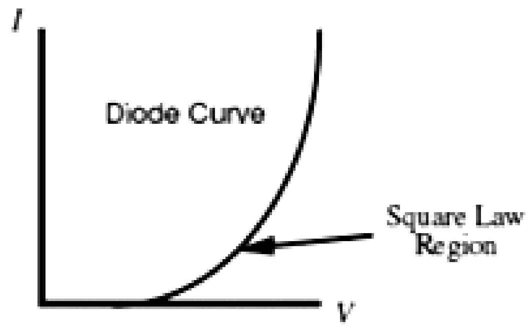


Figure 4 Current-voltage characteristic of diode

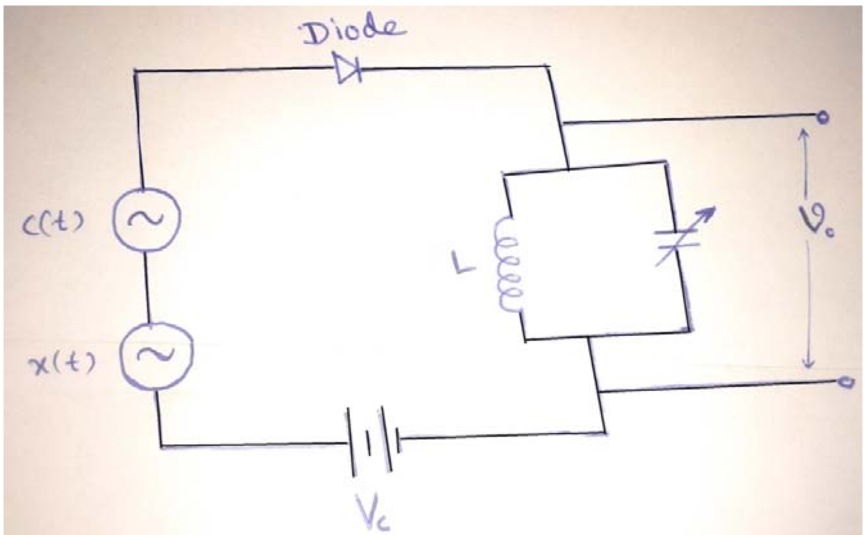


Figure 5 Square Law Diode Modulator

The non-linear current voltage relationship can be written in general as:

$$i = av + bv^2$$

In this application $v = c(t) + x(t)$

So

$$i = a[\mathrm{ACos}(2\pi f_c t) + x(t)] + b[\mathrm{ACos}(2\pi f_c t) + x(t)]^2$$

$$\Rightarrow i = a\,\mathrm{ACos}(2\pi f_c t) + a\,x(t) + bA^2\,\mathrm{Cos}^2(2\pi f_c t) + b\,x^2(t) + 2bA\,x(t)\,\mathrm{Cos}(2\pi f_c t)$$

$$\Rightarrow i = \boxed{a\,\mathrm{ACos}(2\pi f_c t)} + a\,x(t) + \frac{bA^2}{2}\mathrm{Cos}(2\pi(2f_c)t) + \frac{bA^2}{2} + b\,x^2(t) + \boxed{2bA\,x(t)\,\mathrm{Cos}(2\pi f_c t)}$$

Out of the above frequency terms, only the boxed terms have the frequencies in the passband of the tuned circuit, and hence will be at the output of the tuned circuit. There is carrier frequency term and the sideband term which comprise essentially a DSB+C AM signal.

Demodulation of DSB+C by Square Law Detector

It can be used to detect modulated signals of small magnitude, so that the operating point may be chosen in the non-linear portion of the V-I characteristic of diode. A DC supply voltage is used to get a fixed operating point in the non-linear region of diode characteristics. The output diode current is hence

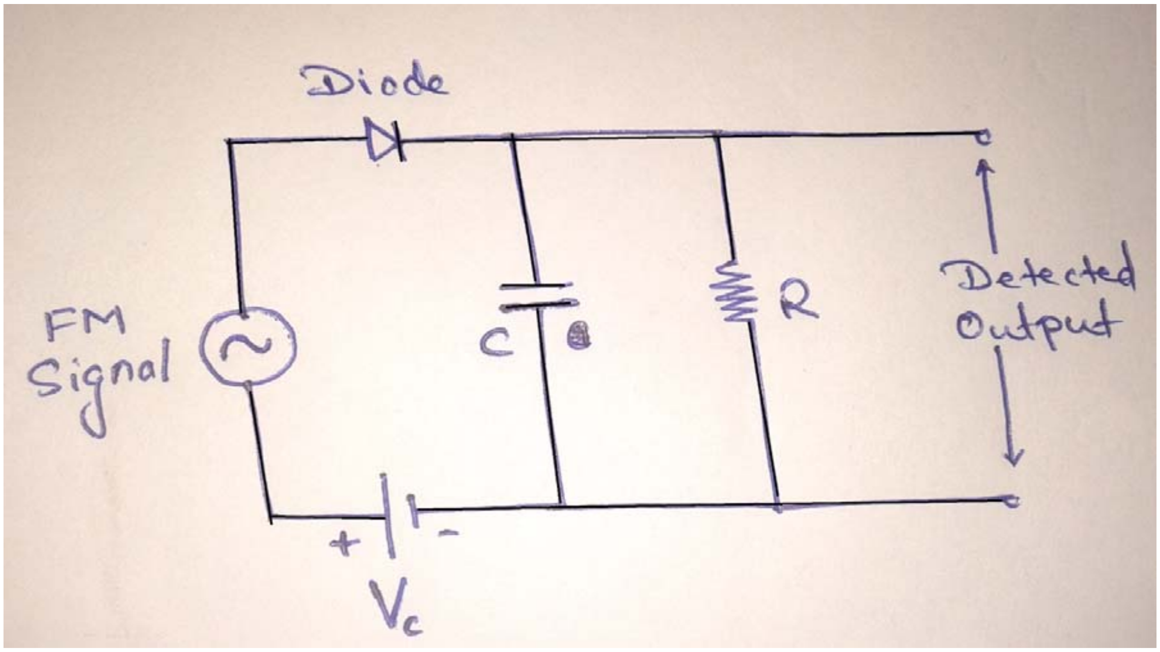


Figure 6 Square Law Detector

given by the non-linear expression:

$$i = av_{FM}(\mathrm{t}) + \mathrm{b}v^2_{FM}(\mathrm{t})$$

Where $v_{FM}(\mathrm{t}) = [\mathrm{A} + x(\mathrm{t})]\mathrm{Cos}(2\pi \mathrm{f}_c \mathrm{t})$

This current will have terms at baseband frequencies as well as spectral components at higher frequencies. The low pass filter comprised of the RC circuit is designed to have cut-off frequency as the highest modulating frequency of the band limited baseband signal. It will allow only the baseband frequency range, so the output of the filter will be the demodulated baseband signal.

Linear Diode Detector or Envelope Detector

This is essentially just a half-wave rectifier which charges a capacitor to a voltage to the peak voltage of the incoming AM waveform. When the input wave's amplitude increases, the capacitor voltage is increased via the rectifying diode quickly, due a very small RC time-constant (negligible *R*) of the charging path. When the input's amplitude falls, the capacitor voltage is reduced by being discharged by a 'bleed' resistor *R* which causes a considerable RC time constant in the discharge path making discharge process a slower one as compared to charging. The voltage across *C* does not fall appreciably during the small period of negative half-cycle, and by the time next positive half cycle appears. This cycle again charges the capacitor *C* to peak value of carrier voltage and thus this process repeats on. Hence the output voltage across capacitor *C* is a spiky envelope of the AM wave, which is same as the amplitude variation of the modulating signal.

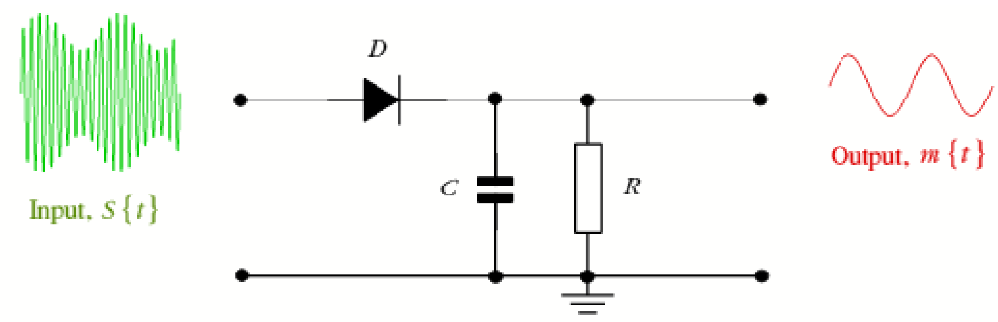


Figure 7 Envelope Detector

Double Sideband Suppressed Carrier(DSB-SC)

If the carrier is suppressed and only the sidebands are transmitted, this will be a way to saving transmitter power. This will not affect the information content of the AM signal as the carrier component of AM signal do not carry any information about the baseband signal variation. A DSB+C AM signal is given by:

$$s_{DSB+C}(t) = ACos(2\pi f_c t) + x(t)Cos(2\pi f_c t)$$

So, the expression for DSB-SC where the carrier has been suppressed can be given as:

$$s_{DSB-SC}(t) = x(t)Cos(2\pi f_c t)$$

Therefore, a DSB-SC signal is obtained by simply multiplying modulating signal *x(t)* with the carrier signal. This is accomplished by a **product modulator** or **mixer**.

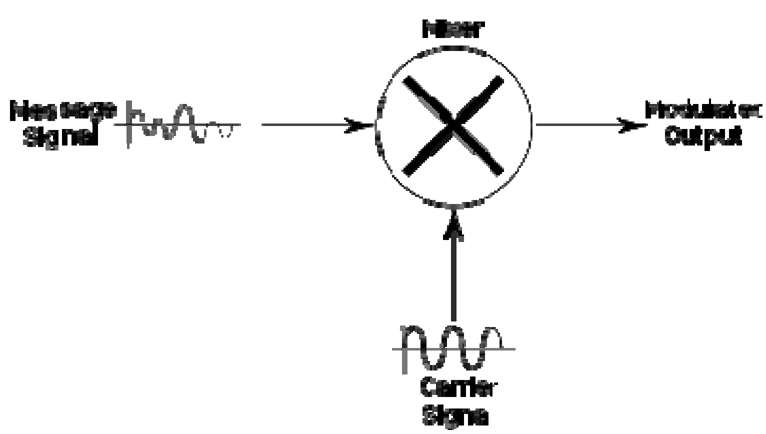


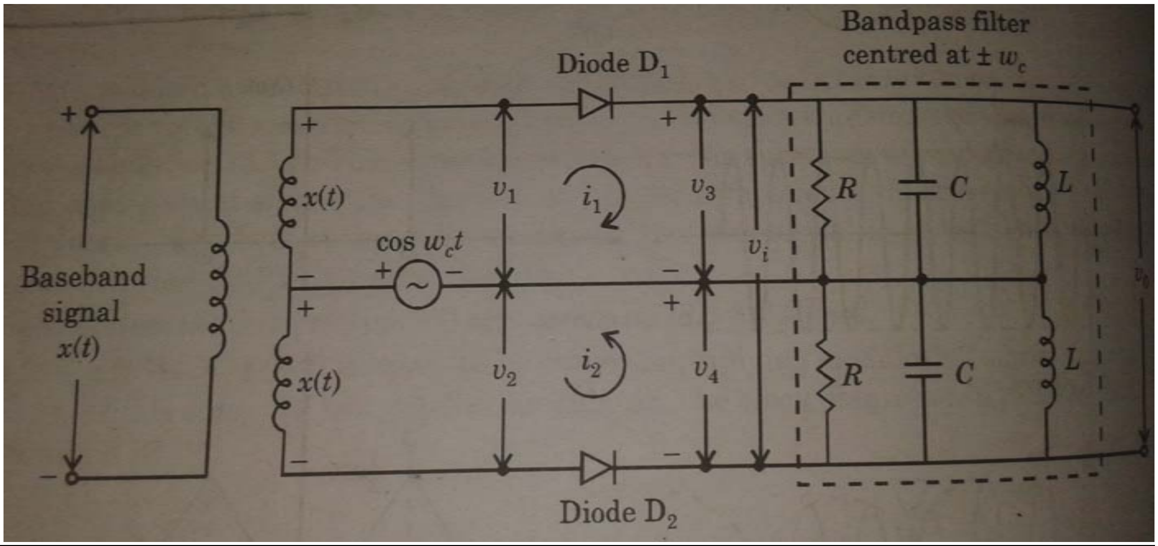
Figure 8 Product Modulator

Difference from the the DSB+C being only the absence of carrier component, and since DSBSC has still both the sidebands, spectral span of this DSBSC wave is still $f_c - f_m$ to $f_c + f_m$, hence has a bandwidth of $2f_m$.

Generation of DSB-SC Signal

A circuit which can produce an output which is the product of two signals input to it is called a product modulator. Such an output when the inputs are the modulating signals and the carrier signal is a DSBSC signal. One such product modulator is a balanced modulator.

Balanced modulator:



$$v_1 = Cos(2\pi f_c t) + x(t)$$
$$v_2 = Cos(2\pi f_c t) - x(t)$$

For diode D_1, the nonlinear v-i relationship becomes:

$$i_1 = av_1 + bv_1^2 = a[Cos(2\pi f_c t) + x(t)] + b[Cos(2\pi f_c t) + x(t)]^2$$

Similarly, for diode D_2,

$$i_2 = av_2 + bv_2^2 = a[Cos(2\pi f_c t) - x(t)] + b[Cos(2\pi f_c t) - x(t)]^2$$

Now, $v_i = v_3 - v_4 = (i_1 - i_2)R$ (substituting for i_1 and i_2)

$$\Rightarrow v_i = 2R[ax(t) + 2\,bx(t)\,Cos(2\pi f_c t)]$$

This voltage is input to the bandpass filter centre frequency f_c and bandwidth $2f_m$. Hence it allows the component corresponding to the second term of the v_i, which is our desired DSB-SC signal.

Demodulation of DSBSC signal

Synchronous Detection: DSB-SC signal is generated at the transmitter by frequency up-translating the baseband spectrum by the carrier frequency f_c. Hence the original baseband signal can be recovered by frequency down-translating the received modulated signal by the same amount. Recovery can be achieved by multiplying the received signal by synchronous carrier signal and then low-pass filtering.

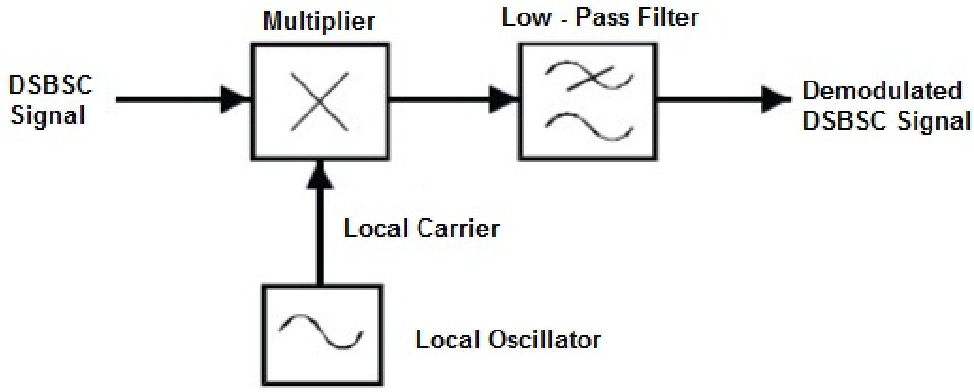


Figure 9 Synchronous Detection of DSBSC

Let the received DSB-SC signal is :

$$r(t) = x(t)Cos(2\pi f_c t)$$

So after carrier multiplication, the resulting signal:

$$e(t) = x(t)\,Cos(2\pi\, f_c t).Cos(2\pi\, f_c t)$$
$$\Rightarrow e(t) = x(t)\,Cos^2(2\pi\, f_c t)$$
$$\Rightarrow e(t) = \frac{1}{2}x(t)\left[1 + Cos(2\pi(2\,f_c)\,t)\right]$$
$$\Rightarrow e(t) = \frac{1}{2}x(t) + \frac{1}{2}x(t)\,Cos(2\pi(2\,f_c)\,t)$$

The low-pass filter having cut-off frequency f_m will only allow the baseband term $\frac{1}{2}x(t)$, which is in the pass-band of the filter and is the demodulated signal.

Single Sideband Suppressed Carrier (SSB-SC) Modulation

The lower and upper sidebands are uniquely related to each other by virtue of their symmetry about carrier frequency. If an amplitude and phase spectrum of either of the sidebands is known, the other sideband can be obtained from it. This means as far as the transmission of information is concerned, only one sideband is necessary. So bandwidth can be saved if only one of the sidebands is transmitted, and such a AM signal even without the carrier is called as Single Sideband Suppressed Carrier signal. It takes half as much bandwidth as DSB-SC or DSB+C modulation scheme.

