Lectures notes

On

Machine Dynamics II

Course Code- BME-317

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BME 317 : Machine Dynamics-II (M)

Module – I

1. **Toothed Gears**: Theory of shape and action of tooth properties and methods of generation of standard tooth profiles, standard proportion , interference and undercutting, methods for eliminating interference ,minimum number of teeth to avoid interference.(7)
2. **Gyroscope**:Gyroscopic couple, plane disc, analysis of forces on bearing due to forced precession of rotating disc mounted on shaft, gyroscopic effect on a two wheel and a four wheel vehicle ,gyroscopic stabilization(6)

Module - II

3. **Cams**: Simple harmonics, constant velocity and acceleration types, displacement, velocity and acceleration of follower ,cams with specified contours.(6)
4. **Governors**: Centrifugal governors – Watt and Porter governors ,spring loaded governors - Hartnell governors , sensitiveness ,stability isochronism, hunting , governer effort and power ,curves of controlling force , effect of friction .(6)

Module - III

5. **Balancing**: Balancing of revolving masses in one plane and different planes ,partial balancing of single cylinder ,engine balancing of multicylinder engine ,v and radial engine ,methods of direct and reverse cranks(5)
6. **Dynamics Of Machine**: Turning moment diagram ,flywheel.(3)

Module - IV

7. **Vibration** : Introduction to vibration, causes of vibration, elimination of vibration, types of vibration – longitudinal, transverse , torsional ; definition of terminology like natural frequency, amplitude , time period , free vibration, forced vibration, resonance, degree of freedom with examples , calculation of natural frequency of undamped single degree of freedom system by Newton's 2nd Law, D-Alembert's principle and energy method, Equivalent spring constant for the system having different types of combination of springs and calculation of their natural frequencies , calculation of natural frequency of single degree of spring – mass system taking mass of spring into account (7)

Text Books:

1. Theory of Machines by S S Ratan , TMH
2. Theory of machine by R K Bansal,Laxmi Pub. Pvt. Ltd.,

Reference Books:

1. Mechanism of Machine theory by Rao and Dulchipati, New Age Publication
2. Theory of Mechanism and Machine by Ghosh and Mallick ,East West Press

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DEPARTMENT OF MECHANICAL ENGG.

LESSON PLAN FOR **Machine Dynamics**

SUBBJECT CODE: **BME- 317, 6th Semester Mechanical**

Lecture	**Topics to be covered**	**Remark**
Lecture 1	Toothed Gears: Introduction, classification of gears, gear terminology.	
Lecture 2	Gear terminology (contd.), law of gearing, velocity of sliding,	
Lecture 3	different forms of teeth, path of contact, arc of contact,	
Lecture 4	Number of pairs of teeth in contact, numerical problems.	
Lecture 5	Interference in involute gears, minimum number of teeth to avoid interference, interference between rack and pinion, numerical problems.	
Lecture 6	Numerical problems on interference, minimum number of teeth.	
Lecture 7	Undercutting, numerical problems.	
Lecture 8	Gyroscope: angular velocity, angular acceleration, gyroscopic couple, numerical problems.	
Lecture 9	Effect of gyroscopic couple on bearings, numerical problems.	
Lecture 10	Stability of 4-wheel automobile, numerical problems	
Lecture 11	Stability of two wheel vehicle, numerical problems	
Lecture 12	Effect of gyroscopic couple on naval ship, numerical problems.	
Lecture 13	Effect of gyroscopic couple on aeroplane.	
Lecture 14	Cams: Introduction, classification of cams, classification of followers, basic terminology used.	
Lecture 15	Motion of follower: Simple harmonics, numerical problems.	
Lecture 16	Constant velocity motion of follower, numerical problems.	
Lecture 17	Uniform acceleration retardation of follower, numerical problems.	
Lecture 18	Cams with specified contours	
Lecture 19	Numerical problems on Cams with specified contours	
Lecture 20	Governors: Introduction, types of governor, centrifugal governors, watt and porter governor, terminology used,	
Lecture 21	Numerical problems on centrifugal governors	
Lecture 22	Spring loaded governors: Hartnell governor, numerical problems.	
Lecture 23	sensitiveness ,stability isochronism, hunting, numerical problems	
Lecture 24	curves of controlling force , effect of friction	

Lecture 25	Numerical problems on governors.	
Lecture 26	Balancing: Balancing of single revolving mass in same plane, balancing of several revolving masses in same plane, several rotating masses in different plane.	
Lecture 27	Numerical problems on revolving masses.	
Lecture 28	Static and dynamic balancing, balancing of reciprocating mass, partial balancing of single cylinder engine, numerical examples.	
Lecture 29	Partial balancing of multi-cylinder engine, numerical problems.	
Lecture 30	Direct and reverse crank method of balancing, numerical problems.	
Lecture 31	Turning moment diagram of flywheel: fluctuation of energy, coefficient of fluctuation of energy, numerical examples.	
Lecture 32	Flywheel: energy stored in a flywheel, dimensions of the flywheel rim, flywheel and punching press. Numerical examples.	
Lecture 33	Numerical examples on flywheel.	
Lecture 34	Vibration: Introduction to vibration, causes of vibration, elimination of vibration, types of vibration – longitudinal, transverse , torsional ; definition of terminology like natural frequency, amplitude , time period	
Lecture 35	Calculation of natural frequency of undamped single degree of freedom system by Newton's 2nd Law, D-Alembert's principle and energy method.	
Lecture 36	Numerical examples on calculation of natural frequency.	
Lecture 37	D-Alembert's principle and energy method, Equivalent spring constant for the system having different types of combination of springs and calculation of their natural frequencies ,	
Lecture 38	Numerical examples on D-Alembert's principle and energy method, and on calculation of natural frequency.	
Lecture 39	calculation of natural frequency of single degree of spring – mass system taking mass of spring into account	
Lecture 40	Numerical examples on calculation of natural frequency of single of spring mass-system.	

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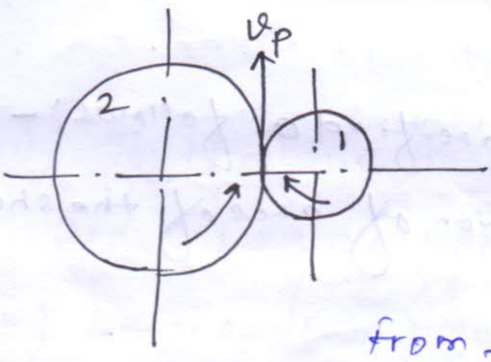
— Toothed Gears —

Gear :- Gears are used to transmit motion from one shaft to another or both a shaft and a slide. This motion transmission is accomplished by means of successive engagement of teeth.

Mechanics of power transmission by Gear :-

Generally two friction wheels can be used to transmit power if the power to be transmitted is very small.



Considering the linear velocity to be v_p, we have

$$v_p = \omega_1 r_1 = \omega_2 r_2 \quad -(1)$$

Also

$$v_p = 2\pi N_1 r_1 = 2\pi N_2 r_2 \quad -(2)$$

from equation (1) and (2)

$$\boxed{\frac{\omega_1}{\omega_2} = \frac{r_2}{r_1} = \frac{N_1}{N_2}}$$

Where N = ~~an~~ rpm

ω = angular velocity (rad/s)

r = radius of wheel.

- It indicates that the speed of two discs rolling together without slipping are inversly propertional to the radii of the discs.
- Incase ~~of~~ to transmit a definite motion of one disc to the another or to prevent slip both the surfaces, both the disce can ~~be~~ have projections on their surfaces. This leads to the formation of teeth on the disc and the motion of surfaces change from rolling to sliding.
- This disce with teeth are called gears.

Classification of Gear :-

Gears can be classified according to the relative position of their shaft axes as follows :-

- Gears which transmit power or motion both parallel axes are spur, helical and herringbone.
- Gears used for joining, intersecting and coplanar shafts are bevel gears.
- Worm and worm gears are used for joining the shafts in different planes.

~~1. Parallel shaft~~

So the gears may be classified as follows :-

I. According to the position of axes of the shafts :-

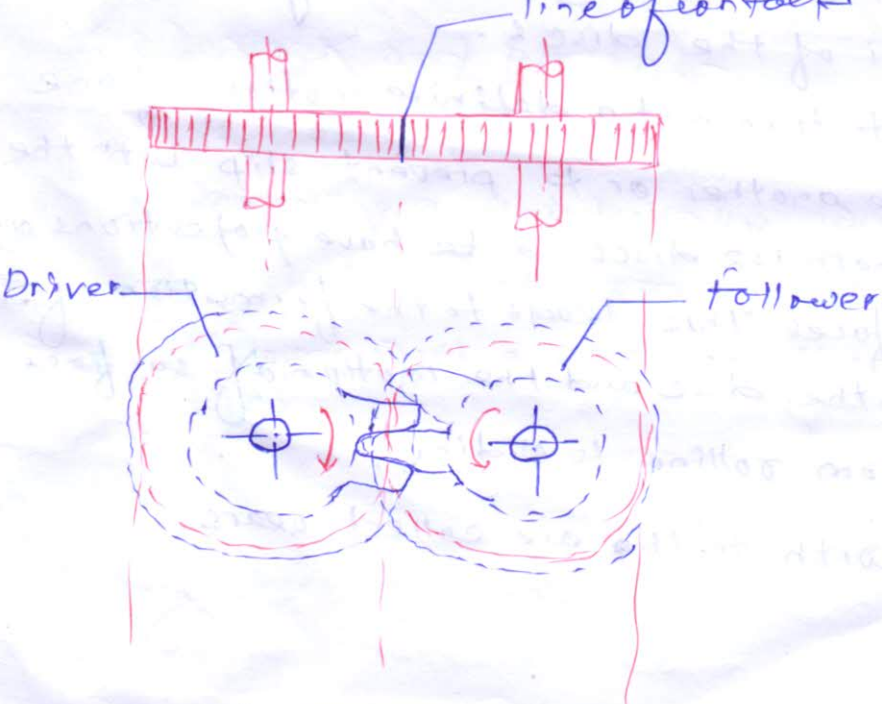
(a) parallel

(b) Intersecting and

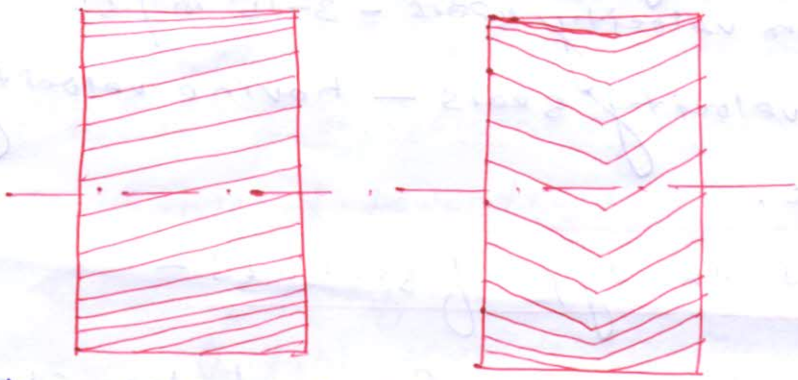
(c) non intersecting and non parallel

(a) parallel shaft :-

(i) spur gear :- spur gears have teeth parallel to the axis of shaft as shown in the figure.