For the case of single-tone baseband signal, the DSB-SC signal will have two sidebands :

The lower side-band: $Cos(2\pi(f_c - f_m)t) = Cos(2\pi f_m t)Cos(2\pi f_c t) + Sin(2\pi f_m t)Sin(2\pi f_c t)$

And the upper side-band: $Cos(2\pi(f_c + f_m)t) = Cos(2\pi f_m t)Cos(2\pi f_c t) - Sin(2\pi f_m t)Sin(2\pi f_c t)$

If any one of these sidebands is transmitted, it will be a SSB-SC waveform:

$$s(t)_{SSB} = Cos(2\pi f_m t)Cos(2\pi f_c t) \pm Sin(2\pi f_m t)Sin(2\pi f_c t)$$

Where (+) sign represents for the lower sideband, and (-) sign stands for the upper sideband. The modulating signal here is $x(t) = Cos(2\pi f_m t)$, so let $x_h(t) = Sin(2\pi f_m t)$ be the Hilbert Transform of $x(t)$. The Hilbert Transform is obtained by simply giving $\left(-\frac{\pi}{2}\right)$ to a signal. So the expression for SSB-SC signal can be written as:

$$s(t)_{SSB} = x(t)Cos(2\pi f_c t) \pm x_h(t)Sin(2\pi f_c t)$$

Where $x_h(t)$ is a signal obtained by shifting the phase of every component present in $x(t)$ by $\left(-\frac{\pi}{2}\right)$.

Generation of SSB-SC signal

Frequency Discrimination Method:

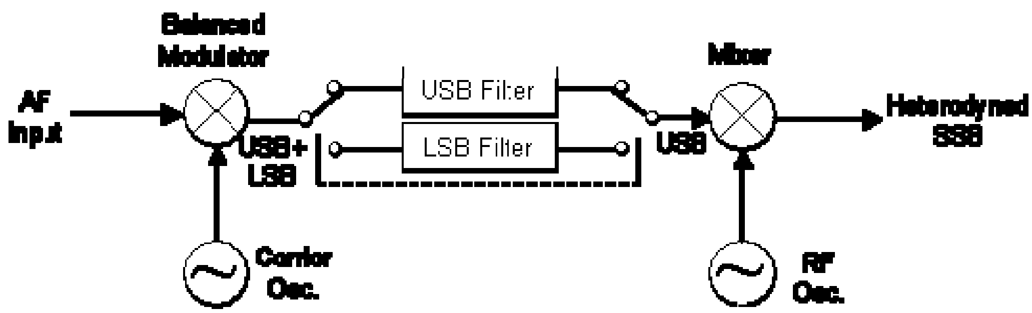


Figure 10 Frequency Discrimination Method of SSB-SC Generation

The filter method of SSB generation produces double sideband suppressed carrier signals (using a balanced modulator), one of which is then filtered to leave USB or LSB. It uses two filters that have different passband centre frequencies for USB and LSB respectively. The resultant SSB signal is then mixed (heterodyned) to shift its frequency higher.

Limitations:

I. This method can be used with practical filters only if the baseband signal is restricted at its lower edge due to which the upper and lower sidebands do not overlap with each other. Hence it is used for speech signal communication where lowest spectral component is 70 Hz and it may be taken as 300 Hz without affecting the intelligibility of the speech signal.
II. The design of band-pass filter becomes quite difficult if the carrier frequency is quite higher than the bandwidth of the baseband signal.

Phase-Shift Method:

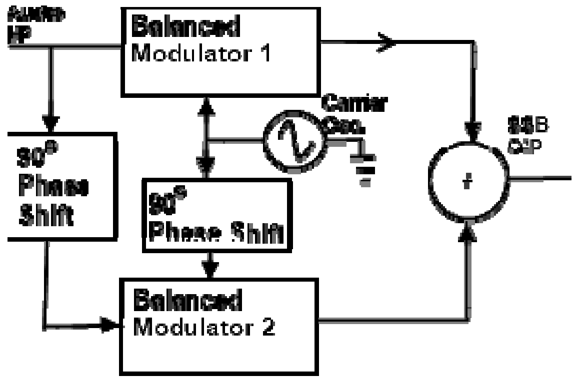
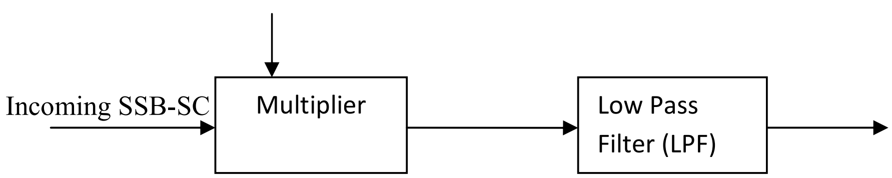


Figure 11 Phase shift method of SSB-SC generation

The phase shifting method of SSB generation uses a phase shift technique that causes one of the side bands to be cancelled out. It uses two balanced modulators instead of one. The balanced modulators effectively eliminate the carrier. The carrier oscillator is applied directly to the upper balanced modulator along with the audio modulating signal. Then both the carrier and modulating signal are shifted in phase by 90o and applied to the second, lower, balanced modulator. The two balanced modulator output are then added together algebraically. The phase shifting action causes one side band to be cancelled out when the two balanced modulator outputs are combined.

Demodulation of SSB-SC Signals:

The baseband or modulating signal x(t) can be recovered from the SSB-SC signal by using synchronous detection technique. With the help of synchronous detection method, the spectrum of an SSB-SC signal centered about , is retranslated to the basedand spectrum which is centered about . The process of synchronous detection involves multiplication of the received SSB-SC signal with a locally generated carrier.



The output of the multiplier will be

or

or

or – –

or – –

When $e_d(t)$ is passed through a low-pass filter, the terms centre at $\pm\omega_c$ are filtered out and the output of detector is only the baseband part i.e. $\frac{1}{2}x(\mathrm{t})$.

Vestigial Sideband Modulation(VSB)

SSB modulation is suited for transmission of voice signals due to the energy gap that exists in the frequency range from zero to few hundred hertz. But when signals like video signals which contain significant frequency components even at very low frequencies, the USB and LSB tend to meet at the carrier frequency. In such a case one of the sidebands is very difficult to be isolated with the help of practical filters. This problem is overcome by the Vestigial Sideband Modulation. In this modulation technique along with one of the sidebands, a gradual cut of the other sideband is also allowed which comes due to the use of practical filter. This cut of the other sideband is called as the *vestige*. Bandwidth of VSB signal is given by :

$$BW = (f_c + f_v) - (f_c - f_m) = f_m + f_v$$

Where $f_m \rightarrow$ bandwidth of bandlimited message signal

$f_v \rightarrow$ width of the vestige in frequency

Angle Modulation

Angle modulation may be defined as the process in which the total phase angle of a carrier wave is varied in accordance with the instantaneous value of the modulating or message signal, while amplitude of the carrier remain unchanged. Let the carrier signal be expressed as:

$$c(\mathrm{t}) = \mathrm{ACos}(2\pi\,\mathrm{f}_c\mathrm{t} + \theta)$$

Where $\phi = 2\pi\,\mathrm{f}_c\mathrm{t} + \theta \rightarrow$ Total phase angle

$\theta \rightarrow$ phase offset

$f_c \rightarrow$ carrier frequency

So in-order to modulate the total phase angle according to the baseband signal, it can be done by either changing the instantaneous carrier frequency according to the modulating signal- the case of *Frequency Modulation,* or by changing the instantaneous phase offset angle according to the modulating signal- the case of *Phase Modulation*. An angle-modulated signal in general can be written as

$$u(t) = ACos(\phi(t))$$

where, $\phi(\mathrm{t})$ is the total phase of the signal, and its instantaneous frequency $f_i(\mathrm{t})$ is given by

$$f_i(t) = \frac{1}{2\pi}\frac{d}{dt}\phi(t)$$

Since *u(t)* is a band-pass signal, it can be represented as

$$u(t) = ACos\left(2\pi f_c t + \theta(t)\right)$$

and, therefore instantaneous frequency f_i becomes :

$$f_i(t) = f_c + \frac{1}{2\pi}\frac{d}{dt}\theta(t)$$

For angle modulation, total phase angle can modulated either by making the instantaneous frequency or the phase offset to vary linearly with the modulating signal.

Let *m(t)* be the message signal, then in a Phase Modulation system we implement to have

$\theta(t) = \theta + k_p m(t)$ and with constant f_c, we get □*(t)* linearly varying with message signal.

and in an Frequency Modulation system letting phase offset θ be a constant, we implement to have

$f_i(t) = \mathrm{f}_c + k_f m(t)$, to get □*(t)* linearly varying with message signal

where k_p and k_f are phase and frequency sensitivity constants.

The maximum phase deviation in a PM system is given by:

$$\Delta\theta_{\max} = k_p \left|m(t)\right|_{\max}$$

And the maximum frequency deviation in FM is given by:

$$\Delta f_{\max} = k_f \left|m(t)\right|_{\max}$$
$$\Delta\omega_{\max} = 2\pi k_f \left|m(t)\right|_{\max}$$

Single Tone Frequency Modulation

The general expression for FM signal is $s(t) = ACos\left(\omega_c t + k_f \int m(\mathrm{t})\,\mathrm{dt}\right)$

So for the single tone case, wheremessage signal is $m(t) = VCos(\omega_m t)$

Then $s(\mathrm{t}) = ACos\left(\omega_c t + \dfrac{k_f V}{\omega_m} Sin(\omega_m t)\right)$

$$\Rightarrow s(\mathrm{t}) = ACos\left(\omega_c t + m_f Sin(\omega_m t)\right)$$

Here $m_f = \dfrac{k_f V}{\omega_m} = \dfrac{\Delta\omega}{\omega_m} \rightarrow$ Modulation Index

Types of Frequency Modulation

High frequency deviation =>High Bandwidth=> High modulation index=>Wideband FM

Small frequency deviation =>Small Bandwidth=> Small modulation index=>Narrowband FM

Carson's Rule

It provides a rule of thumb to calculate the bandwidth of a single-tone FM signal.

$$Bandwidth = 2(\Delta f + f_m) = 2(1 + m_f) f_m$$

If baseband signal is any arbitrary signal having large number of frequency components, this rule can be modified by replacing m_f by deviation ratio D.

$$D = \frac{Peak\ Frequency\ deviation\ corresponding\ maximum\ possible\ amplitude\ of\ m(t)}{Maximum\ frquency\ component\ present\ in\ the\ modulating\ signal\ m(t)}$$

Then the bandwidth of FM signal is given as: $Bandwidth = 2(1+D)f_{\max}$

Spectrum of a Single-tone Narrowband FM signal

A single-tone FM modulated signal is mathematically given as:

$$s(\mathrm{t}) = ACos\left(\omega_c t + m_f Sin(\omega_m t)\right)$$
$$\Rightarrow s(\mathrm{t}) = ACos(\omega_c t)\mathrm{Cos}(m_f Sin(\omega_m t)) - A\mathrm{Sin}(\omega_c t)\mathrm{Sin}(m_f Sin(\omega_m t))$$

Since for narrowband FM modulation index m_f<<1, sowe approximate as:

$$\mathrm{Cos}(m_f Sin(\omega_m t)) \approx 1 \text{ and } \mathrm{Sin}(m_f Sin(\omega_m t)) \approx m_f Sin(\omega_m t)$$

And the expression s(t) becomes:

$$s(\mathrm{t}) = ACos(\omega_c t) - Am_f\,\mathrm{Sin}(\omega_c t)Sin(\omega_m t)$$
$$\Rightarrow s(\mathrm{t}) = ACos(\omega_c t) + \frac{Am_f}{2}\left\{Cos(\omega_c + \omega_m)\mathrm{t} - Cos(\omega_c - \omega_m)\mathrm{t}\right\}$$

The above equation represents the NBFM signal. This representation is similar to an AM signal, except that the lower sideband frequency has a negative sign.

Spectrum of a Single-tone Wideband FM signal

A single-tone FM modulated signal is mathematically given as:

$$s(\mathrm{t}) = ACos\left(\omega_c t + m_f Sin(\omega_m t)\right)$$
$$\Rightarrow s(\mathrm{t}) = ACos(\omega_c t)\mathrm{Cos}(m_f Sin(\omega_m t)) - A\mathrm{Sin}(\omega_c t)\mathrm{Sin}(m_f Sin(\omega_m t))$$

The FM signal can be expressed in the complex envelope form as:

$$s(\mathrm{t}) = \mathrm{Re}\left[Ae^{j\omega_c t + jm_f Sin(\omega_m t)}\right]$$
$$\Rightarrow s(\mathrm{t}) = \mathrm{Re}\left[Ae^{jm_f Sin(\omega_m t)} * e^{j\omega_c t}\right]$$
$$\Rightarrow s(\mathrm{t}) = \mathrm{Re}\left[\tilde{\mathrm{s}}(\mathrm{t}) * e^{j\omega_c t}\right]$$

Where $\tilde{\mathrm{s}}(\mathrm{t}) = Ae^{jm_f Sin(\omega_m t)}$, which is a periodic function of period $\frac{1}{f_m}$.

The Fouries series expansion of this periodic function can be written as:

$$\tilde{s}(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi n f_m t}$$

Where C_n spectral coefficients are given by

$$C_n = f_m \int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} \tilde{s}(t)\, e^{-j2\pi n f_m t}\, dt$$

$$\Rightarrow C_n = A f_m \int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} \left[e^{j m_f Sin(\omega_m t) - j2\pi n f_m t} \right] dt$$

Substituting $x = 2\pi f_m t$, the above equation becomes,

$$C_n = \frac{A}{2\pi} \int_{-\pi}^{\pi} \left[e^{j m_f Sin(x) - jnx} \right] dx$$

As the above expression is in the form of n^{th} order Bessels function of first kind :

$$J_n(m_f) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left[e^{j m_f Sin(x) - jnx} \right] dx,$$

therefore we can write $C_n = A J_n(m_f)$

So, $\tilde{s}(t) = \sum_{n=-\infty}^{\infty} A J_n(m_f) e^{j2\pi n f_m t}$

Hence the FM signal in complex envelope form can be written as:

$$s(t) = A * \text{Re}\left[\sum_{n=-\infty}^{\infty} J_n(m_f) e^{j(2\pi n f_m t + \omega_c t)} \right]$$

$$s(t) = A * \left[\sum_{n=-\infty}^{\infty} J_n(m_f) \text{Cos}(2\pi n f_m t + \omega_c t) \right]$$

This is the Fourier series representation of Wideband Single-tone FM signal. Its Fourier Transform can be written as:

$$S(f) = A * \left[\sum_{n=-\infty}^{\infty} J_n(m_f) \left\{ \delta(f + f_c + n f_m) + \delta(f - f_c - n f_m) \right\} \right]$$

The spectrum of Wideband Single-tone FM signal indicates the following:

1. WBFM has infinite number of sidebands at frequencies $(f_c \pm n f_m)$.

2. Spectral amplitude values depends upon $J_n(\mathrm{m}_f)$.
3. The number of significant sidebands depends upon the modulation index m_f. For $\mathrm{m}_f << 1$, $J_0(\mathrm{m}_f)$ and $J_1(\mathrm{m}_f)$ are only significant, whereas for $\mathrm{m}_f >> 1$, many significant sidebands exists.

Methods of Generating FM wave

Direct FM: In this method the carrier frequency is directly varied inaccordance with the incoming message signal to produce a frequency modulated signal.

Indirect FM: This method was first proposed by Armstrong. In thismethod, the modulating wave is first used to produce a narrow-band FMwave, and frequency multiplication is next used to increase thefrequency deviation to the desired level.

Direct Method or Parameter Variation Method

In this method, the baseband or modulating signal directly modulates the carrier. The carrier signal is generated with the help of an oscillator circuit. This oscillator circuit uses a parallel tuned L-C circuit. Thus the frequency of oscillation of the carrier generation is governed by the expression:

$$\omega_c = \frac{1}{\sqrt{LC}}$$

The carrier frequency is made to vary in accordance with the baseband or modulating signal by making either L or C depend upon to the baseband signal. Such an oscillator whose frequency is controlled by a modulating signal voltage is called as Voltage Controlled Oscillator. The frequency of VCO is varied according to the modulating signal simply by putting shunt voltage variable capacitor (varactor/varicap) with its tuned circuit. The varactor diode is a semiconductor diode whose junction capacitance changes with dc bias voltage. The capacitor C is made much smaller than the varactor diode capacitance C_d so that the RF voltage from oscillator across the diode is small as compared to reverse bias dc voltage across the varactor diode.

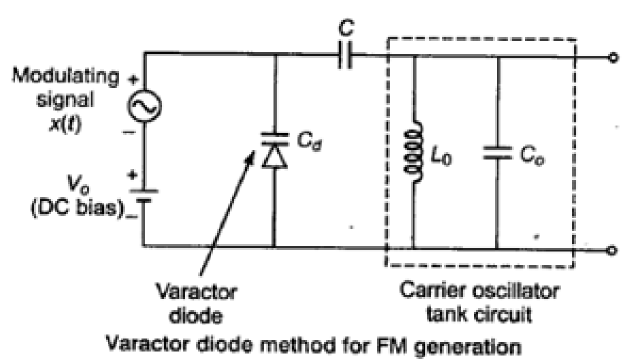


Figure 12 Varactor diode method of FM generation(Direct Method)

$$C_d = \frac{k}{\sqrt{v_D}} = k\left(v_D\right)^{-\frac{1}{2}}$$

$$v_D = V_o + x\left(t\right)$$

$$\omega_i = \frac{1}{\sqrt{L_o\left(C_o + C_d\right)}}$$

$$\Rightarrow \omega_i = \frac{1}{\sqrt{L_o\left(C_o + kv_D^{-\frac{1}{2}}\right)}}$$

Drawbacks of direct method of FM generation:

1. Generation of carrier signal is directly affected by the modulating signal by directly controlling the tank circuit and thus a stable oscillator circuit cannot be used. So a high order stability in carrier frequency cannot be achieved.
2. The non-linearity of the varactor diode produces a frequency variation due to harmonics of the modulating signal and therefore the FM signal is distorted.

Indirect method or Armstrong method of FM generation

A very high frequency stability can be achieved since in this case the crystal oscillator may be used as a carrier frequency generator. In this method, first of all a narrowband FMis generated and then frequency multiplication is used to cause required increased frequency deviation.The narrow band FM wave is then passed through a frequency multiplier to obtain the wide band FM wave. Frequency multiplication scales up the carrier frequency as well as the frequency deviation. The crystal controlled oscillator provides good frequency stability. But this scheme does not provide both the desired frequency deviation and carrier frequency at the same time. This problem can be solved by using multiple stages of frequency multipliers and a mixer stages.

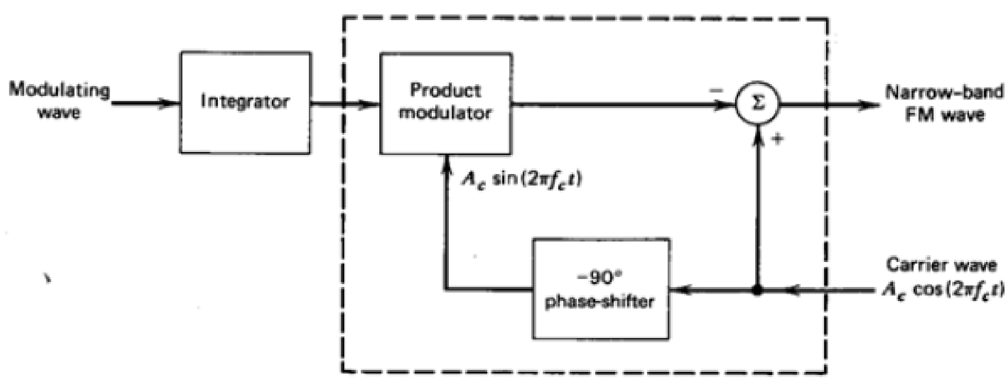


Figure 13 Narrow Band FM Generation

FM Demodulators

In order to be able to demodulate FM, a receiver must produce a signal whose amplitude varies as according to the frequency variations of the incoming signals and it should be insensitive to any amplitude variations in FM signal. Insensitivity to amplitude variations is achieved by having a high gain IF amplifier. Here the signals are amplified to such a degree that the amplifier runs into limiting. In this way any amplitude variations are removed. Generally a FM demodulator is composed of two parts: *Discriminator* and *Envelope Detector.Discriminator* is a frequency selective network which converts the frequency variations in an input signal in to proportional amplitude variations. Hence when it is input with an FM signal, it can produce an amplitude modulated signal. But it does not generally alter the frequency variations which were there in the input signal. So the output of a discriminator is a both frequency and amplitude modulated signal. This signal can be fed to the *Envelope Detector*part of FM demodulator to get back the baseband signal

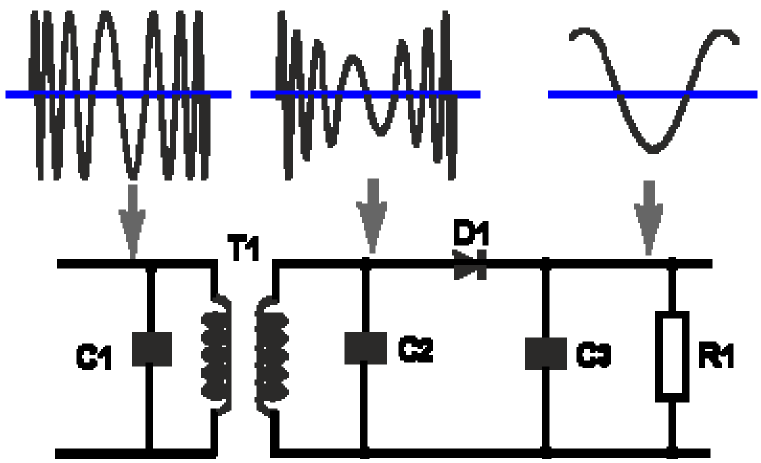


Figure 14 Slope detector

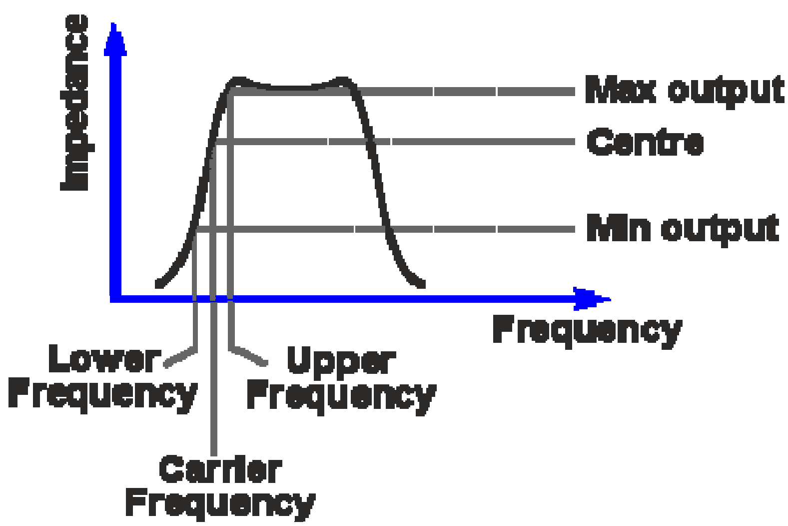


Figure 15 Frequency response of slope detector

<u>*Slope detector*:</u> A very simplest form of FM demodulation is known as *slope detection* or demodulation. It consists of a tuned circuit that is tuned to a frequency slightly offset from the carrier of

the signal.As the frequency of the signals varies up and down in frequency according to its modulation, so the signal moves up and down the slope of the tuned circuit. This causes the amplitude of the signal to vary in line with the frequency variations. In fact, at this point the signal has both frequency and amplitude variations.It can be seen from the diagram that changes in the slope of the filter, reflect into the linearity of the demodulation process.The linearity is very dependent not only on the filter slope as it falls away, but also the tuning of the receiver - it is necessary to tune the receiver frequency to a point where the filter characteristic is relatively linear. The final stage in the process is to demodulate the amplitude modulation and this can be achieved using a simple diode circuit. One of the most obvious disadvantages of this simple approach is the fact that both amplitude and frequency variations in the incoming signal appear at the output. However, the amplitude variations can be removed by placing a limiter before the detector. The input signal is a frequency modulated signal. It is applied to the tuned transformer (T1, C1, C2 combination) which is offset from the centre carrier frequency. This converts the incoming signal from just FM to one that has amplitude modulation superimposed upon the signal. This amplitude signal is applied to a simple diode detector circuit, D1. Here the diode provides the rectification, while C3 removes any unwanted high frequency components, and R1 provides a load.

<u>*PLL FM demodulator / detector:*</u>When used as an FM demodulator, the basic phase locked loop can be used without any changes. With no modulation applied and the carrier in the centre position of the pass-band the voltage on the tune line to the VCO is set to the mid position. However, if the carrier deviates in frequency, the loop will try to keep the loop in lock. For this to happen the VCO frequency must follow the incoming signal, and in turn for this to occur the tune line voltage must vary. Monitoring the tune line shows that the variations in voltage correspond to the modulation applied to the signal. By amplifying the variations in voltage on the tune line it is possible to generate the demodulated signal.The PLL FM demodulator is one of the more widely used forms of FM demodulator or detector these days. Its suitability for being combined into an integrated circuit, and the small number of external components makes PLL FM demodulation ICs an ideal candidate for many circuits these days.

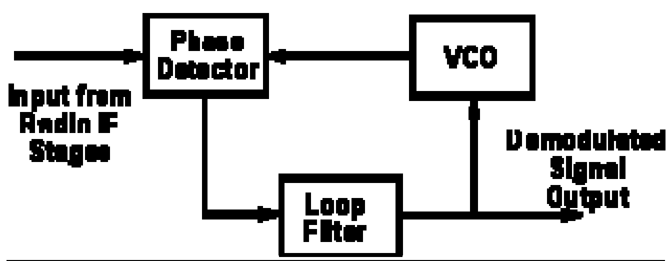


Figure 16 PLL FM Demodulator

Module-III

Sources and types of Noise

Type of noises are

- Thermal Noise
- Shot Noise
- Additive Noise
- Multiplicative Noise (fading)
- Gaussian Noise
- Spike Noise or Impulse Noise

Source of thermal noise are resistive elements in electrical and electronic circuits. Current flowing in conductors can also be an example. Constant agitation at molecular level in all material, which prevails all over the universe, is another example. In brief any source which provides the current is the cause of the thermal energy. Source of shot noise is the solid state semiconductor devices like diode, triode, tetrode, and pentode tubes. The noise which are additive in nature are known as additive noise. This corrupts message signal. Fading occurs because of signal or noise available at destination from multiple paths. White noise is basically approximated by Gaussian noise as its probability density function is Gaussian. Spike noise is observed in FM receivers because of low input SNR.

Frequency Domain Representation Noise

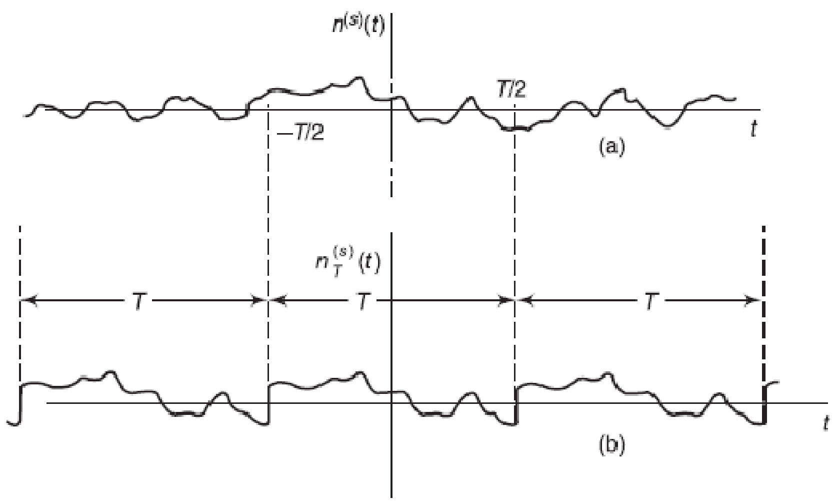


Figure 3.1: (a) A sample noise waveform. (b) A periodic waveform is generated by repeating the interval in (a) from –*T*/2 to *T*/2

n (*t*) is a non periodic complete noise where as $n^{(s)}(t)$ is a sample of it and $n_T^{(s)}(t)$ is a periodic noise as shown in above figure 3.1(b).

$$n_T^{(s)}(t) = \sum_{k=1}^{\infty} (a_k \cos 2\pi k \,\Delta f t + b_k \sin 2\pi k \,\Delta f t) \tag{3.1}$$

$$n_T^{(s)}(t) = \sum_{k=1}^{\infty} c_k \cos (2\pi k \,\Delta f t + \theta_k) \qquad c_k^2 = a_k^2 + b_k^2 \qquad \theta_k = -\tan^{-1} \frac{b_k}{a_k} \tag{3.2}$$

Power Spectrum of Noise

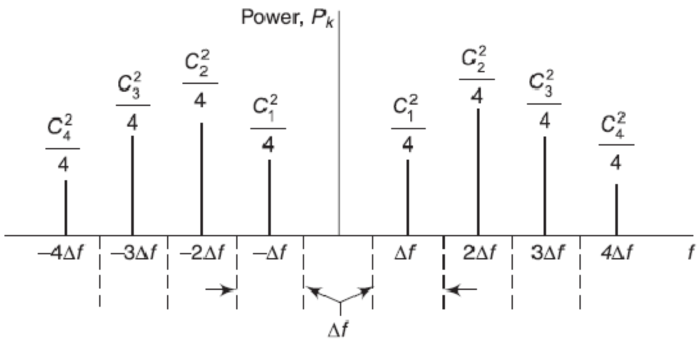


Figure 3.2: The power spectrum of the waveform $n_T^{(s)}$

Power spectral density of noise $n_T^{(s)}$ at $k\Delta f$ or - $k\Delta f$ frequency interval can be written as

$$G_n(k\,\Delta f) \equiv G_n(-k\,\Delta f) \equiv \frac{c_k^2}{4\,\Delta f} = \frac{a_k^2 + b_k^2}{4\,\Delta f} \quad (3.3)$$

Mean Power spectral density $G_n(k\,\Delta f) \equiv G_n(-k\,\Delta f)$ (3.4)

Total power in the interval: $P_k = 2G_n\,(k\,\Delta f)\,\Delta f$ (3.5)

Representation of Noise

Actual noise n (t) which is a non-periodic signal can be represented as

$$n(t) = \lim_{\Delta f \to 0} \sum_{k=1}^{\infty} (a_k \cos 2\pi k\,\Delta f t + b_k \sin 2\pi k\,\Delta f t) \quad (3.6)$$

$$n(t) = \lim_{\Delta f \to 0} \sum_{k=1}^{\infty} c_k \cos(2\pi k\,\Delta f t + \theta_k) \quad (3.7)$$

,

Where, $c_k^2 = a_k^2 + b_k^2$ (3.8)

$$G_n(f) = \lim_{\Delta f \to 0} \frac{\overline{c_k^2}}{4\,\Delta f} = \lim_{\Delta f \to 0} \frac{\overline{a_k^2} + \overline{b_k^2}}{4\,\Delta f} \quad (3.9)$$

Now we can write

$$P(f_1 \to f_2) = \int_{-f_2}^{-f_1} G_n(f)\,df + \int_{f_1}^{f_2} G_n(f)\,df = 2\int_{f_1}^{f_2} G_n(f)\,df \quad (3.10)$$

Total power P_T is $P_T = \int_{-\infty}^{\infty} G_n(f)\,df = 2\int_0^{\infty} G_n(f)\,df$ (3.11)

Spectral Component of Noise

Spectral component of noise at k^{th} instant and within an interval of Δf can be represented as $n_k(t)$ as given below.

$$n_k(t) = a_k \cos 2\pi k\,\Delta f t + b_k \sin 2\pi_k\,\Delta f t \quad (3.12a)$$

$$n_k(t) = c_k \cos(2\pi k\,\Delta f t + \theta_k) \quad (3.12b)$$

Corresponding power can be written as

$$P_k = \overline{[n_k(t)]^2} = \overline{a_k^2} \cos^2 2\pi k \,\Delta f t + \overline{b_k^2} \sin^2 2\pi k \,\Delta f t + \overline{2 a_k b_k} \sin 2\pi k \,\Delta f t \cos 2\pi k \,\Delta f t \tag{3.13}$$

Taking a time t = t_1, such that cos $2\pi k\Delta f$ = 1, we have $P_k = \overline{a_k^2}$, similarly
Taking a time t = t_2, such that cos $2\pi k\Delta f$ = 0, we have $P_k = \overline{b_k^2}$, Hence

$$P_k = 2G_n(k\,\Delta f)\,\Delta f = 2G_n(-k\,\Delta f)\,\Delta f = \overline{a_k^2} = \overline{b_k^2} = \frac{\overline{a_k^2}}{2} + \frac{\overline{b_k^2}}{2} = \frac{\overline{c_k^2}}{2} \tag{3.14}$$

$$\text{Since } \overline{a_k^2} = \overline{b_k^2} \qquad P_k = \overline{a_k^2} + 2\overline{a_k b_k} \sin 2\pi k \,\Delta f t \cos 2\pi k \,\Delta f t \tag{3.15}$$

It is observed that

$$P_k = \overline{a_k^2} \text{ independently of time.} \qquad \overline{a_k b_k} = 0 \qquad n_k(t_1) = a_k \tag{3.16}$$

Let us take two spectral components of noise as given by

$$n_k(t) = a_k \cos 2\pi k \,\Delta f t + b_k \sin 2\pi k \,\Delta f t \tag{3.17a}$$

$$n_l(t) = a_l \cos 2\pi l \,\Delta f t + b_l \sin 2\pi l \,\Delta f t \tag{3.17b}$$

Considering similar analysis as above, we have

$$\overline{a_k a_l} = \overline{a_k b_l} = \overline{b_k a_l} = \overline{b_k b_l} = 0 \tag{3.18}$$

This above explanation indicates noise n (t) is random, Gaussian, and stationary process, wherea_k, b_k, a_l, b_l, are uncorrelated random Gaussian random variables. The probability density function (pdf) of c_k and θ_k can be given as

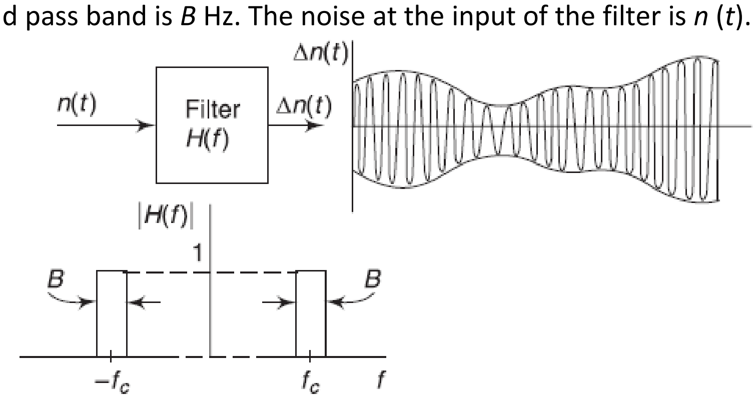
$$f(c_k) = \frac{c_k}{P_k} e^{c_k^2/2P_k} \quad c_k \geq 0 \tag{3.19}$$

$$f(\theta_k) = \frac{1}{2\pi} \quad -\pi \leq \theta_k \leq \pi \tag{3.20}$$

The pdf $f(c_k)$ describes a Reyliegh distribution, where as pdf $f(\theta_k)$ describes a Uniform distribution.