(ii) Helical Gear! Helical gears have teeth inclined to the axis.

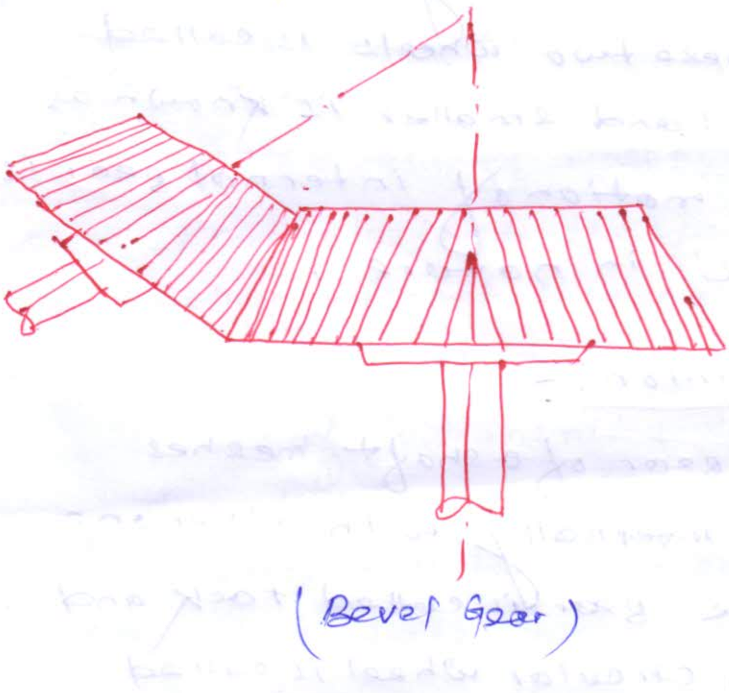


(Single helical Gear) (Double helical Gear)

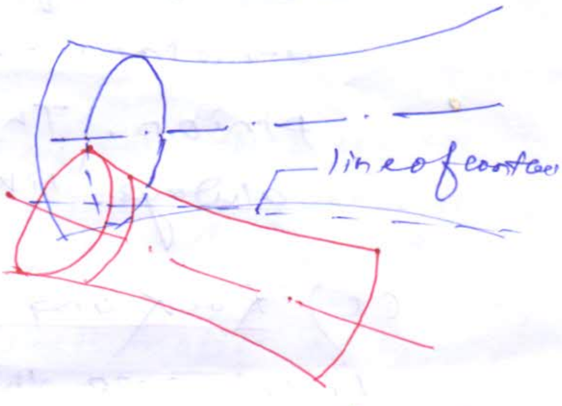
(iii) Herringbone Gear!– Double helical gear are known as herringbone gear.

(b) Intersecting (non parallel) shaft!–

(i) Bevel Gear!– Bevel gears are connected by two non parallel or intersecting but coplanar shafts.



(Bevel Gear)



(Spiral Gear)

(c) Non intersecting and non parallel shaft!–

(i) spiral Gear or skew bevel gear!–

~~The twon~~ spiral gears are connected by two non intersecting and non parallel i.e non coplanar shafts.

2. According to the peripheral velocity of Gears:-

(a) Low velocity gears - having vel. < 3m/s

(b) medium velocity gears - 3-15 m/s.

(c) High velocity gears - having velocity > 15m/s.

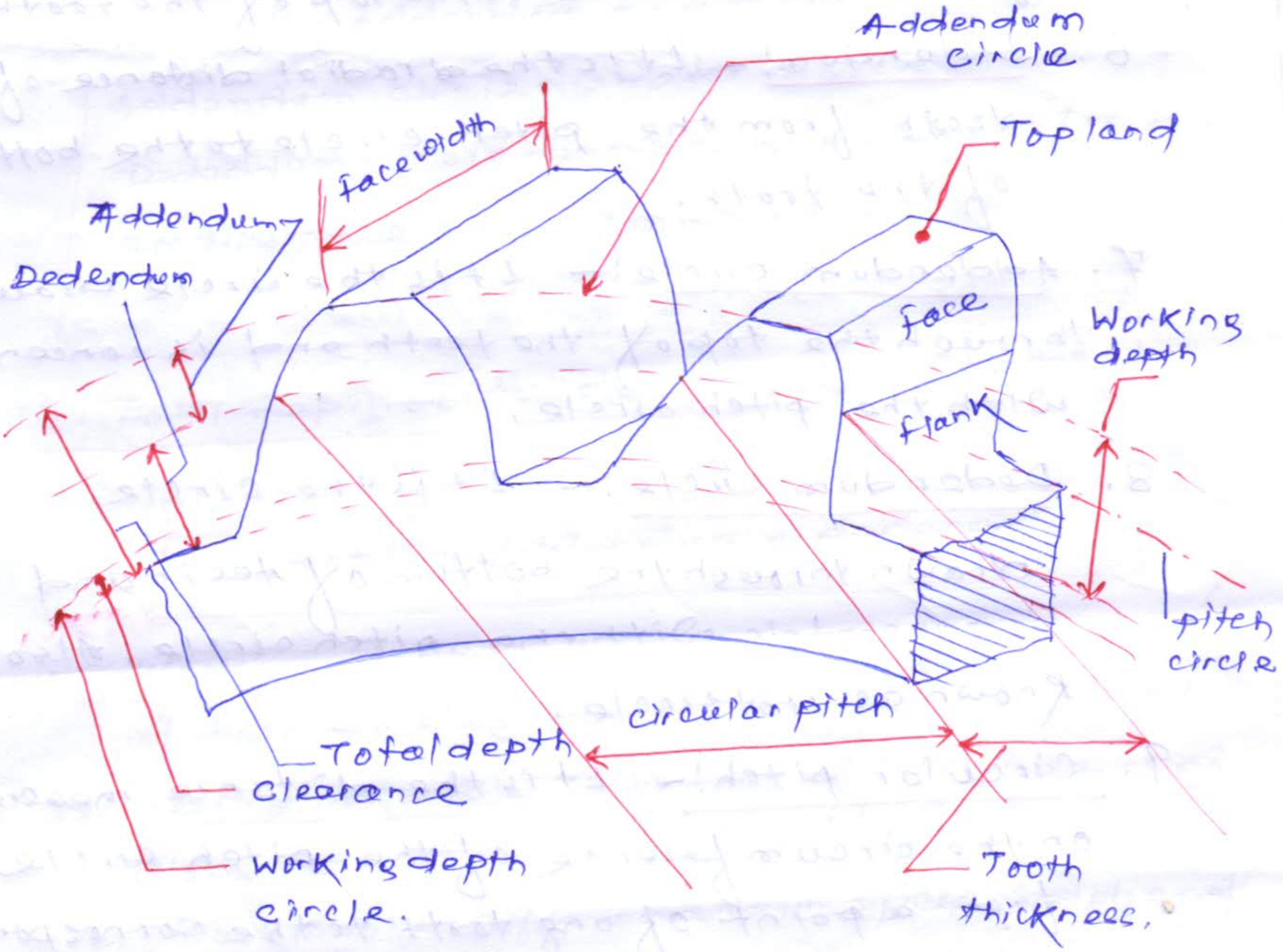
3. According to the type of gearing:-

(a) External Gearing:- Gears of two shafts mesh externally with each other. e.g spur gear. In which the smaller wheel is called pinion. In case of external gearing the motion of two gears is always unlike.

(b) Internal Gearing:- In this case gears of two shafts mesh internally with each other. The larger of these two wheels is called annular wheel and smaller is known as pinion. The motion of internal gear is always 'like' in nature.

(c) Rack and pinion:-
In this case the gear of a shaft meshes externally and internally with other one. The straight line gear is called rack and the gear with a circular wheel is called pinion.

Gear Terminology:-



1. Pitch circle:- An imaginary circle which by pure rolling action, would give the same motion as that of the actual gear.

2. Pitch circle diameter:-

It is the diameter of pitch circle. The size of gear is usually specified by the pitch circle diameter. It is also known as pitch diameter.

3. Pitch point:- The common point of contact betn two pitch circles.

4. Pressure angle or angle of obliquity:- It is the angle betn the common normal

to two gear teeth at the point of contact and the common tangent at pitch point. It is usually denoted by ϕ. Standard pressure angle varies betn $14\frac{1}{2}°$ to $20°$.

5. Addendum! – It is the radial distance of a tooth from the pitch circle to the top of the tooth.

6. Dedendum! – It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.

7. Addendum circle! – It is the circle drawn through the top of the teeth and is concentric with the pitch circle.

8. Dedendum circle! – It is the circle drawn through the bottom of teeth and is concentric with the pitch circle. Also known as root circle.

9. Circular pitch! – It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth.

Circular pitch $\boxed{P_c = \frac{\pi D}{T}}$

where D = pitch circle diameter

T = no of teeth on the wheel.

10. Diametral pitch! –

It is the ratio of number of teeth to the pitch circle diameter in mm. It is denoted by P_d

$$\boxed{P_d = \frac{T}{D} = \frac{\pi}{P_c}}$$

Where T = no of teeth, D = pitch circle diameter.