<u>Narrowband Filter Response to Noise</u>

In the following figure 3.3, the filter used is a narrow band filter with transfer function H (f) and pass band is B Hz. The noise at the input of the filter is n (t).



onse to narrowband noise

The noise $n(t)$ to the filter $H(f)$ is a wideband noise, whereas the noise at the output of the same filter is a narrowband noise $\Delta n(t)$. The amplitude variation of this $\Delta n(t)$ is small as it contains very few harmonics. If we reduce the pass-band B of the filter to a very small value then the variation in amplitude of $\Delta n(t)$ will be small and may be a approximated sinusoidal signal.

Effect of Filter to Noise PSD

The noise sample at the output of the filter can be designated as $n_{k_0}(t)$.

$$H(k\,\Delta f) = |H(k\,\Delta f)|\,e^{j\varphi k} = |H(k\,\Delta f)|/\varphi_k \tag{3.21}$$

$$n_{k_o}(t) = |H(k\,\Delta f)|\; a_k \cos(2\pi k\,\Delta f t + \varphi_k) + |H(k\,\Delta f)|\; b_k \sin(2\pi k\,\Delta f t + \varphi_k) \tag{3.22}$$

Since $|H(k\,\Delta f)|$ is a deterministic function,

$$\overline{[|H(k\,\Delta f)|a_k]^2} = |H(k\,\Delta f)|^2\,\overline{a_k^2} \text{ and } \overline{[|H(k\,\Delta f)|b_k]^2} = |H(k\,\Delta f)|^2\,\overline{b_k^2} \tag{3.23}$$

power P_{k_o} associated with $n_{k_o}(t)$ is $P_{k_o} = |H(k\,\Delta f)|^2\,\dfrac{\overline{a_k^2}+\overline{b_k^2}}{2}$ (3.24)

$$G_{n_o}(k\,\Delta f) = |H(k\,\Delta f)|^2\,G_{n_i}(k\,\Delta f) \tag{3.25}$$

$$G_{n_o}(f) = |H(f)|^2\,G_{n_i}(f) \tag{3.26}$$

Mixing Noise with Sinusoid

Noise $n_k(t)$ mixed with a sinusoidal signal at f_o can be written as

$$n_k(t)\cos 2\pi f_0 t = \frac{a_k}{2}\cos 2\pi(k\,\Delta f + f_0)t + \frac{b_k}{2}\sin 2\pi(k\,\Delta f + f_0)t$$

$$+\frac{a_k}{2}\cos 2\pi(k\,\Delta f - f_0)t + \frac{b_k}{2}\sin 2\pi(k\,\Delta f - f_0)t \tag{3.27}$$

It is already understood that

$$G_n(k\,\Delta f + f_0) = G_n(k\Delta f - f_0) = \frac{G_n(k\,\Delta f)}{4} \tag{3.28}$$

In case of actual noise Δf tends to zero, $k\Delta f$ becomes f and therefore, we can write

$$G_n(f + f_0) = G_n(f - f_0) = \frac{G_n(f)}{4} \tag{3.29}$$

Let us single out two spectral components of noise $n(t)$

$n_k(t) = a_k cos(2\pi k\Delta ft) + b_k sin(2\pi k\Delta ft)$ and (3.30a)

$n_l(t) = a_l cos(2\pi l\Delta ft) + b_l sin(2\pi l\Delta ft)$ (3.30b)

$k\Delta f$ and $l\Delta f$ is chosen in such a manner that $f_o = [(k + l)/2]\Delta f$; this means f_o is in the middle of $k\Delta f$ and $l\Delta f$. Let say $l\Delta f > k\Delta f$. Now we can define two difference frequency components as given below.

$p\Delta f = f_o - k\Delta f = l\Delta f - f_o$. These difference frequency components are also uncorrelated as follows.

$$n_l(t).cos2\pi f_o t = \frac{a_l}{2} cos2\pi(l\Delta f + f_o)t + \frac{b_l}{2} sin2\pi(l\Delta f + f_o)t + \frac{a_l}{2} cos2\pi(l\Delta f - f_o)t + \frac{b_l}{2} sin2\pi(l\Delta f - f_o)t$$

We find the difference frequency components as

$$n_{p1}(t) = \frac{a_k}{2} \cos 2\pi p \Delta ft - \frac{b_k}{2} \sin 2\pi p \Delta ft \quad (3.31a)$$

$$n_{p2}(t) = \frac{a_l}{2} \cos 2\pi p \Delta ft + \frac{b_l}{2} \sin 2\pi p \Delta ft \quad (3.31b)$$

$n_{p1}(t)$ is the difference component due to the mixing of frequencies f_o and $k\Delta f$, while $n_{p2}(t)$ is the difference component due to the mixing of frequencies f_o and $l\Delta f$. Now we are interested to find the expected values of the product of $n_{p1}(t)$ and $n_{p2}(t)$.

Similar to the last explanation, we have

$$\overline{a_k a_l} = \overline{a_k b_l} = \overline{b_k a_l} = \overline{b_k b_l} = 0, \qquad E[n_{p1}(t)n_{p2}(t)] = 0 \quad (3.32)$$

So power at difference frequencies

$$E\{[n_{p1}(t) + n_{p2}(t)]^2\} = E\{[n_{p1}(t)]^2\} + E\{[n_{p2}(t)]^2\} \quad (3.33)$$

Thus mixing noise with a sinusoid signal results in a frequency shifting of the original noise by f_o. The variance of this shifted noise is found by adding the variance of each new noise component. This is also applicable to two shifted power spectral density plots.

Mixing Noise with Noise

$$n_k(t)n_l(t) = \frac{1}{2} c_k c_l \cos[2\pi(k+l)\Delta ft + \theta_k + \theta_l] + \frac{1}{2} c_k c_l \cos[2\pi(k-l)\Delta ft + \theta_k - \theta_l] \quad (3.34)$$

$$P_{k+l} = P_{k-l} = \frac{1}{2}\overline{\left(\frac{1}{2} c_k c_l\right)^2} \quad (3.35)$$

Since c_k and c_l are independent random variables,

$$P_{k+l} = P_{K-l} = \frac{1}{8}\overline{c_k^2}\;\overline{c_l^2} = \frac{1}{2} P_k P_l \tag{3.36}$$

Linear Filtering of Noise

Thermal noise and Shot noise have similar power spectral density which can be approximated as the power spectral density (PSD) of the White noise. This PSD is as shown in figure 3.4.

$$G_n(f) = \frac{\eta}{2}$$

Figure 3.4: Power spectral density of noise

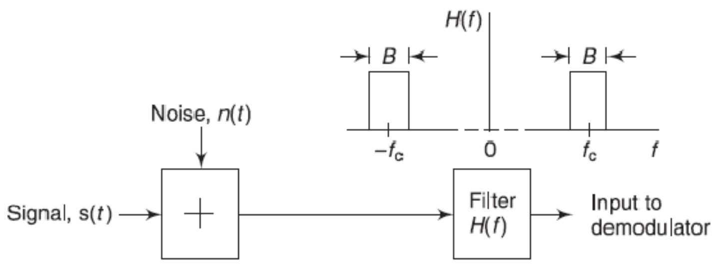


Figure 3.5: A filter is placed before a demodulator to limit the noise power input to the demodulator

In order to minimize the noise power that is presented to the demodulator of a receiving system, a filter is introduced before the demodulator as shown in figure 3.5. The bandwidth B of the filter is made as narrow as possible so as to avoid transmitting any unnecessary noise to the demodulator. For example, in an AM system in which the baseband extends to a frequency of f_M, the bandwidth $B = 2f_M$. In a wideband FM system the bandwidth is proportional to twice the frequency deviation.

Noise and Low Pass Filter

One of the filter most frequently used is the simple RC low-pas filter (LPF). The same RC LPF with a 3 dB cutoff frequency f_c has the transfer function

T.F. of *RC* Low Pass Filter: $H(f) = \dfrac{1}{1 + jf/f_c}$ (3.37)

If PSD of input noise $G_{n_i}(f)$. The PSD of output noise is

$$G_{n_o}(f) = G_{n_i}(f)|H(f)|^2$$

$$G_{n_o}(f) = \frac{\eta}{2}\frac{1}{1+(f/f_c)^2} \tag{3.38}$$

Noise power at the filter output, N_o can be expressed as

$$N_o = \int_{-\infty}^{\infty} G_{n_o}(f)\, df = \frac{\eta}{2}\int_{-\infty}^{\infty} \frac{df}{1+(f/f_c)^2} \tag{3.39}$$

noting that $\int_{-\infty}^{\infty} dx/(1+x^2) = \pi,$ $\qquad N_o = \frac{\pi}{2}\eta f_c$ (3.40)

Ideal Low Pass Filter: $\quad H(f) = \begin{cases} 1 & |f| \le B \\ 0 & \text{elsewhere} \end{cases}$ (3.41)

$$G_{n_o}(f) = \begin{cases} \frac{\eta}{2} & -B \le f \le B \\ 0 & \text{elsewhere} \end{cases} \qquad N_o = \eta B \tag{3.42}$$

Noise and Band Pass Filter

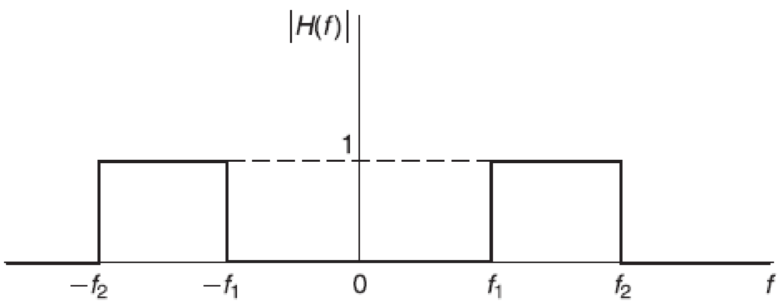


Figure 3.6: A rectangular band-pass filter

$$N_o = 2\frac{\eta}{2}\,(f_2 - f_1) = \eta\,(f_2 - f_1) \tag{3.43}$$

Noise and Differentiator

Transfer function of a differentiator is: $\quad H(f) = j2\pi\tau f$

If white noise with $G_{n_i}(f) = \eta/2$ is applied at the input

$$G_{n_o}(f) = |H(f)|^2 G_{n_i}(f) = 4\pi^2\tau^2 f^2 \frac{\eta}{2} \tag{3.44}$$

If the differentiator is followed by a rectangular low pass filter having a bandwidth B.

Noise power at the output of the LPF is

$$N_o = \int_{-B}^{B} 4\pi^2\tau^2 f^2 \frac{\eta}{2}\, df = \frac{4\pi^2}{3}\eta\tau^2 B^3 \tag{3.45}$$

Noise and Integrator

Transfer function of an integrator is: $$H(f) = \frac{1}{j\omega\tau} - \frac{e^{-j\omega T}}{j\omega\tau} = \frac{1-e^{-j\omega T}}{j\omega\tau} \quad (3.46)$$

$$\text{with } \omega = 2\pi f, \qquad |H(f)|^2 = \left(\frac{T}{\tau}\right)^2 \left(\frac{\sin \pi Tf}{\pi Tf}\right)^2 \quad (3.47)$$

$$N_o = \int_{-\infty}^{\infty} \frac{\eta}{2} |H(f)|^2\, df = \frac{\eta}{2}\left(\frac{T}{\tau}\right)^2 \int_{-\infty}^{\infty} \left(\frac{\sin \pi Tf}{\pi Tf}\right)^2 df \; = \frac{\eta T}{2\tau^2} \quad (3.48)$$

Noise Bandwidth

The noise bandwidth (B_N) is defined as the bandwidth of an idealized (rectangular) filter which passes the same noise power as does the real filter. As per the definition we can find $B_N = (\pi/2)f_o$, where f_o is the frequency at which the transfer function of the actual filter is centered.

Quadrature components of Noise

It is sometimes more advantageous to represent Narrowband noise centred around f_0 as

$$n(t) = n_c(t)\cos 2\pi f_0 t - n_s(t)\sin 2\pi f_0 t \quad (3.49)$$

These $n_c(t)$ and $n_s(t)$ are known as quadrature component of noise.

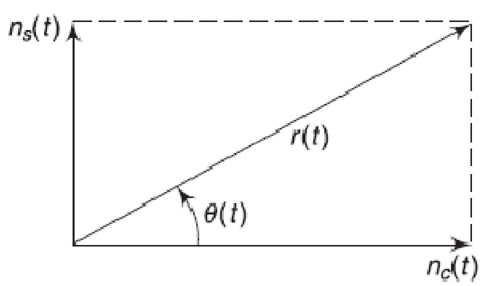


Figure 3.7: Quadrature components of noise

Now as per the initial notation

$$n(t) = \lim_{\Delta f \to 0} \sum_{k=1}^{\infty} (a_k \cos 2\pi k\, \Delta f t + b_k \sin 2\pi k\, \Delta f t) \quad (3.50)$$

$$n(t) = \lim_{\Delta f \to 0} \sum_{k=1}^{\infty} \{a_k \cos 2\pi[f_0 + (k-K)\,\Delta f]t + b_k \sin 2\pi[f_0 + (k-K)\,\Delta f]t\} \quad (3.51)$$

Where, $K.\Delta f = f_0$, Hence

$$n_c(t) = \lim_{\Delta f \to 0} \sum_{k=1}^{\infty} [a_k \cos 2\pi(k-K)\,\Delta f t + b_k \sin 2\pi(k-K)\,\Delta f t] \quad (3.52)$$

$$n_s(t) = \lim_{\Delta f \to 0} \sum_{k-1}^{\infty} [a_k \sin 2\pi(k-K)\,\Delta f t - b_k \cos 2\pi(k-K)\,\Delta f t] \quad (3.53)$$

$$r(t) = [n_c^2(t) + n_s^2(t)]^{1/2} \qquad \theta(t) = \tan^{-1}[n_s(t)/n_c(t)] \quad (3.54)$$

A. M. Receiver

This receiver as shown in figure 3.8 is capable of processing an amplitude modulated carrier and recovering the baseband signal. The modulated RF carrier + noise is received by the receiving antenna and submitted to Radio frequency (RF) amplifier. After a number of operations as indicated in the same figure 3.8, finally baseband signal with some small noise is obtained at the output of the receiver.