11. Module:-

It is the ratio of pitch circle diameter in mm. to the no. of teeth. It is ueeually denoted by m and can be expressed as $\boxed{m = \frac{D}{T}}$

12. Clearance:- It is the radial distance ~~from~~ between the addendum and dedendum of a tooth.

$$\boxed{\begin{aligned} &\text{Addendum circle diameter} = d+2m \\ &\text{Dedendum circle diameter} = d-2\times 1.157\,m \\ &\text{so clearance} = 1.157\,m - m \\ &= 0.157\,m. \end{aligned}}$$

13. Total depth or full depth:- It is the total radial depth of the troth space.

$$\boxed{\text{full depth} = \text{Addendum} + \text{dedendum}}$$

14. Working depth:- It is the sum of addendums of two meshing gears.

15. Tooth thickness:- Thickness of tooth measured along the pitch circle.

16. Backlash:- Difference betn space width and and the tooth thickness.

17. Space width:- width of tooth space along the pitch circle.

18. Face width:- Width of gear tooth along the gear axis.

19. Top land:- Top surface of tooth.

20. Bottom land:- Bottom surface of tooth betn the adjacent fillet.

21. Face:- Tooth surface betn the pitch circle and the top land.

22. Flank:- Tooth surface betn the pitch circle and bottom land including fillet.

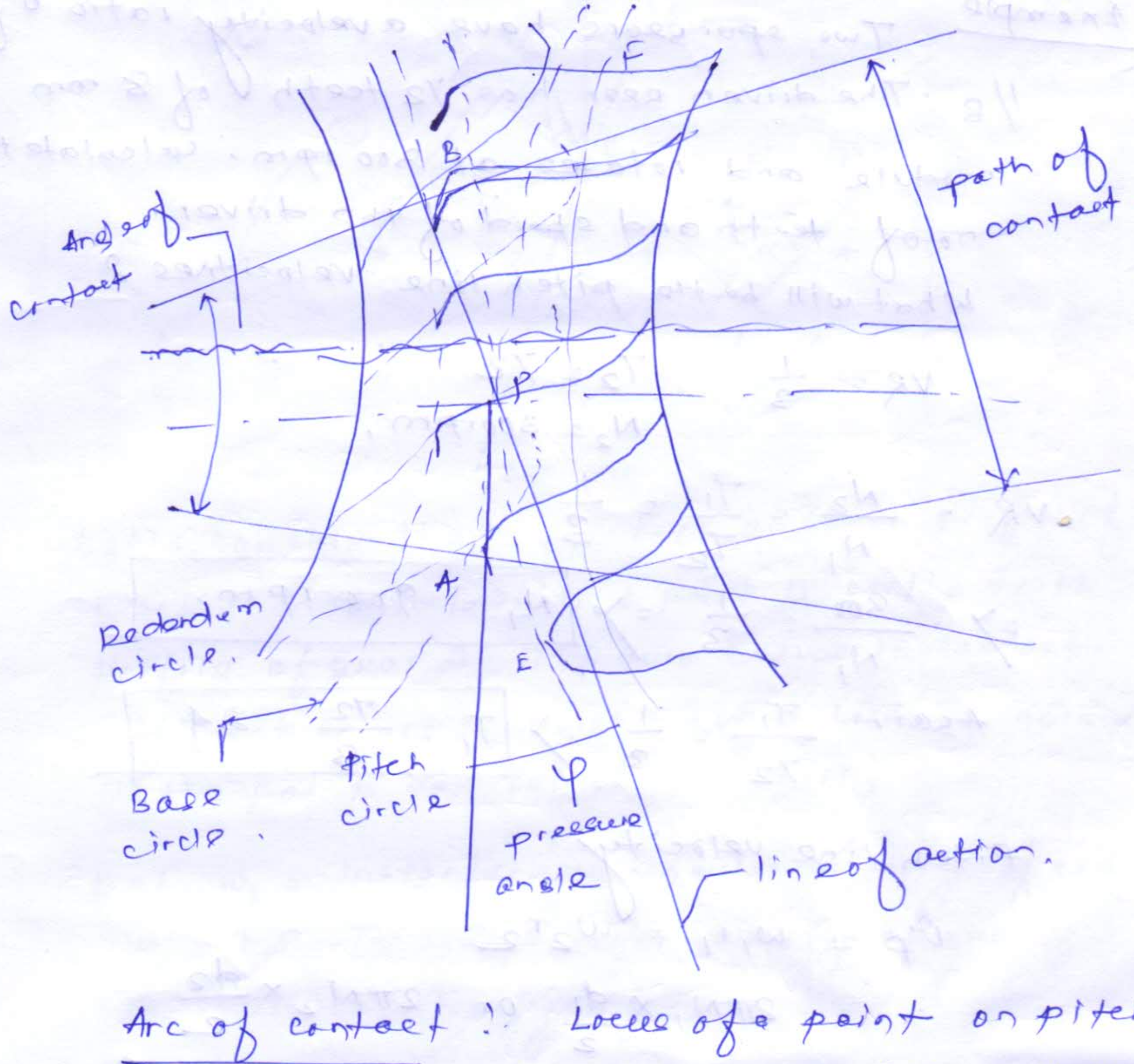
23. Fillet :- Curved portion of the tooth flank at the root circle.

24. Line of action or Pressure line :-
The force which the driving tooth exerts on the driven tooth, is along a line from the pitch point to the point of contact of the two ~~tooth~~ teeth. This line is also the common normal at the point of contact of meshing gears and is ~~also~~ known as the line of action or pressure line.

~~25. Pressure angle or angle of obliquity (~~

Line of action or Pressure line :-

The force which the driving tooth exerts on the driven tooth, is along a line from the pitch point to the point of contact of the two teeth. This line is also the common normal at the point of contact of mating gears and is known as the line of action or pressure line.



Arc of contact :- Locus of a point on pitch circle from the begining to the end of engagement of two mating gears is known as the arc of contact. In the above fig. APB or EPF is the arc of contact.

Arc of contact is divided into two sub-portions

(i) Arc of approach :- It is the portion of the arc of contact from the beginning of engagement to pitch point ~~to~~ ~~the~~ i.e length AP or EP.

(ii) Arc of Recess :- Portion of arc of contact ~~to~~ from the pitch point to the end of end of engagement is called arc of recess. i.e length PB or ~~PE~~. PF.

Example Two spur gears have a velocity ratio of 1/3. The driven gear has 72 teeth of 8 mm module and rotates at 300 rpm. Calculate the no of teeth and speed of the driver. What will be the pitch line velocities ?

$$VR = \frac{1}{3} \qquad T_2 = 72$$
$$N_2 = 300\,rpm,$$

$$VR = \frac{N_2}{N_1} = \frac{T_1}{T_2} = \frac{1}{3}$$

$$\Rightarrow \frac{300}{N_1} = \frac{1}{3} \Rightarrow \boxed{N_1 = 900\ rpm.}$$

$$\text{Again } \frac{T_1}{72} = \frac{1}{3} \Rightarrow \boxed{T_1 = \frac{72}{3} = 24}$$

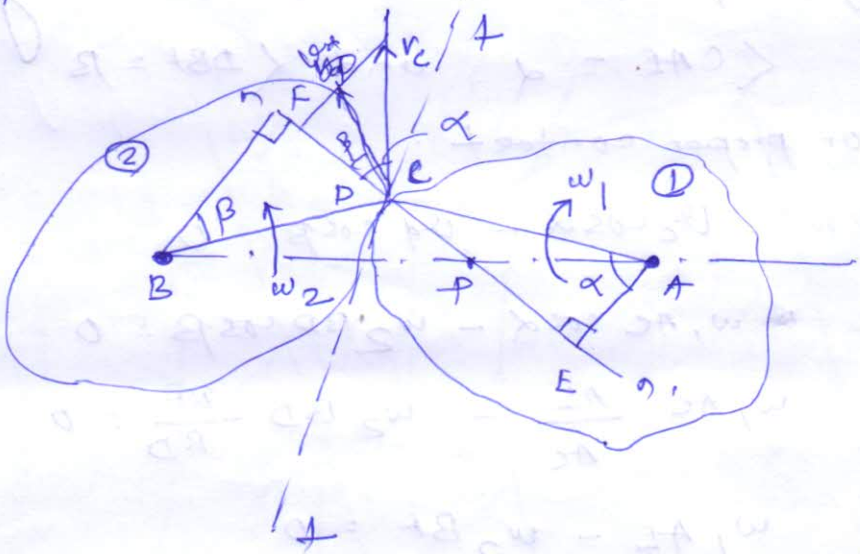
Pitch line velocity

$$v_p = \omega_1 r_1 = \omega_2 r_2$$
$$= 2\pi N_1 \times \frac{d_1}{2} \text{ or } 2\pi N_2 \times \frac{d_2}{2}$$
$$= 2\pi N_1 \times \frac{mT_1}{2} \text{ or } 2\pi N_2 \times \frac{mT_2}{2}$$
$$= \cancel{2}\pi \times 900 \times \frac{8 \times 24}{\cancel{2}}$$
$$= 9047.8\ mm/s \text{ or } \boxed{9.0478\ m/s}$$

Law of Gearing

The law of gearing states ~~that~~ the condition which must be fullfilled by the gear tooth profiles to maintain a constant angular velocity ratio betn two gears.

- And for constant angular velocity ratio of two gears, the common normal at the point of contact of two mating teeth must pass through the pitch point.



Let's consider that point C on tooth profile of gear 1 is in contact with a point D on the tooth profile of gear 2. The two curves in contact at point C or D must have a common normal at the point and let it be n-n,

Let ω_1 = Instantaneous angular velocity of gear 1

ω_2 = Instantaneous angular velocity of gear 2,

v_c = linear velocity of C

v_d = linear velocity of D.

Then $v_c = \omega_1 AC$ in a direction $\perp r$ to AC or at an angle α to n-n

$v_d = \omega_2 BD$ in a direction $\perp r$ to BD or at angle β to n-n.

The relative motion betn the surfaces along the common normal n-n must be zero to avoid the separation, or the penetration of the two teeth into each other.

Now component of v_c along n-n = $v_c \cos\alpha$

component of v_d along n-n = $v_d \cos\beta$

relative motion along n-n = $v_c \cos\alpha - v_d \cos\beta$

Now drawing ⊥r AE and BF on n-n from points A and B respectively, then

$$\angle CAE = \alpha \quad \text{and} \quad \angle DBF = \beta$$

for proper contact:

$$v_c \cos\alpha - v_d \cos\beta = 0.$$

or $\omega_1 AC \cos\alpha - \omega_2 BD \cos\beta = 0$

$$\omega_1 AC \frac{AE}{AC} - \omega_2 BD \frac{BF}{BD} = 0$$

$$\omega_1 AE - \omega_2 BF = 0$$

$$\Rightarrow \boxed{\frac{\omega_1}{\omega_2} = \frac{BF}{AE} = \frac{BP}{AP}}$$

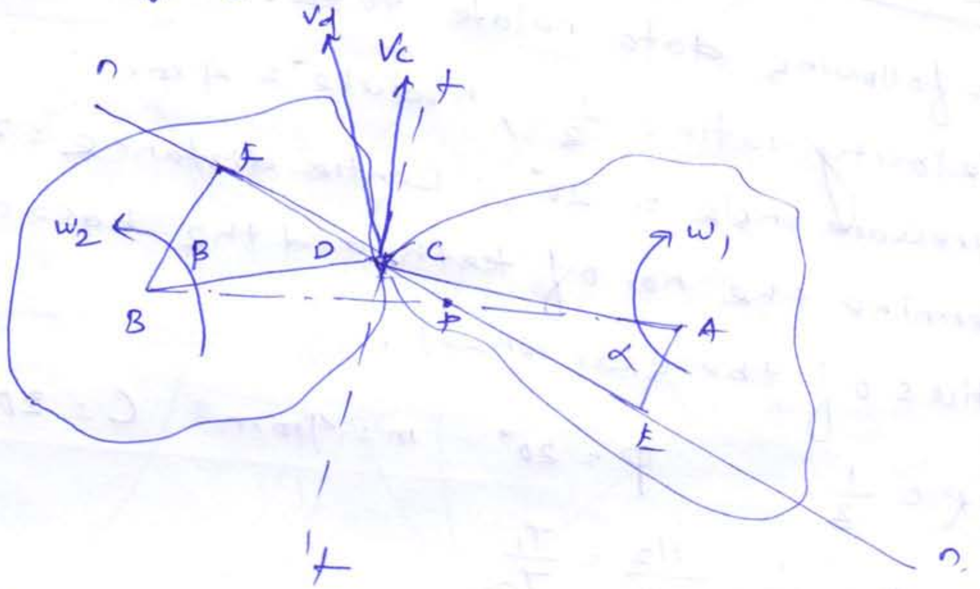
(∵ Δs AEP and BFP are similar)

Gear Materials :-

The materials used for manufacturing of gears depends upon the strength and service conditions like wear, noise etc.