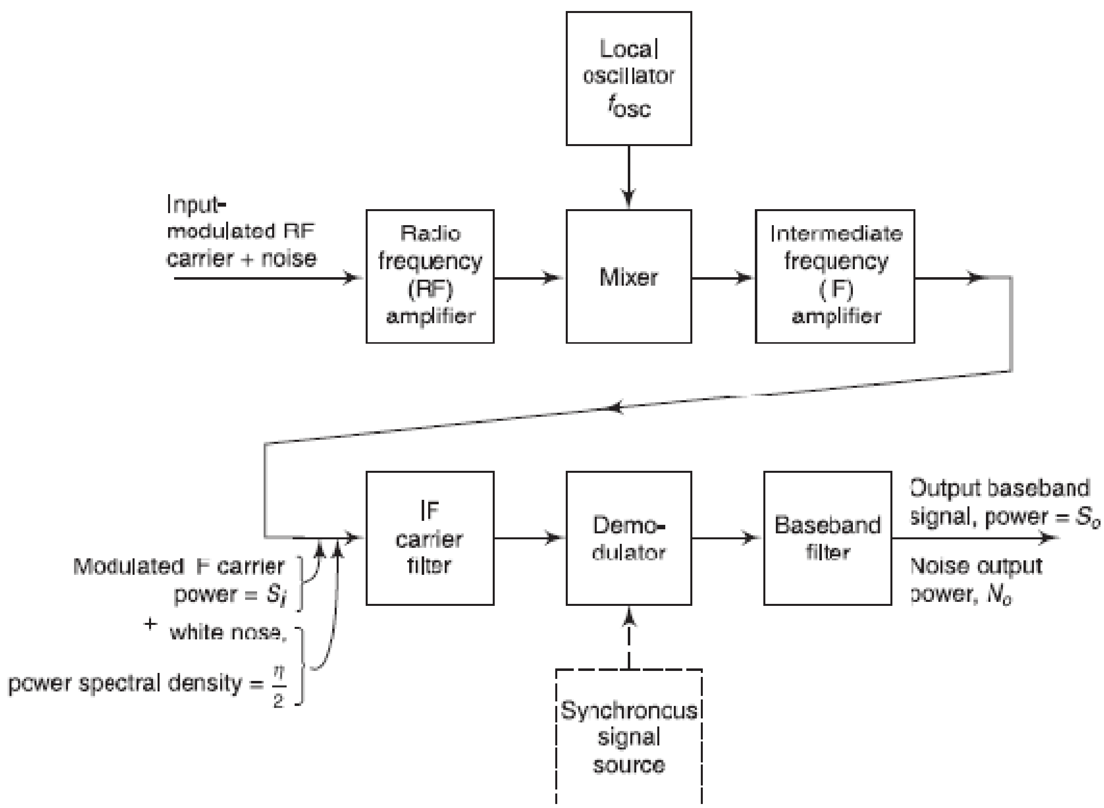


Figure 3.8: A receiving system for amplitude modulated signal

Superheterodyne principle

In early days TRF receivers were used to detect the baseband signal from modulated RF signal. The performance of such receiver varies as the incoming RF frequency varies. This is because it uses single conversion technique. Later double conversion technique (frequency of incoming RF signal changes two times) is used by some receiver as shown in figure 3.8. These are known as superheterodyne receiver. The main idea behind the design of such receiver is that: whatever may be the frequency of the incoming RF signal, the output after first conversion always produces a fixed frequency known as intermediate frequency. Due to this the performance of receiver remains same for all type of incoming RF signal.

Calculation of Signal power and noise power in SSB-SC

SSB-SC: Signal Power

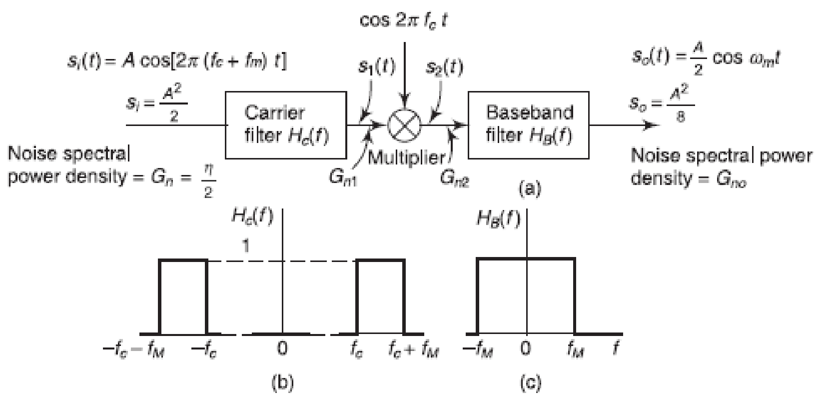


Figure 3.9: (a) A synchronous demodulator operating on a single-sideband single-tone signal. (b) The bandpass range of the carrier filter. (c) The passband of the lowpass baseband filter.

$$s_i(t) = A\cos[2\pi(f_c + f_m)t] \tag{3.55}$$

Output of multiplier is

$$s_2(t) = s_1(t)\cos\omega_c t = \frac{A}{2}\cos[2\pi(2f_c + f_m)t] + \frac{A}{2}\cos 2\pi f_m t \tag{3.56}$$

Output of baseband filter can be written as

$$s_o(t) = \frac{A}{2}\cos 2\pi f_m t \tag{3.57}$$

The input signal power is

$$S_i = \frac{A^2}{2} \tag{3.58}$$

The output signal power is

$$S_o = \frac{1}{2}\left(\frac{A}{2}\right)^2 = \frac{A^2}{8} = \frac{S_i}{4} \tag{3.59}$$

$$\frac{S_o}{S_i} = \frac{1}{4} \tag{3.60}$$

Noise Power

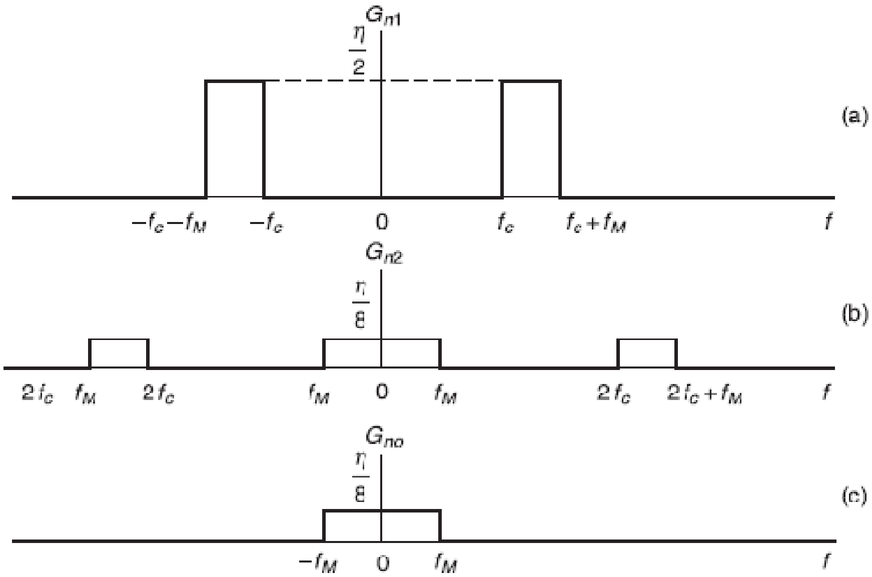


Figure 3.10: Spectral densities of noises in SSB demodulator. (a) Density G_{n1} of noise input to multiplier. (b) Density G_{n2} of noise output of multiplier. (c) Density G_{no} of noise output of baseband filter.

$$N_o = 2f_M \frac{\eta}{8} = \frac{\eta f_m}{4} \tag{3.61}$$

SNR, $$\frac{S_o}{N_o} = \frac{S_i/4}{\eta f_M/4} = \frac{S_i}{\eta f_M} \tag{3.62}$$

Calculation of Signal power and noise power in DSB-SC

When a baseband signal of frequency f_M is transmitted over a DSB-SC system, the bandwidth of the carrier filter must be 2 f_M rather than f_M. Thus, along with signal the input noise in the frequency range $f_c - f_M$ to $f_c + f_M$ will contribute to the output noise, rather than only in the range of f_c to $f_c + f_M$ as in SSB case.

DSB-SC: Signal Power:

$$s_i(t) = \sqrt{2}\, A \cos 2\pi f_m t \cos 2\pi f_c t$$

$$= \frac{A}{\sqrt{2}} \cos\,[2\pi(f_c + f_m)t] + \frac{A}{\sqrt{2}} \cos\,[2\pi(f_c - f_m)t] \tag{3.63}$$

$$s_o'(t) = \frac{A}{2\sqrt{2}} \cos 2\pi f_m t \tag{3.64}$$

$$s_o''(t) = \frac{A}{2\sqrt{2}} \cos 2\pi f_m t \tag{3.65}$$

$$s_o(t) = s'(t) + s''(t) = \frac{A}{\sqrt{2}} \cos 2\pi f_m t \tag{3.66}$$

$$S_o = \frac{A^2}{4} = \frac{S_i}{2} \tag{3.67}$$

DSB-SC: Noise Power

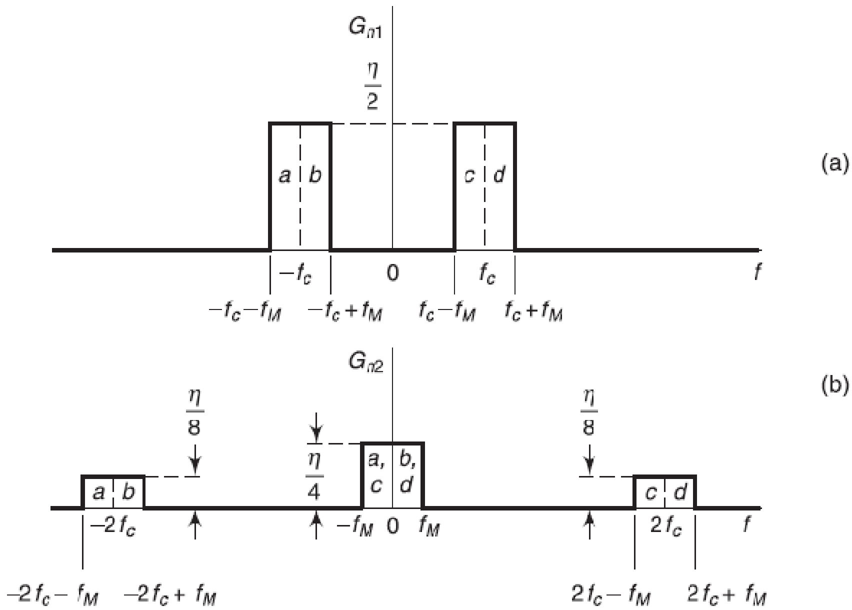


Figure 3.11: Spectral densities of noise in DSB demodulation. (a) Density G_{n1} of noise at output of IF filter. (b) Density G_{n2} of noise output of baseband filter.

$$N_o = \frac{\eta}{4}(2f_M) = \frac{\eta f_M}{2} \tag{3.68}$$

$$\text{SNR,} \quad \frac{S_o}{N_o} = \frac{S_i}{\eta f_M} \tag{3.69}$$

DSB-SC: Arbitrary Modulated Signal:

$$s_i(t) = m(t) \cos 2\pi f_c t \tag{3.70}$$

$$S_i \equiv \overline{s_i^2(t)} = \overline{m^2(t) \cos^2 2\pi f_c t} = \frac{1}{2}\overline{m^2(t)} + \frac{1}{2}\overline{m^2(t) \cos(4\pi f_c t)} \tag{3.71}$$

$$S_i \equiv \overline{s_i^2(t)} = \frac{1}{2}\overline{m^2(t)} \tag{3.72}$$

$$S_o = \frac{\overline{m^2(t)}}{4} \tag{3.73}$$

$$S_o = \frac{S_i}{2} \tag{3.74}$$

$$n(t) = n_c(t) \cos 2\pi f_c t - n_s(t) \sin 2\pi f_c t \tag{3.75}$$

$$G_{n_c}(f) = G_{ns}(f_c + f) + G_{n1}(f_c - f) \tag{3.76}$$

In the frequency range $|f| \le f_M$, $G_{n1}(f_c + f) = G_{n1}(f_c - f) = \eta/2$. (3.77)

$$G_{n_c}(f) = G_{n_s}(f) = \eta \qquad |f| \le f_M \tag{3.78}$$

$$n(t) \cos 2\pi f_c t = \frac{1}{2} n_c(t) + \frac{1}{2} n_c(t) \cos 4\pi f_c t - \frac{1}{2} n_s(t) \sin 4\pi f_c t \tag{3.79}$$

$$n_o(t) = \frac{1}{2} n_c(t) \tag{3.80}$$

$$G_{no}(f) = \frac{1}{4} G_{n_c}(f) = \frac{\eta}{4} \qquad -f_M \le f \le f_M \tag{3.81}$$

$$N_o = \frac{\eta}{4}\, 2f_M = \frac{\eta f_M}{2} \tag{3.82}$$

<u>Calculation of Signal power and noise power in DSB-C</u>

DSB-C: Arbitrary Modulated Signal:

Let us consider the case, where a carrier accompanies the double sideband signal. Demodulation is achieved synchronously as in SSB-SC and DSB-SC. We note that the carrier increases the total input signal power but makes no contribution to the output signal power. We know that

$$\frac{S_o}{N_o} = \frac{S_i^{(SB)}}{\eta f_M} \tag{3.83}$$

Suppose that the received signal is

$$\begin{aligned} s_i(t) &= A[1 + m(t)] \cos 2\pi f_c t \\ &= A \cos 2\pi f_c t + Am(t) \cos 2\pi f_c t \end{aligned} \tag{3.84}$$

The carrier power, $P_c = A^2/2$; The sidebands are contained in the term $Am(t) \cos 2\pi f_c t$. The power associated with the term is $(A^2/2)\overline{m^2(t)}$, where $\overline{m^2(t)}$ is the time average of the square of the modulating waveform.

We now have the total input power S_i as given by

$$S_i = P_c + S_i^{(SB)} = \frac{A^2}{2} + \frac{A^2}{2}\overline{m^2(t)} = \frac{A^2}{2}\left[1 + \overline{m^2(t)}\right] = P_c\left[1 + \overline{m^2(t)}\right] \tag{3.85}$$

$$\frac{S_i^{(SB)}}{S_i} = \frac{\left(A^2/2\right)\overline{m^2(t)}}{\left(A^2/2\right)\{1 + \overline{m^2(t)}\}} \tag{3.86a}$$

$$S_i^{(SB)} = \frac{\overline{m^2(t)}}{1 + \overline{m^2(t)}} S_i \tag{3.86b}$$

$$\frac{S_o}{N_o} = \frac{\overline{m^2(t)}}{1 + \overline{m^2(t)}} \frac{S_i}{\eta f_M} \tag{3.87}$$

In terms of the carrier power $P_c \equiv A^2/2$,

$$\frac{S_o}{N_o} = \overline{m^2(t)} \frac{P_c}{\eta f_M} \tag{3.88}$$

If the modulation is sinusoidal, with $m(t) = m \cos 2\pi f_m t$

$$s_i(t) = A(1 + m \cos 2\pi f_m t) \cos 2\pi f_c t \quad (3.89)$$

In this case $\overline{m^2(t)} = m^2/2$ and

SNR, $$\frac{S_o}{N_o} = \frac{m^2}{2+m^2}\frac{S_i}{\eta f_M} \quad (3.90)$$

Figure of Merit:

$$\gamma \equiv \frac{S_o/N_o}{S_i/N_M} \quad (3.91a)$$

$$\gamma = \begin{cases} 1 & \text{SSB-SC} \\ 1 & \text{DSB-SC} \\ \dfrac{\overline{m^2(t)}}{1+\overline{m^2(t)}} & \text{DSB} \\ \dfrac{m^2}{2+m^2} & \text{DSB with sinusoidal modulation} \end{cases} \quad (3.91b)$$

<u>The Square Law Demodulator and Threshold:</u>

DSB-SC as well as DSB-C can be demodulated using square law demodulator. This avoids requirement of synchronous carrier as in case of synchronous detector, which is costlier. But in case of synchronous detector there is no threshold i.e. as S_i/N_M decreases by a factor of α, the S_o/N_o is also decreases by a factor of α. Therefore, figure of merit γ is independent of S_i/N_M. In case of nonlinear demodulator as S_i/N_M decreases, there is a point, a threshold at which the S_o/N_o decreases more rapidly than does the S_i/N_M. This threshold often makes the limits to the usefulness of the demodulator.

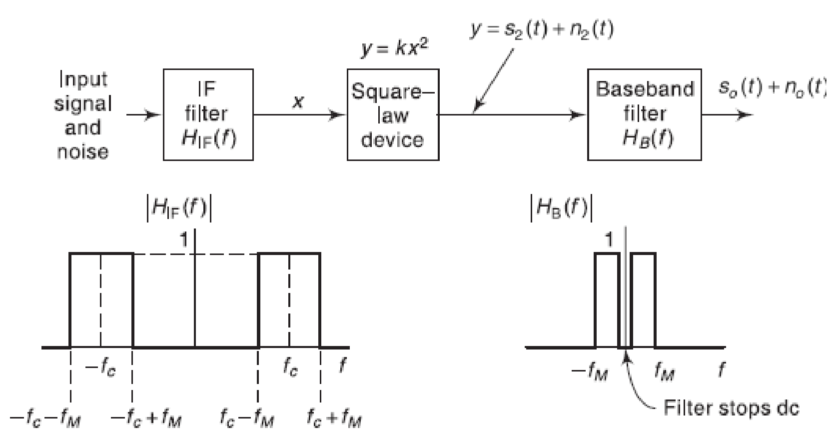


Figure 3.12: The square-law AM demodulator

$$x(t) = A[1 + m(t)] \cos \omega_c t + n(t) \tag{3.92}$$

$$y(t) = \lambda\{A[1 + m(t)] \cos \omega_c t + n(t)\}^2 \tag{3.93}$$

$$s_2(t) = \lambda A^2 m(t) \left[1 + \frac{m(t)}{2}\right] \tag{3.94}$$

$$n_2(t) = 2\lambda A n(t)[1 + m(t)] \cos \omega_c t + \lambda n^2(t) \tag{3.95}$$

assuming $|m(t)| \ll 1$

$$s_2(t) \approx \lambda A^2 m(t) \tag{3.96}$$

$$n_2(t) \approx 2\lambda A n(t) \cos \omega_c t + \lambda n^2(t) \tag{3.97}$$

$$S_o = \lambda^2 A^4 \overline{m^2(t)} \tag{3.98}$$

noise power N_o', due to the term $2\lambda A n(t) \cos \omega_c t$

$$N_o' = 4\lambda^2 A^2 \frac{\eta}{4} 2f_M = 2\lambda^2 A^2 \eta f_M \tag{3.99}$$

noise power N_o'' which results from the term $\lambda n^2(t)$

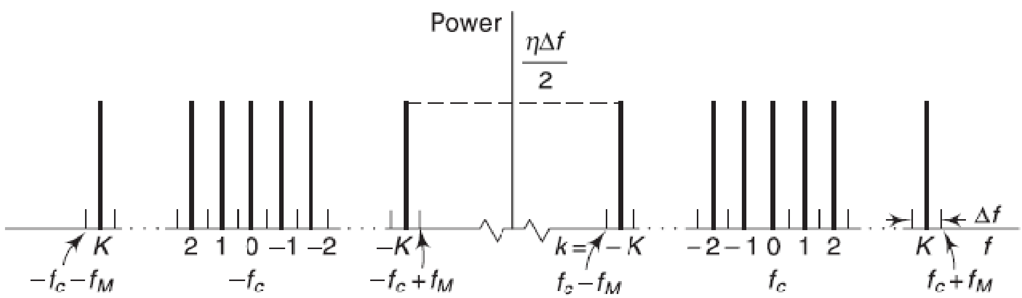


Figure 3.13: The spectral range $|f - f_c| \leq f_M$ of the noise $n(t)$ of power spectral density $\eta/2$ is divided into intervals Δf. The power in each interval is represented approximately by a single spectral line of power $\eta\, \Delta f/2$.

$$n(t) = \sum_{k=-K}^{+K} c_k \cos [(2\pi f_c + k\, \Delta f)t + \theta_k] \tag{3.100}$$

with $G_n(k\, \Delta f) = \eta/2$, we have

$$\overline{c_k^2} = 2\eta\, \Delta f \tag{3.101}$$

$$n_{k,\rho}(t) = c_k \cos [(2\pi f_c + k\, \Delta f)t + \theta_k] + c_{k+\rho} \cos \{[2\pi f_c + (k + \rho)\, \Delta f]t + \theta_{k+\rho}\} \tag{3.102}$$

$$n_\rho(t) = c_k c_{k+\rho} \cos (2\pi\rho\, \Delta f t + \theta_{k+\rho} - \theta_k) \tag{3.103}$$

$$\text{since } \overline{c_k^2} = \overline{c_{k+\rho}^2} \tag{3.104}$$

$$P_\rho \equiv \overline{n_\rho^2(t)} = \frac{1}{2}\overline{c_k^2}\,\overline{c_{k+\rho}^2} = 2(\eta\, \Delta f)^2 \tag{3.105}$$

$$2G_{n^2}(\rho\ \Delta f)\ \Delta f = (2K - \rho)2(\eta\ \Delta f)^2 \tag{3.106}$$

$$G_{\lambda n^2} = \lambda^2\eta^2(2f_M - f) \tag{3.107}$$

$$N_o'' = 3\lambda^2\eta^2 f_M^2 \tag{3.108}$$

total output-noise power

$$N_o = N_o' + N_o'' = 2\lambda^2\eta f_M A^2 + 3\lambda^2\eta^2 f_M^2 \tag{3.109}$$

SNR, $$\frac{S_o}{N_o} = \frac{\overline{A^4 m^2(t)}}{2\eta f_M A^2 + 3\eta^2 f_M^2} \tag{3.110}$$

$$\frac{S_o}{N_o} = \overline{m^2(t)}\frac{P_c}{N_M}\frac{1}{1 + \frac{3}{4}(N_M / P_c)} \tag{3.111}$$

Above threshold, when P_c/N_M is very large,

$$\frac{S_o}{N_o} = \overline{m^2(t)}\frac{P_c}{N_M} \tag{3.112}$$

Below threshold, when $P_c/N_M \ll 1$,

$$\frac{S_o}{N_o} = \frac{4}{3}\overline{m^2(t)}\left(\frac{P_c}{N_M}\right)^2 \tag{3.113}$$

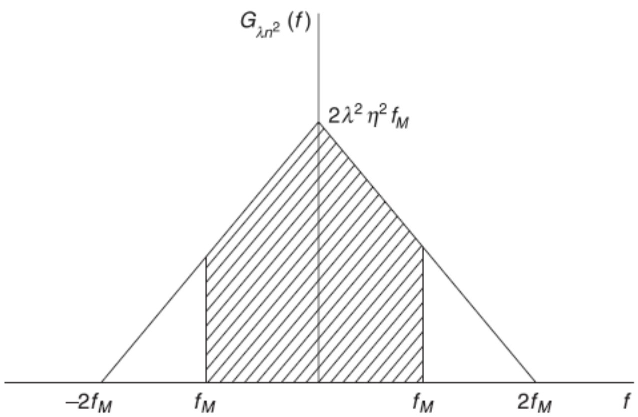


Figure 3.14: Plot of power spectral density $G_{\lambda n^2}(f)$ in baseband region.

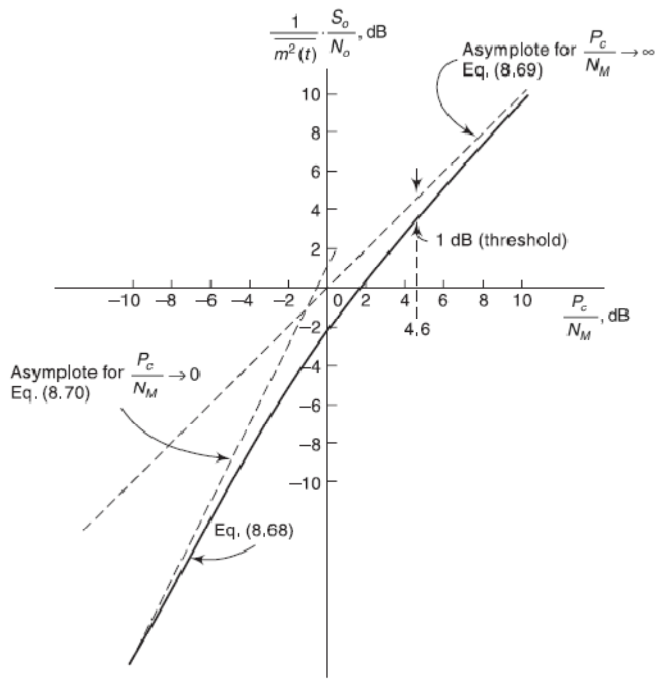


Figure 3.15: Performance of a square-law demodulator illustrating the phenomena of threshold

The solid line in figure 3.15 is applicable to the equation as in (3.111). The dashed line which passes through the center of the axis is applicable to the equation as in (3.112). The third line in the left and dashed is applicable to the equation as in (3.113).

From the figure 3.15, it is clear that as P_c/N_M decreases, the demodulator performance curve fall progressively further away from the straight line plot corresponding to P_c/N_M very large (i.e. applicable to synchronous detector).

Let's say we choose the performance curve of square law demodulator falls away by 1 dB from performance curve of synchronous demodulator. This is achieved at Pc/N_M = 4.6 dB i.e. $P_c = 2.884\ N_M$.

If P_c/N_M is taken more than 2.9 then the difference in ordinate value will be less than 1 dB and it is still better. When $\overline{m^2(t)} \ll 1$, then $S_i \cong P_c$. So, we can say $S_i \geq 2.9\ N_M$.

The Envelop Demodulator and Threshold:

This envelope demodulator can be used when $|m(t)| < 1$. Let us take quadrature component expression of noise. (3.114)

$$n(t) = n_c(t) \cos \omega_c t - n_s(t) \sin \omega_c t$$

If the noise $n(t)$ has a PSD of $\eta/2$ in the range of $|f - f_c| \leq f_M$ and is zero elsewhere. Then $n_c(t)$ and $n_s(t)$ have the PSD of η in the frequency range of $-f_M$ to f_M. At the demodulator i/p, the i/p signal and noise is

$$\begin{aligned} s_1(t) + n_1(t) &= A[1 + m(t)] \cos \omega_c t + n_c(t) \cos \omega_c t - n_s(t) \sin \omega_c t \\ &= \{A[1 + m(t)] + n_c(t)\} \cos \omega_c t - n_s(t) \sin \omega_c t \end{aligned} \quad (3.115)$$

The output signal plus noise just prior to base-band filtering is the envelope (phasor sum)

$$s_2(t) + n_2(t) = \{(A[1 + m(t)] + n_c(t))^2 + n_s^2(t)\}^{1/2}$$

$$= \{A^2[1 + m(t)]^2 + 2A[1 + m(t)]n_c(t) + n_c^2(t) + n_s^2(t)\}^{1/2} \quad (3.116)$$

Assuming then that $|n_c(t)| \ll A$ and $|n_s(t)| \ll A$,

$$s_2(t) + n_2(t) \approx \{A^2[1 + m(t)]^2 + 2A[1 + m(t)]n_c(t)\} 1/2$$

$$= A[1 + m(t)]\left\{1 + \frac{2n_c(t)}{A[1+m(t)]}\right\}^{1/2} \quad (3.118)$$

$$s_2(t) + n_2(t) \approx A[1 + m(t)] + n_c(t) \quad (3.119)$$

$$\gamma \equiv \frac{S_o/N_o}{S_i/N_M} = \frac{\overline{m^2(t)}}{1+\overline{m^2(t)}} \quad (3.120)$$

The γ here is same as the γ obtained using synchronous demodulator. To make a comparison with the square law demodulator, we assume $\overline{m^2(t)} \ll 1$. In this case as before $S_i \cong P_c$ and equation (3.120) reduces equation (3.112).

A threshold can be considered by understanding that the synchronous demodulator, the square law demodulator, and the envelop demodulator all performs equally well provided $\overline{m^2(t)} \ll 1$. Like square law demodulator, the envelop demodulator exhibits a threshold. As the input SNR decreases a point is reached where the SNR at the output decreases more rapidly than the input. The calculation of SNR is quite complex, we can simply state the result that for $S_i/N_M \ll 1, and\ \overline{m^2(t)} \ll 1$

$$\frac{S_o}{N_o} = \frac{\overline{m^2(t)}}{1.1}\left(\frac{S_i}{N_M}\right)^2 \quad (3.121)$$

Since both square-law demodulation and envelope demodulation exhibit a threshold, a comparison is of interest. We had assumed in square-law demodulation that $\overline{m^2(t)} \ll 1$. Then, as noted above, $S_i \cong A^2/2 = P_c$ the carrier power, and Eq. (8.77) becomes

$$\frac{S_o}{N_o} = \frac{\overline{m^2(t)}}{1.1}\left(\frac{P_c}{N_M}\right)^2 \quad (3.122)$$

which is to be compared with Eq. (8.70) giving S_o/N_o below threshold for the square-law demodulator.

Comparison:

(i) Square law demodulator has lower threshold

(ii)It also performs better below threshold

Module – IV

Noise in Frequency Modulation System:

An FM Receiving System

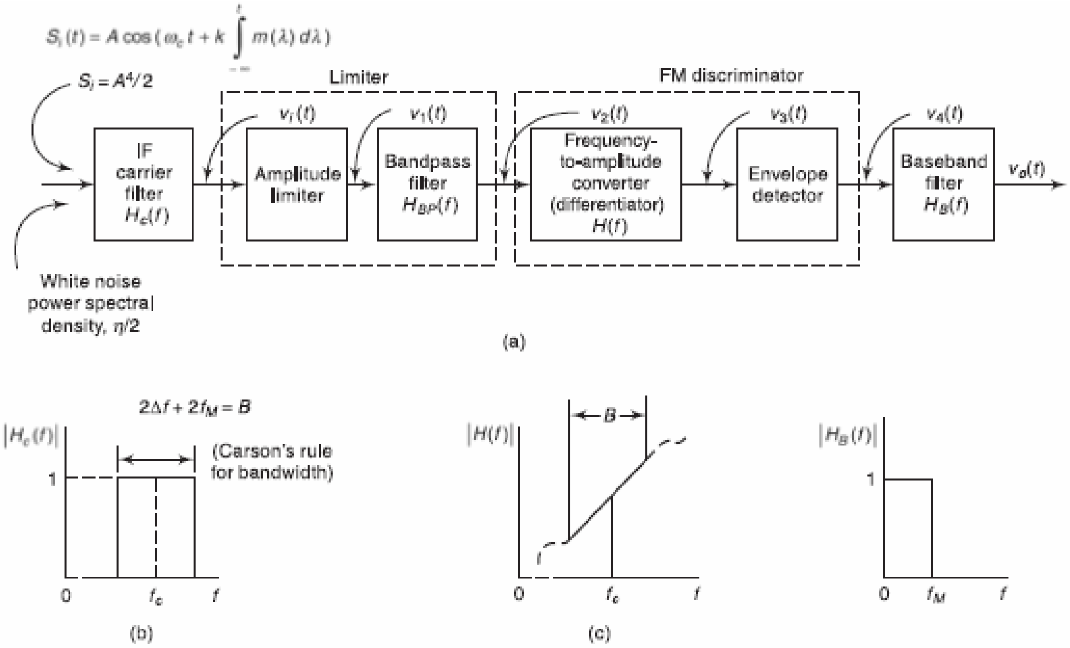


Figure 4.1: A limiter-discriminator used to demodulate an FM signal

Limiter and Discriminator:

$$V_1(t) = \begin{cases} V_i(t) & 0 \le t \le t_1 \\ A_L & t_1 \le t \le t_2 \\ V_i(t) & t_2 \le t \le t_3 \\ -A_L & t_3 \le t \le t_4 \\ V_i(t) & t_4 \le t \le T \end{cases}$$

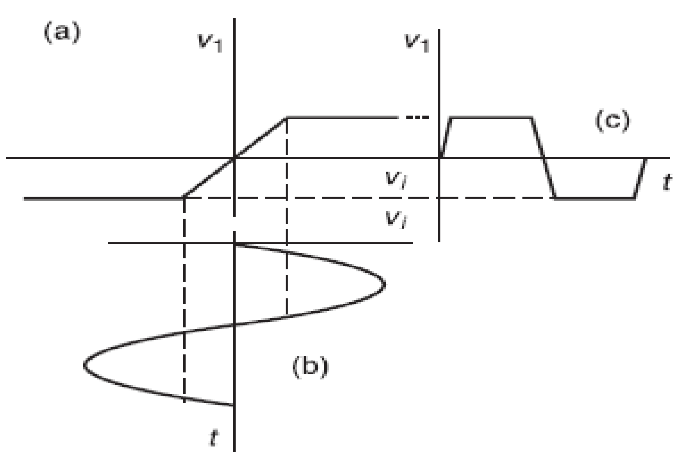


Figure 4.2: a) A limiter input-output characteristics. b) A cycle of the input carrier. c) The output waveform.

Limiter is to suppress amplitude variation noise. Discriminator gives at output an amplitude variation according to instantaneous frequency of input. This is as shown in figures 4.1 1nd 4.2.