- Gears may be metallic or non metallic. commercially available metallic gears are made of cast iron, steel and bronze.
- Non metallic gears are made of synthetic resins, wood; etc.

y of sliding:-



If the curved surfaces of the two teeth of the gears 1 and 2 are to remain in contact, one can have a sliding motion relative to the other along the common tangent t-t at C or D.

Component of v_c along t-t = $v_c \sin\alpha$

Component of v_D along t-t = $v_D \sin\beta$.

Velocity of sliding = $v_c \sin\alpha - v_d \sin\beta$

$= \omega_1 \cdot AC \cdot \frac{EC}{AC} - \omega_2 BD \cdot \frac{FD}{BD}$

$= \omega_1 EC - \omega_2 FD$

$= \omega_1 (EP + PC) - \omega_2 (FP - PD)$

$= \omega_1 EP + \omega_1 PC - \omega_2 FP + \omega_2 PD$

$= (\omega_1 + \omega_2) PC + \omega_1 EP - \omega_2 FP$

$= \boxed{(\omega_1 + \omega_2) PC}$ $\quad (\because \omega_1 EP = \omega_2 FP)$

= sum of angular velocities × Distance betn the pitch point and the point of contact.

Example - 2

The following data relate to two meshing gears:

velocity ratio $= \frac{1}{3}$, module $= 4$ mm

pressure angle $= 20^\circ$, Centre distance $= 200$ mm

Determine the no. of teeth and the base circle radius of the gear wheel.

$VR = \frac{1}{3}$ $\quad \phi = 20^\circ$ $\quad m = 4$ mm $\quad C = 200$ mm

(i) $VR = \frac{1}{3} = \frac{N_2}{N_1} = \frac{T_1}{T_2}$

$\Rightarrow \boxed{T_2 = 3T_1}$

And centre distance $C = \frac{d_1 + d_2}{2} = \frac{m(T_1 + T_2)}{2}$

$\Rightarrow 200 = \frac{4(T_1 + T_2)}{2} = \frac{4(T_1 + 3T_1)}{2}$

$\Rightarrow 200 = 8T_1 \Rightarrow \boxed{T_1 = 25}$

$\Rightarrow \boxed{T_2 = 75}$

Now we have $d_1 = mT_1 = 4 \times 25 = 100$ mm

$d_2 = mT_2 = 4 \times 75 = 300$ mm.

Base circle radius of driven wheel

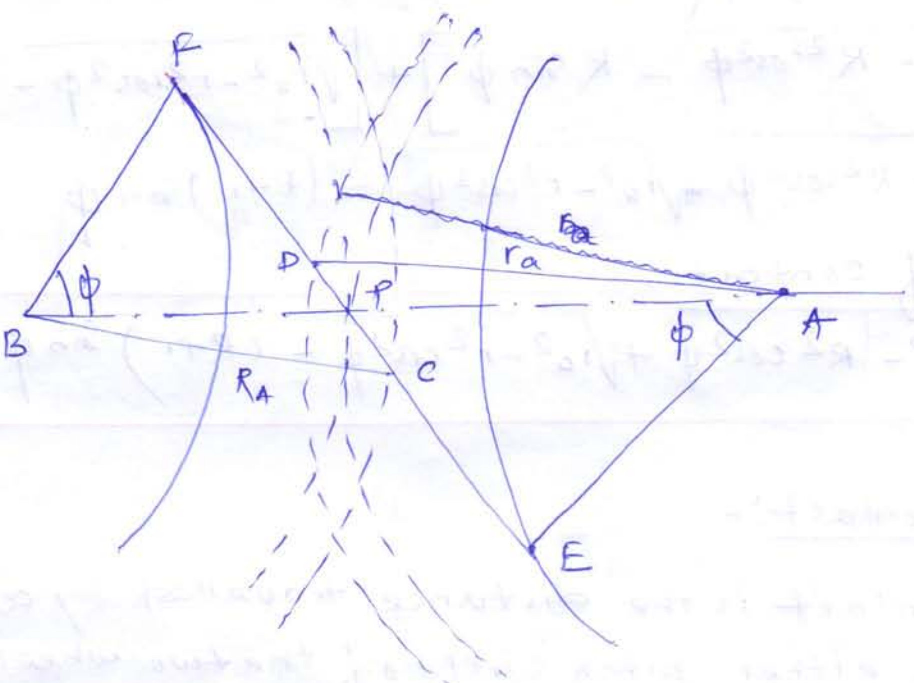
$= \frac{d_2}{2} \cos\phi = 150 \cos 20^\circ$

$= 140.95 \simeq \boxed{141 \text{ mm}}$

Path of contact

Let two gear wheel with centres A and B are in contact with each other. The gear wheel 1 is the driver and is rotating in clockwise direction and the driven i.e the gear wheel 2 is rotating in anticlockwise direction.

EF ~~EF~~ is the common tangent to their base circles.



contact of the two teeth is made where addendum circle of the gear meet the line of action EF.

CD is then the path of contact.

Let r = pitch circle radius of driving gear

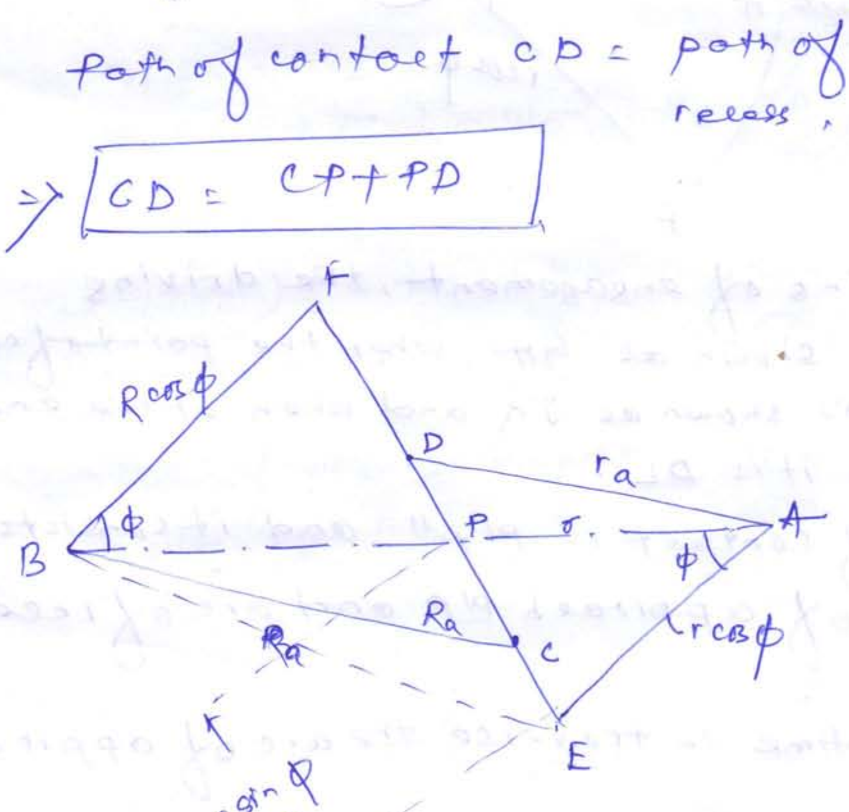
R = pitch circle radius of driven gear.

r_a = addendum circle radius of driving gear

R_a = addendum circle radius of driven gear.

Path of contact CD = path of approach + path of recess.

$\Rightarrow \boxed{CD = CP + PD}$



Now, we have $CD = CP + PD$

$$= (CF - PF) + (DE - PE)$$

$$= \left[\sqrt{R_a^2 - R^2\cos^2\phi} - R\sin\phi\right] + \left[\sqrt{r_a^2 - r^2\cos^2\phi} - r\sin\phi\right]$$

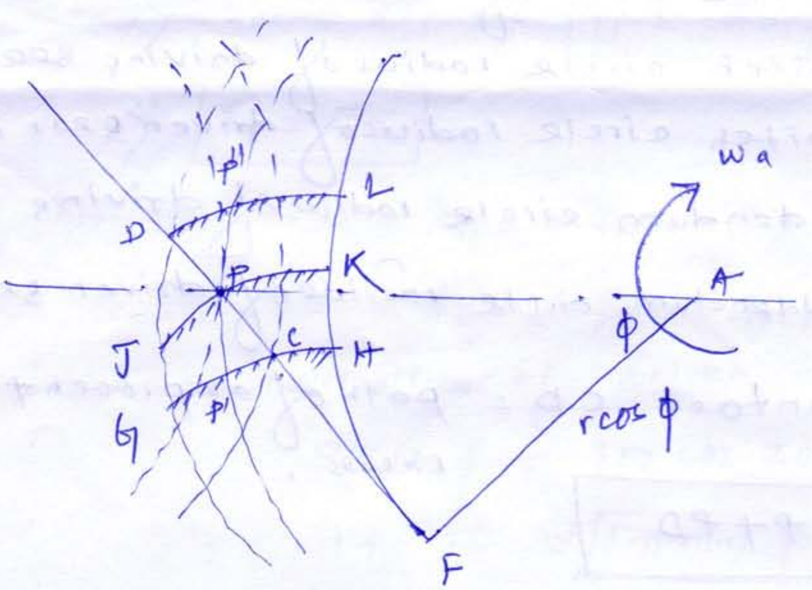
$$= \sqrt{R_a^2 - R^2\cos^2\phi} + \sqrt{r_a^2 - r^2\cos^2\phi} - (R+r)\sin\phi$$

So path of contact

$$\boxed{CD = \sqrt{R_a^2 - R^2\cos^2\phi} + \sqrt{r_a^2 - r^2\cos^2\phi} - (R+r)\sin\phi}$$

Arc of contact:-

Arc of contact is the distance travelled by a point on either pitch circle of the two wheels during the period of contact of a pair of teeth.



At the begining of engagement, the driving Involute is shown as GH, when the point of contact is at P, it's shown as JK and when at the end of engagement it is DL.

The arc of contact is P'P'' and it consists of the arc of approach P'P and arc of recess PP''.

Let t_a = time to traverse the arc of approach, Now arc of approach

$p'p$ = Tangential velocity of p' x time of approach.