The baseband signal is recovered by passing the amplitude modulated waveform through an envelope detector.

frequency-to-amplitude converter

$$H(j\omega) = j\sigma\omega \tag{4.1}$$

$$\sigma\frac{d}{dt} \Leftrightarrow j\sigma\omega \tag{4.2}$$

$$v_3(t) = \sigma\frac{d}{dt}v_2(t) \tag{4.3}$$

suppose that the voltage $v_2(t)$ applied to the converter is

$$v_2(t) = A_L \cos[\omega_c t + \phi(t)] \tag{4.4}$$

Here A_L is the *limited* amplitude of the carrier so that A_L is fixed and independent of the input amplitude, and $\omega_c t + \varphi(t)$ is the instantaneous phase.

$$v_3(t) = -\sigma A_L\left[\omega_c + \frac{d}{dt}\phi(t)\right]\sin[\omega_c t + \phi(t)] \tag{4.5}$$

using $\alpha \equiv \sigma A_L$,

output of the envelope detector

$$v_4(t) = \sigma A_L = \left[\omega_c + \frac{d}{dt}\phi(t)\right] = \alpha\omega_c + \alpha\frac{d}{dt}\phi(t) \tag{4.6}$$

SNR Calculation:

Signal Power:

Consider that the input signal to the IF carrier filter of figure 4.1 is

$$s_i(t) = A\cos\left[\omega_c t + k\int_{-\infty}^{t} m(\lambda)\,d\lambda\right] \tag{4.7}$$

Bandwidth $B = 2\Delta f + 2f_M$ (4.8)

The signal is $s_2(t)$ [corresponding to $v_2(t)$] given by

$$s_2(t) = A_L\cos\left[\omega_c t + k\int_{-\infty}^{t} m(\lambda)\,d\lambda\right] \tag{4.9}$$

$$\phi(t) = k\int_{-\infty}^{t} m(\lambda)\,d\lambda \tag{4.10}$$

We find foe the output of the discriminator

$$s_4(t) = \alpha\omega_c + \alpha k m(t) \tag{4.11}$$

Baseband filter rejects the DC component and passes the signal component

output signal is $S_o(t) = \alpha k m(t)$, and the output-signal power is

$$S_o = \alpha^2 k^2 \overline{m^2(t)} \tag{4.12}$$

Noise Power:

The carrier and noise at the limiter input are

$$\begin{aligned} v_i(t) &= A\cos\omega_c t + n_c(t)\cos\omega_c t - n_s(t)\sin\omega_c t \\ &= [A + n_c(t)]\cos\omega_c t - n_s(t)\sin\omega_c t \end{aligned} \tag{4.13}$$

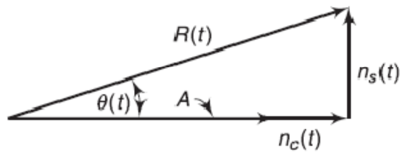


Figure 4.3: A Phasor diagram of the terms in above equation (4.13)

$$R(t) = \sqrt{[A + n_c(t)]^2 + [n_s(t)]^2} \tag{4.14}$$

$$\theta(t) = \tan^{-1} \frac{n_s(t)}{A + n_c(t)} \tag{4.15}$$

$$v_i(t) = R(t) \cos [w_c t + \theta(t)] \tag{4.16}$$

We ignore the time-varying envelope $R(t)$, since all time variations are removed by the limiter. Output of the limiter-band-pass filter, $v_2(t) = A_L \cos[\omega_c t + \theta(t)]$, where A_L is a constant. Assume that we are operating under the condition of high-input SNR such that $|n_c(t)| \leq A$ and $|n_s(t)| \leq A$ (4.17)

$\tan\theta \approx \theta$ for small θ,

$$\theta(t) \approx \frac{n_s(t)}{A} \tag{4.18}$$

$$v_2(t) = A_L \cos\left[\omega_c t + \frac{n_s(t)}{A}\right] \tag{4.19}$$

$$v_4(t) = \alpha\left[\omega_c + \frac{1}{A}\frac{d}{dt} n_s(t)\right] \tag{4.20}$$

input to the baseband filter

$$v_4(t) = \frac{\alpha}{A}\frac{d}{dt} n_s(t) \tag{4.21}$$

$$H(j\omega) = j\alpha\omega/A \tag{4.22}$$

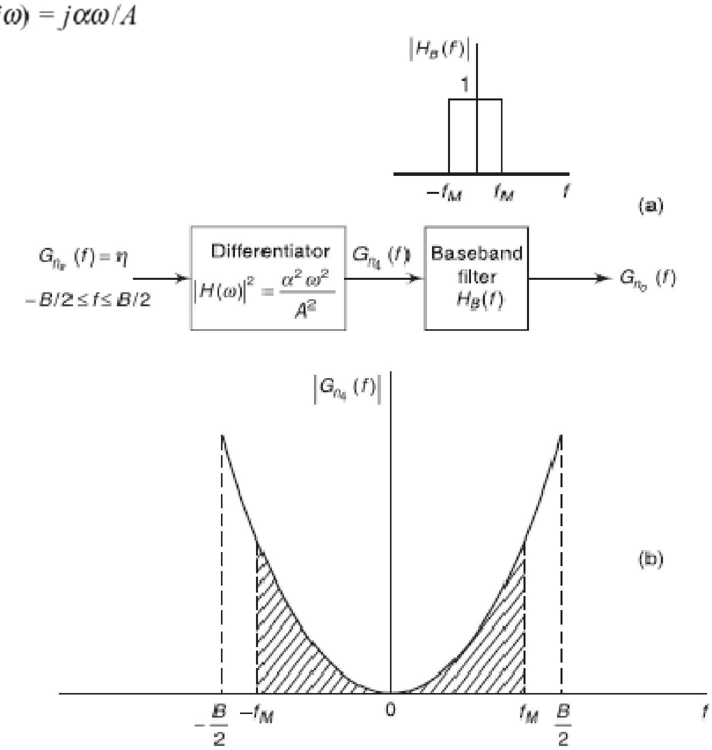


Figure 4.4 (a) Indicating the operations performed by the discriminator and baseband filter on the noise output of the limiter. (b) The variation with frequency of the power spectral density at the output of an FM demodulator.

$$G_{n4}(f) = \frac{\alpha^2 \omega^2}{A^2} \eta \qquad |f| \le \frac{B}{2} \tag{4.23}$$

Output-noise power

$$N_o = \int_{-f_M}^{f_M} G_{n4}(f)\, df$$

$$= \frac{\alpha^2 \eta}{A^2} \int_{-f_M}^{f_M} 4\pi^2 f^2\, df$$

$$= \frac{8\pi^2}{3} \frac{\alpha^2 \eta}{A^2} f_M^3 \tag{4.24}$$

SNR, $\frac{S_o}{N_o} = \frac{\alpha^2 k^2 \overline{m^2(t)}}{(8\pi^2/3)(\alpha^2 \eta / A^2) f_M^3} = \frac{3}{4\pi^2} \frac{k^2 \overline{m^2(t)}}{f_M^2} \frac{A^2/2}{\eta f_M}$ (4.25)

Let us consider that the modulating signal $m(t)$ is sinusoidal and produces a frequency deviation Δf. then the input signal s_i(t)

$$s_i(t) = A \cos\left(\omega_c t + \frac{\Delta f}{f_m} \sin 2\pi f_m t\right) \tag{4.26}$$

$$km(t) = 2\pi\, \Delta\pi\, \Delta f \cos 2\pi f_m t \tag{4.27}$$

$$k^2 \overline{m^2(t)} = \frac{4\pi^2 (\Delta f)^2}{2} = 2\pi^2 (\Delta f)^2 \tag{4.28}$$

$$\frac{S_o}{N_o} = \frac{3}{2} \left(\frac{\Delta f}{f_M}\right)^2 \frac{A^2/2}{\eta f_M} = \frac{3}{2} \beta^2 \frac{S_i}{N_M} \tag{4.29}$$

$$\gamma_{\mathrm{FM}} \equiv \frac{S_o / N_o}{S_i / N_M} \equiv \frac{3}{2} \beta^2 \tag{4.30}$$

<u>Comparison: FM and AM</u>

Let us compare the result for sinusoidal 100% modulation

$$\frac{\gamma_{\mathrm{FM}}}{\gamma_{\mathrm{AM}}} = \frac{9}{2} \beta^2 \tag{4.31}$$

FM is better if $\beta \cong \sqrt{2}/3 \cong 0.5$ or more. But this comes at the cost of higher bandwidth as

$$B_{\mathrm{FM}} = 2(\beta + 1) f_M \tag{4.32}$$

For$\beta_{\mathrm{FM}} \approx 2\beta f_M$ and bandwidth of AM system is $\beta_{\mathrm{AM}} = 2f_M$,

$$\frac{\gamma_{\mathrm{FM}}}{\gamma_{\mathrm{AM}}} = \frac{9}{2} \left(\frac{B_{\mathrm{FM}}}{B_{\mathrm{AM}}}\right)^2 \tag{4.33}$$

Several authors to make the comparison not on the basis of equal power but rather on the basis of *equal signal power measured when the modulation m*(t) = 0. In this case, as it can be easily verified, we find that the above equation (4.33) can be replaced by

$$\frac{\gamma_{\mathrm{FM}}}{\gamma_{\mathrm{AM}}} = 3\beta^2 \tag{4.34}$$

SNR Improvement: Pre-emphasis and de-emphasis

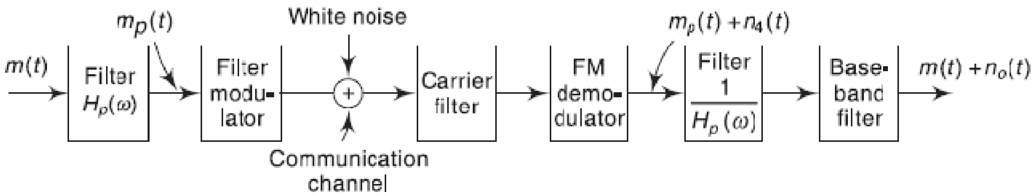


Figure 4.5: Pre-emphasis and de-emphasis in an FM system

$$P_m = \int_{-f_M}^{f_M} G_m(f)df = \int_{-f_M}^{f_M} |H_p(f)|^2 G_m(f)\ df \tag{4.35}$$

$$N_{od} = \left(\frac{\alpha}{A}\right)^2 4\pi^2\eta \int_{-f_M}^{f_M} f^2 \left|\frac{1}{H_p(f)}\right|^2 df \tag{4.36}$$

$$N_o/N_{od} \equiv \mathscr{R} \tag{4.37}$$

$$\mathscr{R} = \frac{(\alpha/A)^2(4\pi^2\eta)\int_{-f_M}^{f_M} f^2 df}{(\alpha/A)^2(4\pi^2\eta)\int_{-f_M}^{f_M} f^2/|H_p(f)|^2 df} = \frac{f_M^3/3}{\int_0^{fM} f^2 df/|H_p(f)|^2} \tag{4.38}$$

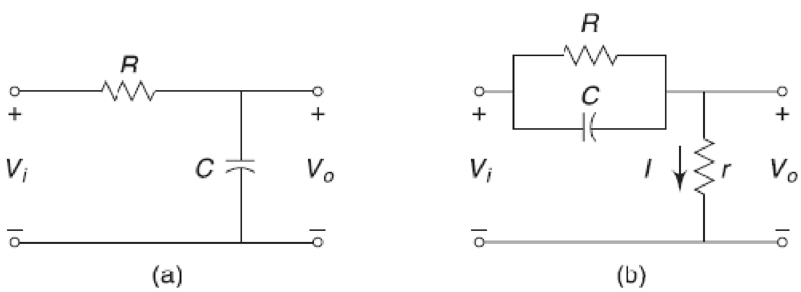


Figure 4.6: (a) Deemphasis network and (b) Preemphasis network

$$H_d(f) = \frac{1}{1 + jf/f_1} \tag{4.39}$$

$$f_1 = 1/2\pi RC \tag{4.40}$$

$$H_p(f) = \frac{r}{R}\,(1 + j\omega CR) = \frac{r}{R}\left(1 + j\frac{f}{f_1}\right) \tag{4.41}$$

$$H_p(f).H_d(f) = r/R = \text{constant} \tag{4.42}$$

The improvement in signal-to-noise ratio which results from pre-emphasis depends on the frequency dependence of the PSD of the baseband signal. Let us assume that the PDF of a typical audio signal, say music, may reasonably be represented as having a frequency dependence given by

$$G_m(f) = \begin{cases} G_0 \dfrac{1}{1+(f/f')^2} & |f| \le f_M \\ 0 & \text{elsewhere} \end{cases} \tag{4.43}$$

preemphasis network so that $f_1 = f'$

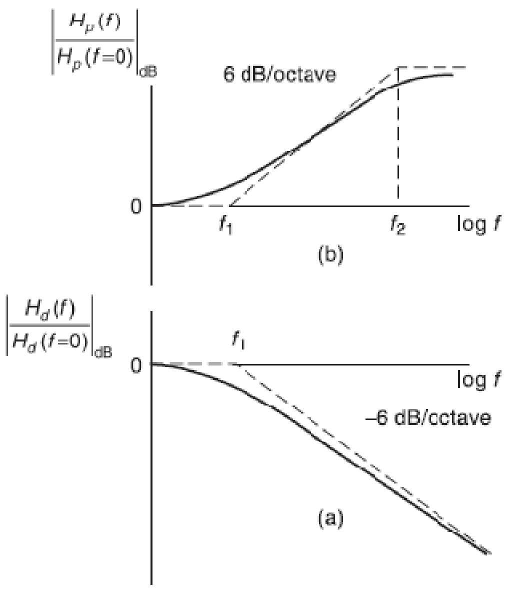


Figure 4.7: Normalized logarithmic plots of the frequency characteristics of a) the de-emphasis network and b) the pre-emphasis network

$$H_p(f) = K\left(1 + j\frac{f}{f_1}\right) \tag{4.44}$$

$$P_m = \int_{-f_M}^{f_M} \frac{G_0\, df}{1+(f/f_1)^2} = \int_{-f_M}^{f_M} K^2 G_0 df \tag{4.45}$$

Integrating and solving for K^2

$$K^2 = \frac{f_1}{f_M} \tan^{-1}\frac{f_M}{f_1} \tag{4.46}$$

$$\mathscr{R} = \frac{\tan^{-1}(f_M/f_1)}{3(f_1/f_M)[1-(f_1/f_M)\tan^{-1}(f_M/f_1)]} \tag{4.47}$$

When $f_M/f_1 \gg 1$

$$\mathscr{R} \cong \frac{\pi}{6}\frac{f_M}{f_1} \tag{4.48}$$

In commercial FM broadcasting f_1 = 2.1 kHz, while f_M may reasonably taken as = 15 kHz

$\mathscr{R} \cong 4.7$ corresponding to 6.7 dB improvement

Multiplexing:

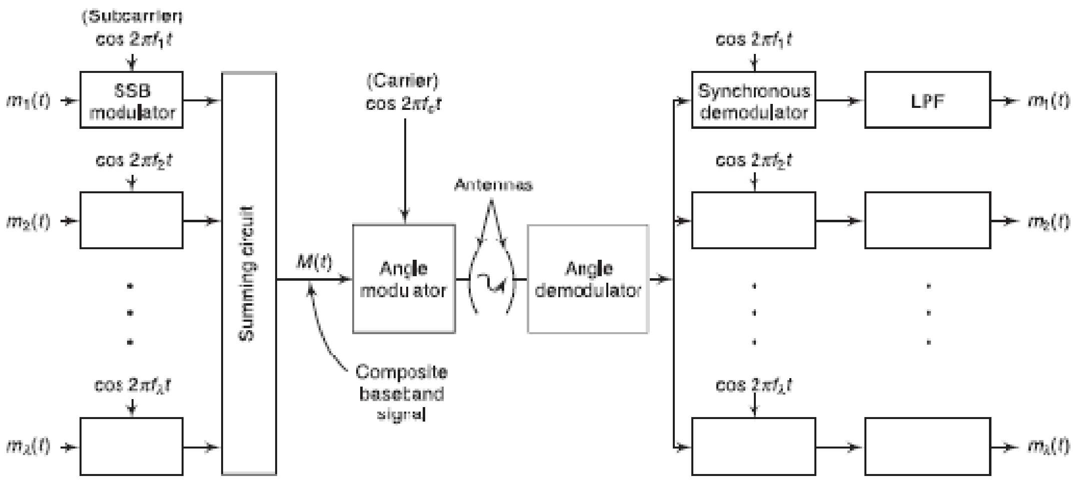


Figure 4.8: A system of frequency division multiplexing

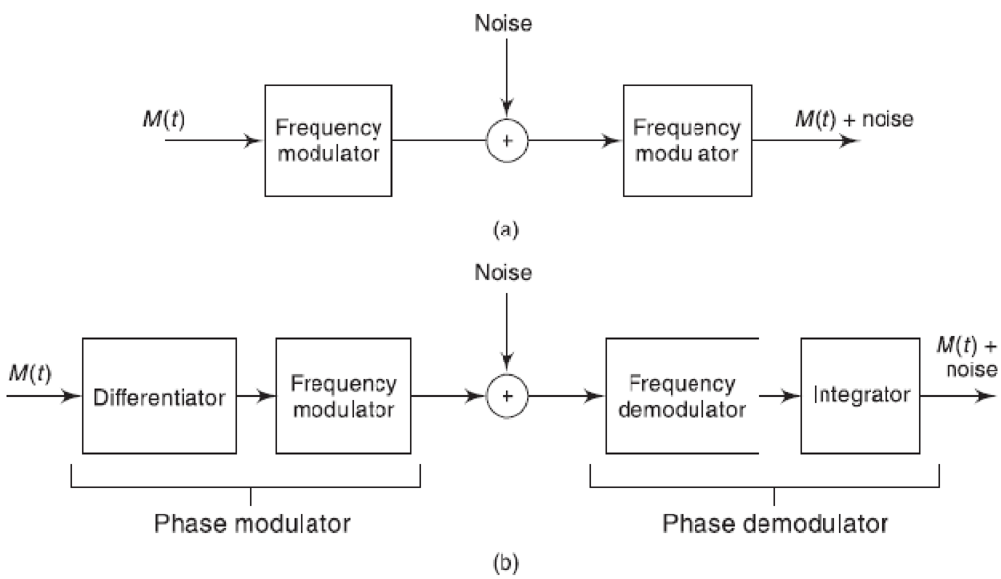


Figure 4.9: Comparison of an FM system in (a) with a phase modulation system in (b)

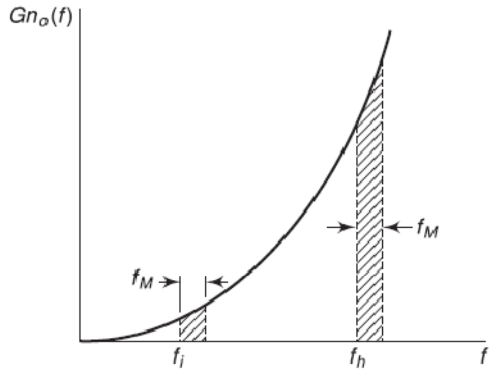


Figure 4.10: To illustrate that in the multiplex system of figure 4.8 using FM, channels associated with high carrier frequencies are noisier than those associated with lower frequencies.

$\theta(t) = n_s(t)/A$ is the phase-modulation noise. Since $\theta(t)$ and $n_s(t)$ are directly related, the form of the power spectral density of is identical.

The quadratic nature of noise power in FM makes it inferior to PM for higher carrier frequencies. In PM, noise power in each channel is same.

Assuming that both channels (*a*) and (*b*), are constrained to use the same bandwidth. The frequency range of the topmost channel of the composite signal $M(t)$ extends from $(N\text{-}1)f_M$ to Nf_M is the frequency range of an individual in the absence of de-emphasis, the noise output of the top channel

$$N_{o,\text{top}} = 2\frac{\alpha^2\eta}{A^2}\int_{(N-1)f_M}^{Nf_M} 4\pi^2 f^2\, df$$

$$\simeq \frac{8\pi^2\alpha^2\eta N^2 f_M^3}{A^2} \tag{4.49}$$

$$|H_p(f)|^2 = 4\pi^2\tau^2 f^2 \tag{4.50}$$

$$|H_d(f)|^2 = \frac{1}{|H_p(f)|^2} = \frac{1}{4\pi^2\tau^2 f^2} \tag{4.51}$$

The condition of equal bandwidth requires that

$$\tau^2 = \frac{3}{4\pi^2 N^2 f_M^2} \tag{4.52}$$

$$N_{od,\text{top}} = 2\frac{\alpha^2\eta}{A^2}\int_{(N-1)f_M}^{Nf_M} \frac{4\pi^2 f^2\, df}{|H_p(f)|^2} \tag{4.53}$$

$$\mathscr{R}_{\text{top}} = \frac{N_{o,\text{top}}}{N_{od,\text{top}}} = 3(=4.8\text{ dB}) \tag{4.54}$$

Effect of Transmitter Noise

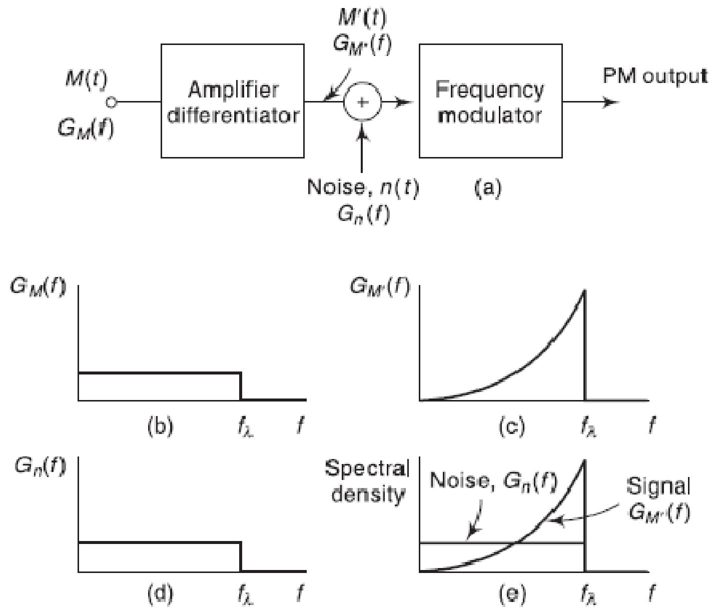


Figure 4.11: (a) A PM system in which noise is introduced before transmission. (b) The spectral density of the system. (c) The spectral density of the signal after differentiation. (d) The spectral density of the noise. (e) Comparison of spectral densities of signal and noise at input to modulator.

A network similar to the pre-emphasis circuit of figure 4.6(b) is suitable. In practice the 4.8 dB advantage quoted above for PM over FM is not realized. The advantage is more nearly 3 to 4 dB.

Threshold in Frequency Modulation:

$$\left[\frac{S_o}{N_o}\right]_{\text{dB}} = \left[\frac{S_i}{N_M}\right]_{\text{dB}} + 10\log\frac{3}{2}\beta^2 \tag{4.55}$$

Experimentally it is determined that the FM system exhibits a threshold.

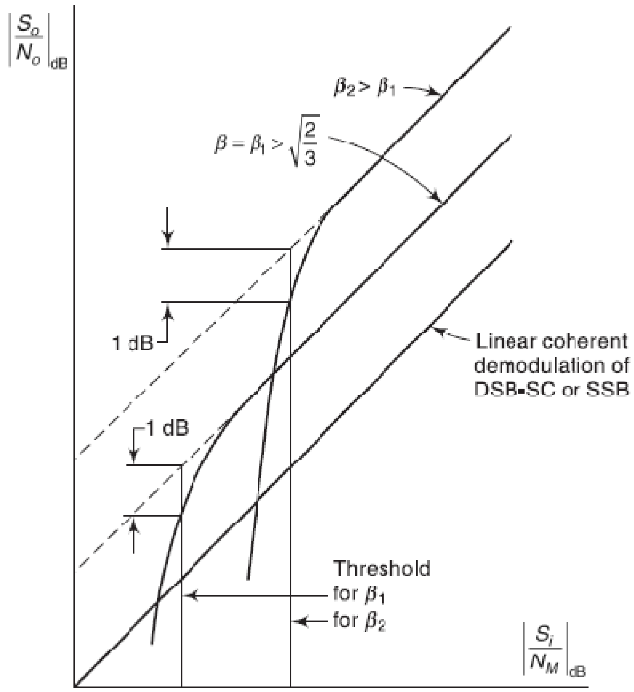


Figure 4.12: Plots of output SNR against input SNR for linear modulation and demodulation and also for an FM system illustrating the phenomenon of threshold in FM.

The threshold value of S_i/N_M is arbitrarily taken to be the value at which S_o/N_o falls 1 dB below the dashed extension.

For larger β the threshold is also higher.

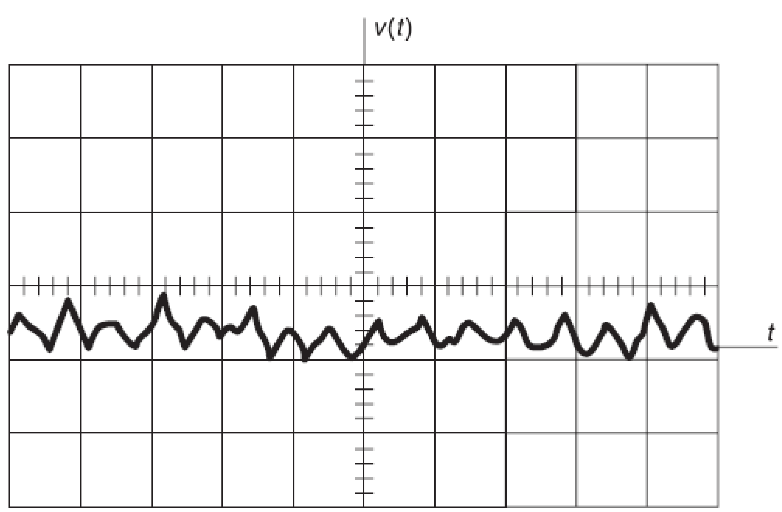


Figure 4.13: Thermal noise at discriminator output

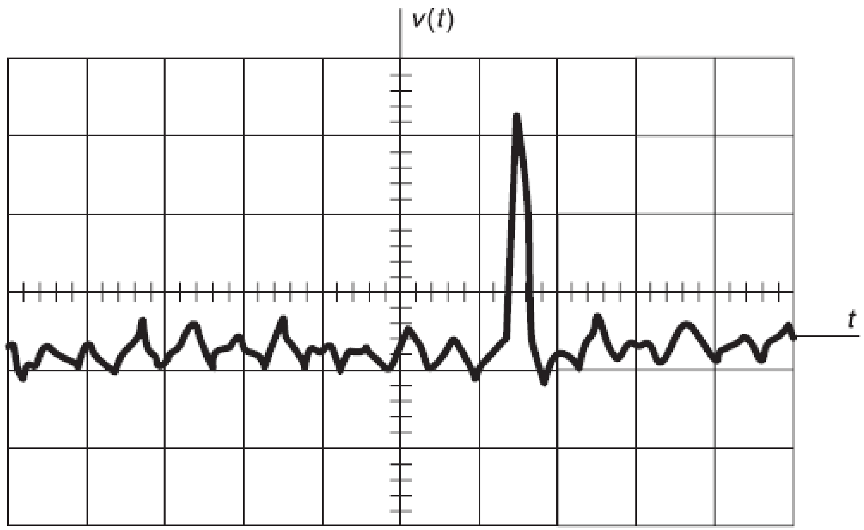


Figure 4.14: A spike superimposed on a background of smooth (thermal) noise

The onset of threshold may be observed by examining the noise output of an FM discriminator on a CRO. A *spike* or *impulse* noise appears (with clicking sound) in the background thermal-type noise, usually referred to as *smooth* noise.

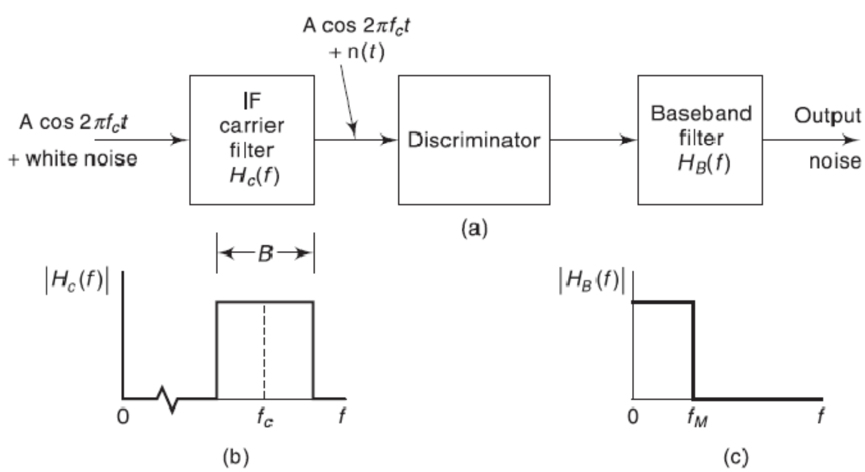
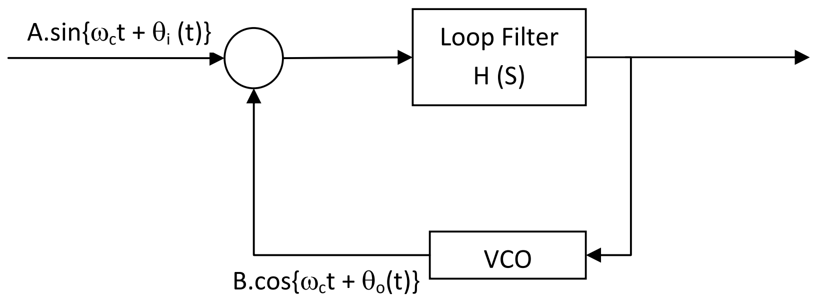


Figure 4.15 (a) An FM discriminator and associated filters. (b) The bandpass range of the carrier filter. (c) The passband of the baseband filter.

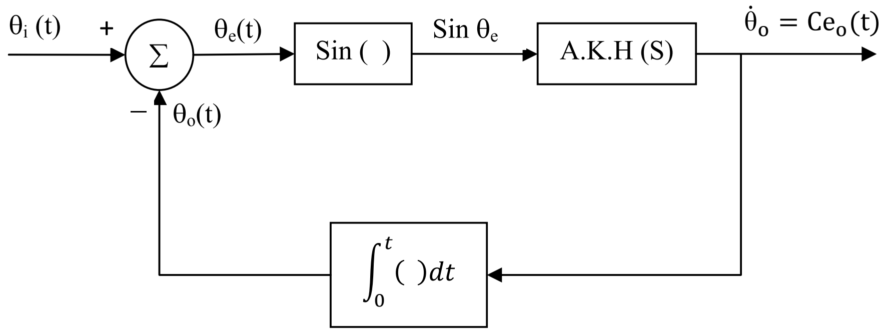
Phase Lock Loop (PLL)

The PLL is an important circuit which helps to detect the original signal from a frequency modulated signal corrupted by noise. The operation of this device has been properly explained in Module II.

In fact PLL is very popular because of their low cost and superior performance, especially when SNR is low. FM demodulation using PLL is the most widely used method today. We know PLL tracks the incoming signal angle and instantaneous frequency.



a)



b)

Figure 4.16 a) Phase Lock Loop (PLL) b) Equivalent circuit of PLL

The free running frequency of VCO is set at the carrier frequency ω_c. The instantaneous frequency of the VCO can be given by

$$\omega_{VCO} = \omega_c + C.e_o(t) \tag{4.56}$$

If the VCO output is $B.\cos\{\omega_c t + \theta_o(t)\}$, then the instantaneous frequency can be represented as $\omega_{VCO} = \omega_c + \dot{\theta}_o(t)$ (4.57)

This means, $\dot{\theta}_o(t) = Ce_o(t)$ (4.58)

In the above equations C and B are constants of PLL.

The multiplier output in figure 4.16 a) is $AB.\sin(\omega_c t + \theta_i)\cos(\omega_c t + \theta_o) = (AB/2)[\sin(\theta_i - \theta_o) + \sin(2\omega_c t + \theta_i + \theta_o)]$. The term $(AB/2).\sin(2\omega_c t + \theta_i + \theta_o)$ is suppressed by the loop filter (LPF). Hence the effective input to the is $(AB/2).\sin\{\theta_i(t) - \theta_o(t)\}$. If h(t) is the unit impulse response of the loop filter, then

$$e_o(t) = h(t) \star \frac{1}{2}AB\sin\{\theta_i(t) - \theta_o(t)\} = \frac{1}{2}AB\int_0^T h(t-x)\sin\{\theta_i(x) - \theta_o(x)\}dx \tag{4.59}$$

But, $\dot{\theta}_o(t) = Ce_o(t)$, therefore $\dot{\theta}_o(t) = AK\int_0^t h(t-x)\sin\theta_e(x)\,dx$ (4.60)

Where, K = (CB/2) and $\theta_e(t)$ is the phase error and defined by $\theta_e(t) = \theta_i(t) - \theta_o(t)$ i.e. $\theta_o(t) = \theta_i(t) - \theta_e(t)$.

FM carrier is $A.\sin\{\omega_c t + \theta_i(t)\}$

Where, $\theta_i(t) = K_f \int_{-\infty}^{t} m(\alpha)d\alpha$ (4.61)

Hence, $\theta_o(t) = K_f \int_{-\infty}^{t} m(\alpha)d\alpha - \theta_e(t)$ (4.62)

When θ_e is very small, then $e_o(t) = \frac{1}{C}\dot{\theta}_o(t) \simeq \frac{K_f}{C}m(t)$ (4.63)

Thus PLL works as a FM demodulator. If the incoming signal is phase modulated wave, then, $\theta_o(t) = \theta_i(t) = K_p m(t)$ and $e_o(t) = K_p\dot{m}(t)/C$. In this case we need to integrate $e_o(t)$ to obtain the desired signal.