$= \omega_a r \times t_a$

$= \omega_a \times (r\cos\phi) \cdot \frac{1}{\cos\phi} \cdot t_a$

$= (\text{tangential velocity of } H)\, t_a \cdot \frac{1}{\cos\phi}$

$(\because AF = AH)$

$= \frac{\text{Arc } HK}{\cos\phi}$

$= \frac{\text{Arc } FK - \text{Arc } FH}{\cos\phi}$

$= \frac{FP - FC}{\cos\phi}$

$= \frac{CP}{\cos\phi}$

Similarly we can have arc of ~~reces~~ recess

$pp'' = \frac{PD}{\cos\phi}$

so arc of contact $= \frac{CP}{\cos\phi} + \frac{PD}{\cos\phi} = \frac{CP + PD}{\cos\phi}$

$\Rightarrow \boxed{\text{arc of contact} = \frac{CD}{\cos\phi}}$

Number of pairs of teeth in contact:-

The arc of contact is the length of the pitch circle traversed by a point on it during the mating of pair of teeth.

So all the teeth lying in betn the arc of contact $p'p''$ will be meshing with teeth on the other wheel.

No of teeth on $p'p''$ = $\frac{\text{arc } p'p''}{\text{circular pitch}}$

$\boxed{n = \frac{CD}{\cos\phi} \cdot \frac{1}{p}}$

For continuous transmission of motion, at least one tooth of one wheel must be in contact with another tooth of the second wheel.

Therefore n must be greater than unity.

Q.1 Two gears in mesh have a module of 8mm and a pressure angle of 20°. The larger gear has 57 and pinion has 23 teeth. If the addenda on pinion and gear wheel are equal to one module find,

(i) no of pairs of teeth in contact

(ii) angle of action of pinion and gear wheel

(iii) ratio of sliding to rolling velocity at

(a) begining of contact

(b) pitch point

(c) end of contact.

Given data: $\phi = 20°$

$T = 57 \quad t = 23$

addendum = 1 module = 8 mm.

$R = \frac{mT}{2} = \frac{8\times 57}{2} = 228$ mm

$r = \frac{mT}{2} = \frac{8\times 23}{2} = 92$ mm

$r_a = r + m = 92 + 8 = 100$ mm $\quad R_a = 228 + 8 = 236$

(i) $n = \frac{\text{arc of contact}}{\text{circular pitch}}$

$= \left(\frac{\text{Path of contact}}{\cos\phi}\right) \times \frac{1}{\pi m} \quad \left(P_c = \frac{\pi D}{T}\right)$

$= \frac{\left(\sqrt{R_a^2 - R^2\cos^2\phi} - R\sin\phi\right) + \left(\sqrt{r_a^2 - r^2\cos^2\phi} - r\sin\phi\right)}{\cos\phi \times \pi m}$

$= \frac{\sqrt{236^2 - 228^2\cos^2 20} - 228\sin 20 + \sqrt{100^2 - 92^2\cos^2 20} - 92\sin 20}{\cos 20 \times \pi \times 8}$

$= 1.68$

(ii) Angle of action = angle traversed by arc of contact

$$\boxed{\delta_p = \text{arc of contact} \times \frac{360}{2\pi r}}$$

$$= 42.31 \times \frac{360}{2\pi \times 92}$$

$$= 26.3^\circ$$

$$\delta_g = 42.31 \times \frac{360}{2\pi \times 228}$$

$$= 10.63^\circ$$

(iii) (a) at the begining

$$\frac{\text{Sliding velocity}}{\text{Rolling velocity}} = \frac{(w_p + w_g)\ \text{path of approach}}{\text{pitch line velocity.}}$$

$$= \frac{\left(w_p + \frac{23}{57} w_p\right) \times 20.97}{w_p \times 92} = 0.32$$

(b) $\frac{\text{Sliding velocity}}{\text{Rolling velocity}} = 0.$

(c) $\frac{\left(w_p + \frac{23}{57} w_s\right) \times \text{path of recess}}{\text{pitch line velocity}}$

$$= \frac{\left(1 + \frac{23}{57}\right) \times 18.79}{92} = 0.287.$$

We have from previous class!

Path of contact $CD = CP + PD$

path approach = dimension of driven; path of recess = dimension of driving wheel

$$\left[\sqrt{R_a^2 - R^2\cos^2\phi} - R\sin\phi\right] + \left[\sqrt{r_a^2 - r^2\cos^2\phi} - r\sin\phi\right]$$

$$\Rightarrow \boxed{\sqrt{R_a^2 - R^2\cos^2\phi} + \sqrt{r_a^2 - r^2\cos^2\phi} - (R+r)\sin\phi}$$

$$\boxed{\text{arc of contact} = \frac{\text{path of contact}}{\cos\phi}}$$

no of pairs of teeth in contact $n = \left(\frac{CD}{\cos\phi}\right)\cdot\frac{1}{p}$

or $\boxed{n = \frac{\text{arc of contact}}{\text{circular pitch}}}$

Q.1 Two 20° gears have a module pitch of 4mm the numbers of teeth on gear 1 is 40 and gear 2 is 24. If gear 2 rotates at 600 rpm, determine the velocity of sliding when the contact is at the tip of the tooth of gear 2. Take addendum equal to one module. Also find the max^m velocity of sliding.

Given data!. $\phi = 20°$ $m = 4$ mm.

$T = 40$. $t = 24$

$n = 600$ rpm.

addendum = 1 module = 4mm.

$R = \frac{mT}{2} = \frac{4\times40}{2} = 80$ mm

$R_a = 80+4 = 84$ mm.

$r = \frac{mt}{2} = \frac{4\times24}{2} = 48$ mm.

$r_a = 48+4 = 52$ mm

Now $\frac{N}{n} = \frac{t}{T} \Rightarrow N = 600\times\frac{24}{40} = \boxed{360 \text{ rpm}}$

(i) in case the pinion (gear 2) is driver.

We have path of recess

We have from previous class!

Path of contact $CD = CP + PD$

path approach = dimension of driven — path of recess = dimension of driving gear

$$\left[\sqrt{R_a^2 - R^2\cos^2\phi} - R\sin\phi\right] + \left[\sqrt{r_a^2 - r^2\cos^2\phi} - r\sin\phi\right]$$

$$\Rightarrow \boxed{\sqrt{R_a^2 - R^2\cos^2\phi} + \sqrt{r_a^2 - r^2\cos^2\phi} - (R+r)\sin\phi}$$

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$r_a = 48 + 4 = 52$ mm

Now $\frac{N}{n} = \frac{t}{T} \Rightarrow N = 600 \times \frac{24}{40} = \boxed{360 \text{ rpm}}$

(i) in case the pinion (gear 2) is driver.

We have path of recess

$= \sqrt{r_a^2 - r^2\cos^2\phi} - r\sin\phi$

$\sqrt{52^2 - 48^2\cos^2 20} - 48\sin 20 = 9.45$ mm.

Now velocity of Sliding

$= (\omega_1+\omega_2)\cdot$ path of recess.

$= 2\pi \, (N+n) \times 9.45$

$= 57049$ mm/min

$= \boxed{950.8 \text{ mm/sec}}$

<u>in case the gear wheel is driver</u>

path of approach

$= \sqrt{r_a^2 - r^2\cos^2\phi} - r\sin\phi$

$= 9.458$ mm.

$V_s = (\omega_1+\omega_2)\, 9.458$

$= \boxed{950.8 \text{ mm/s.}}$

<u>Maxm sliding velocity</u>

sliding velocity will be maxm when the path of contact is maxm.

Now considering the pinion to be driver.

We have <u>path of approach</u>

$= \sqrt{R_a^2 - R^2\cos^2\phi} - R\sin\phi$

$= \sqrt{84^2 - 80^2\cos^2 20^\circ} - 80\sin 20$

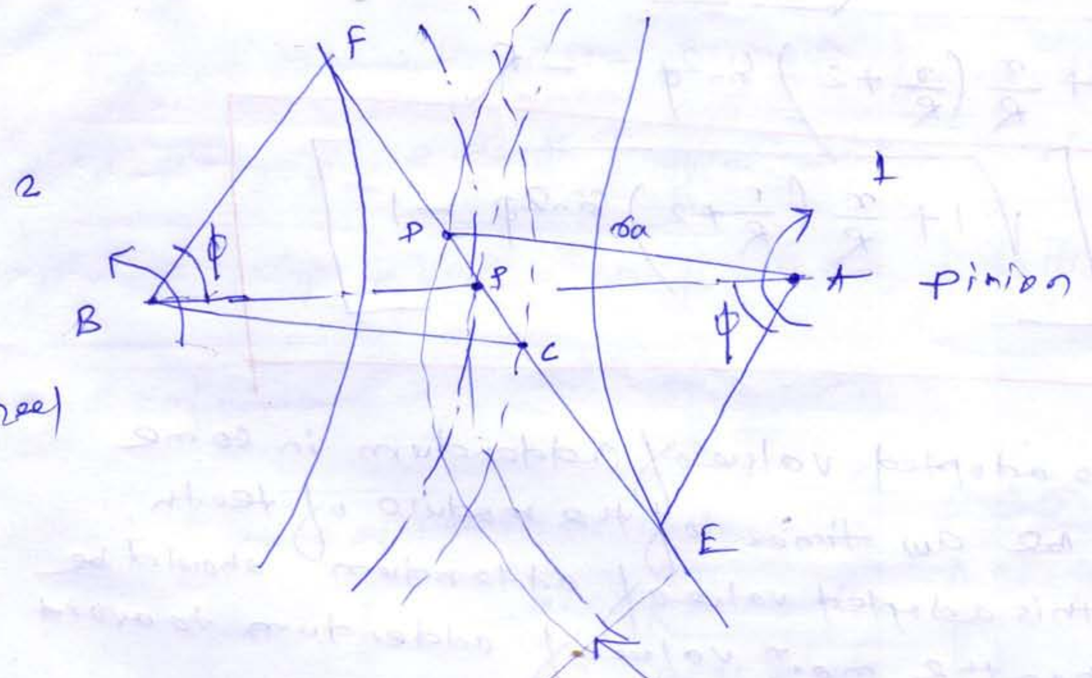
$= 10.117$ mm.

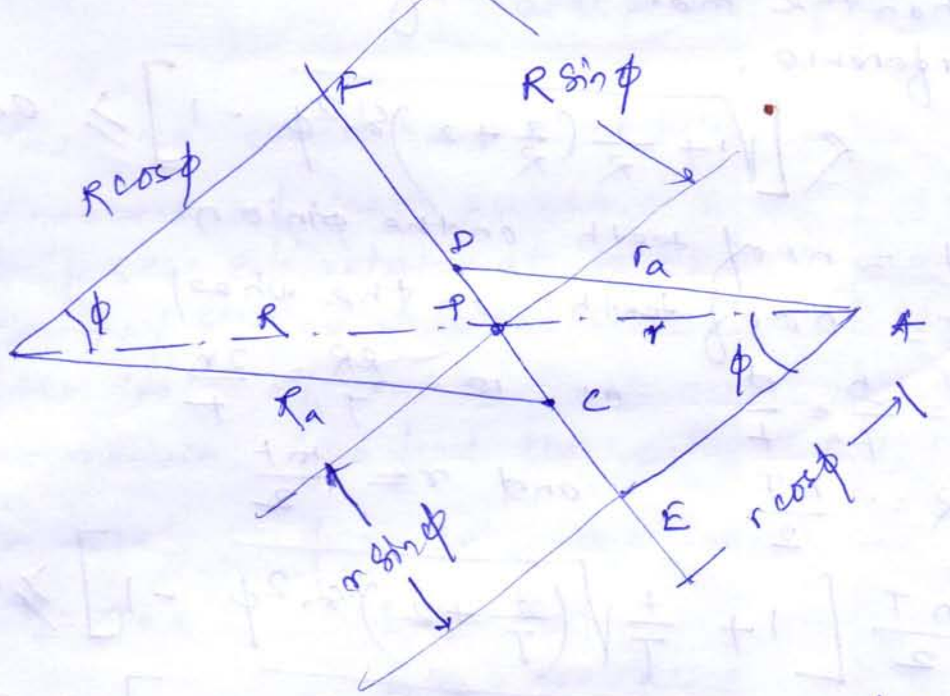
Maxm sliting velocity $= (\omega_1+\omega_2) \times$ maxm path of contact

$= 2\pi\,(600+360) \times 10.117$ (mm/min)

$= \boxed{1017.1 \text{ mm/s.}}$

Min^m no of teeth :-





The max^m value of addendum radius of the wheel to avoid interference can be upto BE

$$(BE)^2 = (BF)^2 + (FE)^2$$

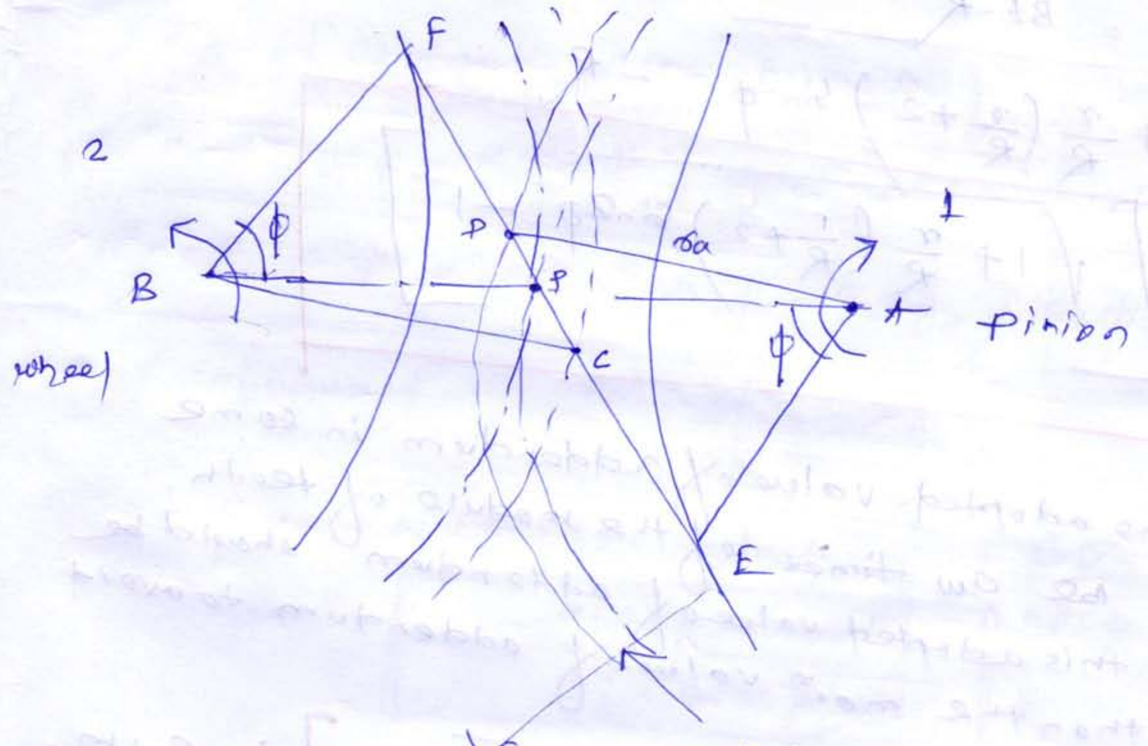
$$= (BF)^2 + (FP + PE)^2$$

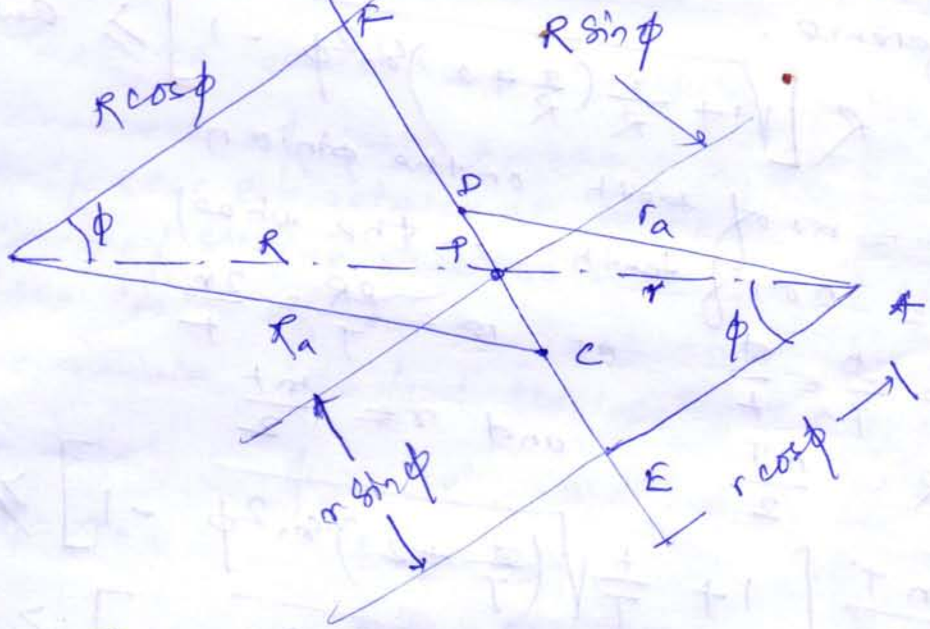
$$= (R\cos\phi)^2 + (R\sin\phi + r\sin\phi)^2$$

$$= R^2\cos^2\phi + R^2\sin^2\phi + r^2\sin^2\phi + 2rR\sin^2\phi$$

$$= R^2(\cos^2\phi + \sin^2\phi) + (r^2 + 2rR)\sin^2\phi$$

$$= R^2\left[1 + \left(\frac{r^2}{R^2} + \frac{2r}{R}\right)\sin^2\phi\right]$$





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$$= R^2(\cos^2\phi + \sin^2\phi) + (r^2 + 2rR)\sin^2\phi$$

$$= R^2\left[1 + \left(\frac{r^2}{R^2} + \frac{2r}{R}\right)\sin^2\phi\right]$$

$$\boxed{= R\sqrt{1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2\phi}}$$

$$= BE - R$$

$$R\sqrt{1+\frac{r}{R}\left(\frac{r}{R}+2\right)\sin^2\phi} - R$$

$$a_w = \boxed{R\left[\sqrt{1+\frac{r}{R}\left(\frac{r}{R}+2\right)\sin^2\phi} - 1\right]}$$

Let the adopted value of addendum in some cases be a_w times of the module of teeth. Then this adopted value of addendum should be less than the max value of addendum to avoid interference.

i.e. $$R\left[\sqrt{1+\frac{r}{R}\left(\frac{r}{R}+2\right)\sin^2\phi} - 1\right] \geq a_w m,$$

let t = no of teeth on the pinion

T = no of teeth on the wheel

$$m = \frac{D}{T} = \frac{d}{t} \quad \text{or} \quad m = \frac{2R}{T} = \frac{2r}{t}$$

$$R = \frac{mT}{2} \quad \text{and} \quad r = \frac{mt}{2}$$

so $$\frac{mT}{2}\left[1+\frac{t}{T}\sqrt{\left(\frac{t}{T}+2\right)\sin^2\phi} - 1\right] \geq a_w m,$$

or $$\frac{T}{2}\left[\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1\right] \geq a_w$$

where $G = \frac{T}{t}$ (gear ratio)

or $$T \geq \frac{2a_w}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1}$$

in the limit.

$$\boxed{T = \frac{2a_w}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1}}$$

And max value of addendum

$$= BE - R$$

$$R\sqrt{1+\frac{r}{R}\left(\frac{r}{R}+2\right)\sin^2\phi} - R$$

$$a_w = \boxed{R\left[\sqrt{1+\frac{r}{R}\left(\frac{r}{R}+2\right)\sin^2\phi} - 1\right]}$$

Let the adopted value of addendum in some cases be a_w times of the module of teeth, Then this adopted value of addendum should be less than the max value of addendum to avoid interference.

i.e $R\left[\sqrt{1+\frac{r}{R}\left(\frac{r}{R}+2\right)\sin^2\phi} - 1\right] \geq a_w m.$

let t = no of teeth on the pinion

T = no of teeth on the wheel

$m = \frac{D}{T} = \frac{d}{t}$ or $m = \frac{2R}{T} = \frac{2r}{t}$

$R = \frac{mT}{2}$ and $r = \frac{mt}{2}$

so $\frac{mT}{2}\left[1+\frac{t}{T}\sqrt{\left(\frac{t}{T}+2\right)\sin^2\phi} - 1\right] \geq a_w m,$

or $\frac{T}{2}\left[\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1\right] \geq a_w$

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in the limit.

$$\boxed{T = \frac{2a_w}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1}}$$

(i) when addendum is equal to module

$a_w = 1$

$$\boxed{T = \frac{2}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi}-1}}$$

(ii) for equal no of teeth on the pinion and wheel, $G = 1$

$$\boxed{T_{min} = \frac{2}{\sqrt{1+3\sin^2\phi}-1}}$$

for a pressure angle of 20° i.e $\phi = 20°$

$$T_{min} = \frac{2}{\sqrt{1+3\sin^2 20°}-1} = 12.31 \simeq 13$$

Q. 2 Two 20° involute spur gears mesh externally and give a velocity ratio of 3. Module is 3 mm and addendum is equal to 1.1 module. If the pinion rotates at 120 rpm, determine

(i) minm no of teeth on each wheel to avoid interference.

(ii) no of pairs of teeth in contact.

Given data:

$\phi = 20°$ $\qquad$ $N_p = 120$ rpm.

$VR = 3$ $\qquad$ addendum $= 1.1$ m,

$m = 3$ $\qquad$ $a_w = 1.1$

$$T = \frac{2a_w}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi}-1}$$

$$= \frac{2\times 1.1}{\sqrt{1+\frac{1}{3}\left(\frac{1}{3}+2\right)\sin^2 20}-1} = 49.44$$

$T = 51$

and $t = \frac{51}{3} = 17$

(ii) no of pairs of teeth in contact

$$n = \left(\frac{\text{arc of contact}}{\text{circular pitch}}\right) = \left(\frac{\text{path of contact}}{\cos\phi}\right) \times \frac{1}{\pi m}$$

or

$$n = \frac{\sqrt{R_a^2 - R^2\cos^2\phi} - R\sin\phi + \sqrt{r_a^2 - r^2\sin^2\phi} - r\sin\phi}{\cos\phi \times \pi m}$$

We have

$R = \frac{mT}{2} = \frac{3\times 51}{2} = 76.5\,mm$

$R_a = R + 1.1m = 76.5 + 3.3 = 79.8\ mm$

$r = \frac{mt}{2} = \frac{3\times 17}{2} = 25.5\,mm$

$r_a = 25.5 + (1.1\times 3) = 28.8\,mm$

$$n = \frac{\sqrt{(79.8)^2 - (76.5\cos 20)^2} - 76.5\sin 20 + \sqrt{(28.8)^2 - (25.5\cos 20)^2} - 25.5\sin 20}{\cos 20^\circ \times \pi \times 3}$$

$= 1.78$

~~Thus one pair of teeth will always be in contact and 78% of time~~

Q.2 two 20° involute spur gears have a module of 10 mm. The addendum is equal to module. larger gear has a teeth of 40 while the pinion has a teeth of 20. will the gear interfere with the pinion.

$\phi = 20^\circ \quad T = 40 \quad t = 20 \quad m = 10$

$a_w = 10.$

$R = \frac{mT}{2} = \frac{10\times 40}{2} = 200\,mm$

$R_a = 200+10 = 210$ mm

$r = \frac{mt}{2} = \frac{10\times 20}{2} = 100$ mm

$r_a = 100+10 = 110$ mm.

let pinion be the driver.

path of approach,

$\sqrt{R_a^2 - R^2 \cos^2\phi} - R\sin\phi$

$= \sqrt{210^2 - (200\cos 20)^2} - 200\sin 20.$

$= 25.3$ mm.

to avoid interference max^m length of path of approach will be PE

$PE = r\sin\phi = 100\sin 20 = 34.2$ mm.

Since actual path of approach is less than max^m limit there will be no interference

Q.3 two 20° spur gears have a module of 10 the addendum is one module, the

$T = 50 \quad t = 13.$

Does interference occurs if it occurs what value of pressure angle will eliminate interfere.

$\phi = 20 \quad T = 50 \quad t = 13$

$m = 10 \quad a_w = 10.$

$R = \frac{mT}{2} = \frac{10\times 50}{2} = 250$ mm

$R_a = 250+10 = 260$ mm

$r = \frac{mt}{2} = \frac{10\times 13}{2} = 65$ mm.

$r_a = 65+10 = 75$

$R_{a\,max} = \sqrt{(R\cos\phi)^2 + (R\sin\phi + r\sin\phi)^2}$

Since $R_{reflected}$ > R_{min}
interference will occur.

$$260 = \sqrt{(R\cos\phi)^2 + (R\sin\phi + r\sin\phi)^2}$$

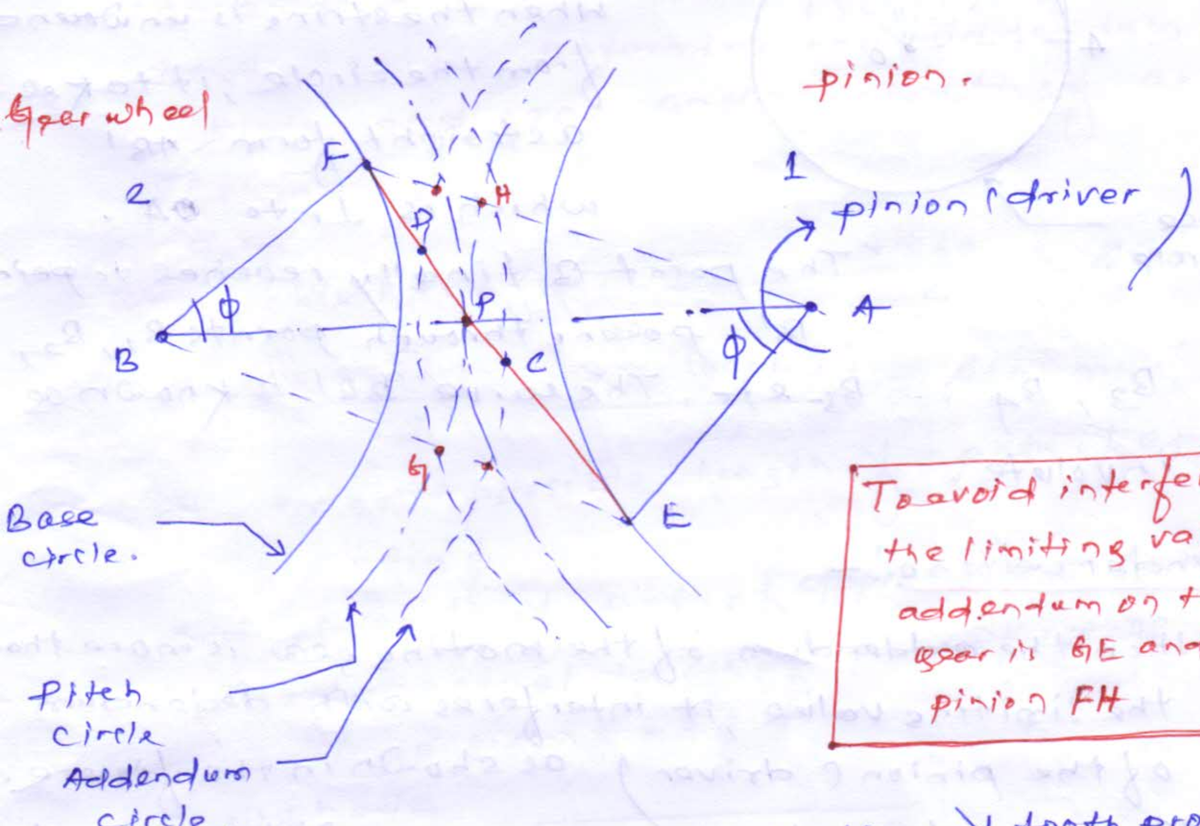
$$260^2 = 250^2\cos^2\phi + 315^2(1-\cos^2\phi)$$

$$\cos\phi = 0.928$$

$$\boxed{\phi = 21.88^\circ}$$

Interference in Involute Gears :-

Power transmission betn two mating gears is along the line of action or the common normal to the two involutes at the point of contact. The common normal is also a common tangent to the two base circles and it passes through the pitch point.

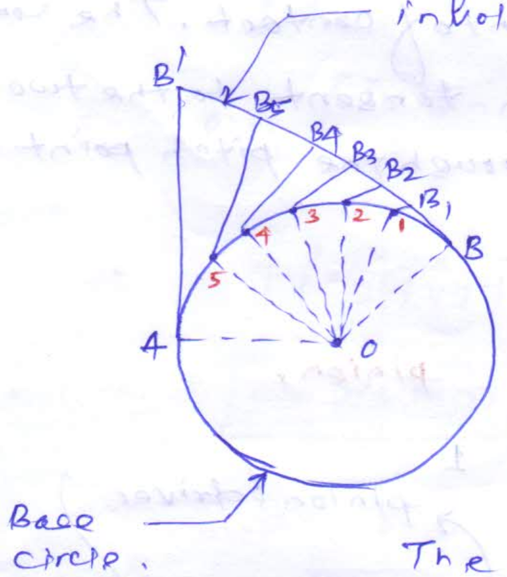


To avoid interference the limiting value of addendum on the gear is GE and on the pinion FH

- At any instant, the portions of tooth profile which are in contact must be involute so that the line of action does not deviate. If any of the two surfaces is not involute, the two surfaces would not ~~tech~~ touch each other tangentially and the transmission of power would not be proper.
- This mating of two non-conjugate (non-involute) teeth is known as interference. Because of interference the two mating teeth do not slide properly and rough action and binding occurs, resulting different velocities and

Involute tooth profile:-

An involute is defined as the locus of a point on a straight line which rolls without slipping on the circumference of a circle.

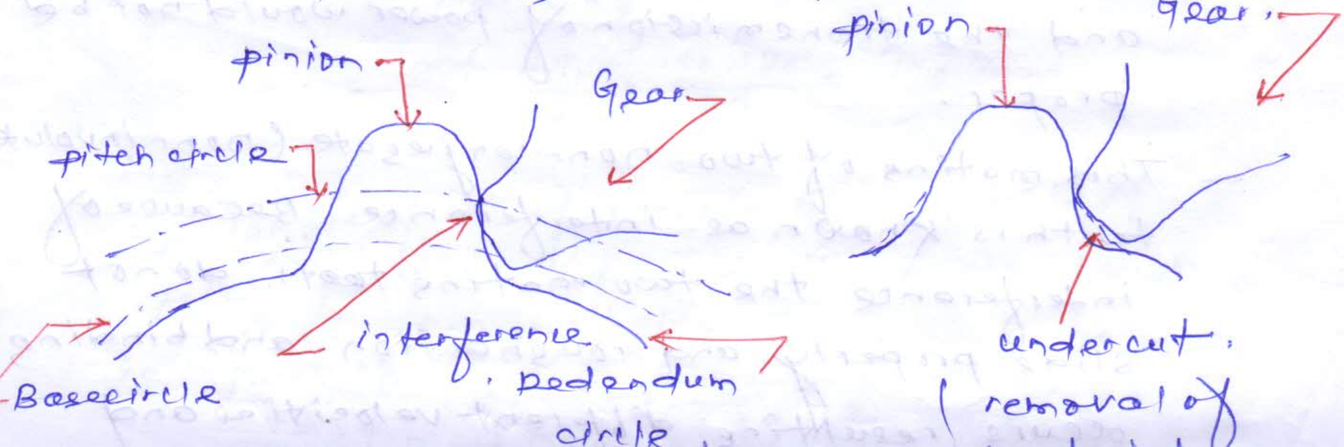


Considering the figure, a string AB is wrapped on the circumference of a circle. When the string is unwound from the circle, it takes a straight form AB' which is ⊥r to OA.

The point B finally reaches to point B' passing through points B_1, B_2, B_3, B_4 ... B_5 etc. The curve BB' is known as involute.

Undercutting:-

When the addendum of the mating gear is more than the limiting value, it interferes with dedendum of the pinion (driver) as shown in the figure. The tip of the tooth on the gear will undercut the tooth on the mating pinion at the root and remove part of the involute profile of tooth on the gear. This phenomenon is called undercutting.



Q.5 Following data relates to the meshing of two involute gears:

No. of teeth on the gear wheel = 60,
pressure angle = 20°
Gear ratio = 1.5
speed of gear wheel = 100 rpm
module = 8 mm

if the addendum on each wheel is such that the path of approach and the path of recess on each side are 40% of max possible length each. Determine the addendum of pinion and gear and the length of arc of contact.

Solu^n: We have $R = \frac{mT}{2} = \frac{8\times 60}{2} = 240$ mm

$r = \frac{mt}{2} = \frac{8\times(60/1.5)}{2} = 160$ mm,

Now max possible length of path of approach
$= r\sin\phi$

Actual length of path of approach $= 0.4\times r\sin\phi$

Similarly actual length of path of recess
$= 0.4\times R\sin\phi$

So we have

$0.4 r\sin\phi = \sqrt{R_a^2 - (R\cos\phi)^2} - R\sin\phi$

$\Rightarrow 0.4\times 160\sin 20° = \sqrt{R_a^2 - (240\cos 20)^2} - 240\sin 20°$

$\Rightarrow 21.889 = \sqrt{R_a^2 - 50862.08} - 82.084$

$\Rightarrow R_a^2 - 50862.08 = 10810.384$

$\Rightarrow \boxed{R_a = 248.3 \text{ mm}}$

so addendum of wheel $= R_a - R = \boxed{8.3 \text{ mm}}$

Also

$0.4\times R\sin\phi = \sqrt{r_a^2 - (r\cos\phi)^2} - r\sin\phi$

$\Rightarrow 0.4\times 240\sin 20 = \sqrt{r_a^2 - (160\cos 20)^2} - 160\sin 20$

$\Rightarrow r_a = 173.95 \simeq 174$ mm

So addendum of pinion $= r_a - r$

$= \boxed{14\text{ mm}}$

Arc of contact $= \dfrac{\text{path of contact}}{\cos\phi}$

$= 0.4 \left(\dfrac{r \sin\phi + R \sin\phi}{\cos\phi} \right)$

$= 0.4 \left(240 + 160 \right) \dfrac{\sin 20}{\cos 20}$

$= \boxed{58.23\text{ mm}}$ (ans)

Q. Two mating gear wheels have 20 and 40 involute teeth of 10 mm module and 20° pressure angle. The addendum of each wheel is to be made of such a length that the line of contact on each side of the pitch point has half the max^m possible length. Determine the addendum height for each gear wheel and the length of line of contact. If the smaller wheel rotates at 250 rpm find the velocity of point of contact.

Given data:

$T_1 = 20 \quad T_2 = 40 \quad m = 10$ mm $\quad \phi = 20°$

Now pitch circle radius of larger gear

$R = \dfrac{mT_2}{2} = \dfrac{10 \times 40}{2} = 200$ mm

pitch circle radius of pinion

$r = \dfrac{mT_1}{2} = \dfrac{10 \times 20}{2} = 100$ mm,

Now

length of line of contact on each side of pitch point = half of max^m possible length.

So path of approach

$r \sin\phi = \sqrt{R_A^2 - (R\cos\phi)^2} - R\sin\phi$

path of relese

$$\frac{R\sin\phi}{2} = \sqrt{r_a^2-(r\cos\phi)^2} - r\sin\phi$$

so length of path of contact $= \frac{(R+r)}{2}\sin\phi$

And path of approach

$$\sqrt{R_A^2-(R\cos\phi)^2} - R\sin\phi = \frac{r\sin\phi}{2}$$

$$\Rightarrow \sqrt{R_A^2-(200\cos 20)^2} - 200\sin 20^\circ = 50\sin 20^\circ$$

$$\Rightarrow \sqrt{R_A^2-35320.89} - 68.404 = 17.1$$

$$\Rightarrow R_A = 206.47 \text{ mm.}$$

so addendum of gear wheel $= R_A - R = \boxed{6.47 \text{ mm}}$

path of recess

$$\sqrt{r_a^2-(r\cos\phi)^2} - r\sin\phi = \frac{R\sin\phi}{2}$$

$$\Rightarrow \sqrt{r_a^2-(100\cos 20)^2} - (100\sin 20) = \frac{200}{2}\sin 20^\circ$$

$$\Rightarrow \sqrt{r_a^2-8830.23} - 34.202 = 34.202$$

$$\Rightarrow r_a = 116.22 \text{ mm}$$

Addendum of pinion $= r_a - r = \boxed{16.22 \text{ mm}}$

length of path of contact $= \left(\frac{R+r}{2}\right)\sin\phi = \boxed{51.303 \text{ mm}}$

length of arc of contact $= \frac{\text{length of path of contact}}{\cos\phi}$

$$= \frac{51.303}{\cos 20} = \boxed{54.6 \text{ mm.}}$$

Angular velocity of pinion $\omega_1 = \frac{2\pi N_1}{60} = \frac{2\pi \times 250}{60}$

$$= 26.18 \text{ rad/s.}$$

angular velocity of gear wheel

$$\omega_2 = \omega_1 \times \frac{T_1}{T_2} \quad \left(\because \frac{\omega_1}{\omega_2} = \frac{T_2}{T_1}\right)$$

$$= 26.18 \times \frac{20}{40} = \boxed{13.09 \text{ rad/sec.}}$$

Velocity of sliding

$V_s = (\omega_1+\omega_2) \times$ length of path of approach

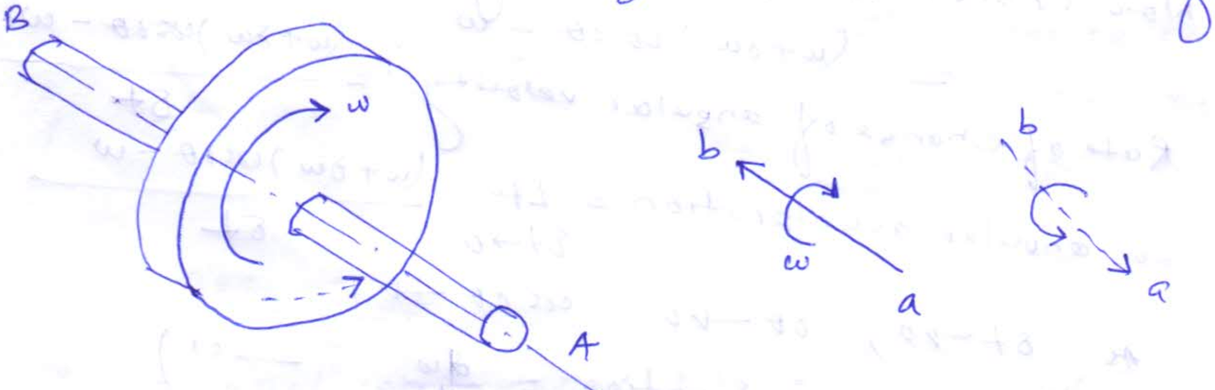
$$= (26.18+13.09) \times \frac{r\sin\phi}{2}$$

$$= 39.27 \times 50\sin 20^\circ = 672 \text{ mm/sec}$$

Angular velocity :-

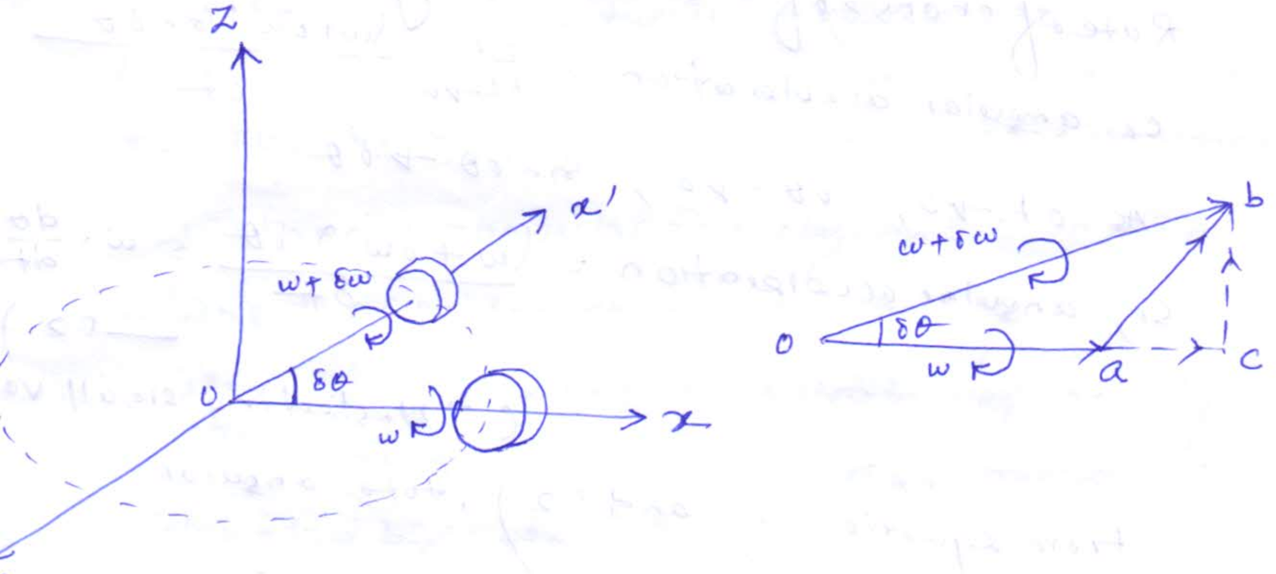
The angular velocity of a rotating body is specified by — magnitude of the velocity
— the direction of axis of rotation
— sense of rotation i.e. clockwise or anticlockwise.

for example let's consider a rotor rotating in clockwise direction when viewed from A end. It's sense of



direction of the vector is from a to b is according to the screw rule.

Angular acceleration :-



Let's consider a rotor rotates about the horizontal axis ox at a speed of ω rad/s in a direction as shown in the figure. Let oa represents its angular velocity.

— Now, if the magnitude of the angular velocity changes to (ω + δω) and the direction of the axis of spin to ox' (in time δt), the vector ob would represent

The change in angular velocity can be represented by joining ab. This vector ab can be resolved into two components

(i) component ac represents the ~~ang~~ change in angular velocity in a plane $\perp$r to x axis

(ii) component cb represents the change in angular velocity in a plane $\perp$r to y axis.

Now change in angular velocity ac

$$= (\omega + \delta\omega)\cos\delta\theta - \omega$$

Rate of change of angular velocity $= \dfrac{(\omega+\delta\omega)\cos\delta\theta - \omega}{\delta t}$

So angular acceleration $= \lim\limits_{\delta t \to 0} \dfrac{(\omega+\delta\omega)\cos\delta\theta - \omega}{\delta t}$

As $\delta t \to 0$, $\delta\theta \to 0$ $\cos\delta\theta \to 1$

So angular acceleration $= \dfrac{d\omega}{dt}$ —— (1)

Now, change of angular velocity cb

$$= (\omega + \delta\omega)\sin\delta\theta$$

Rate of change of angular velocity $= \dfrac{(\omega+\delta\omega)\sin\delta\theta}{\delta t}$

So angular acceleration $= \lim\limits_{\delta t \to 0} \dfrac{(\omega+\delta\omega)\sin\delta\theta}{\delta t}$

As $\delta t \to 0$, $\delta\theta \to 0$, $\sin\delta\theta \to \delta\theta$

So, angular acceleration $= \dfrac{(\omega+\delta\omega)\,\delta\theta}{\delta t} = \omega \cdot \dfrac{d\theta}{dt}$

—— (2)

($\because$ Neglecting small values)

From equation (1) and (2), total angular acceleration

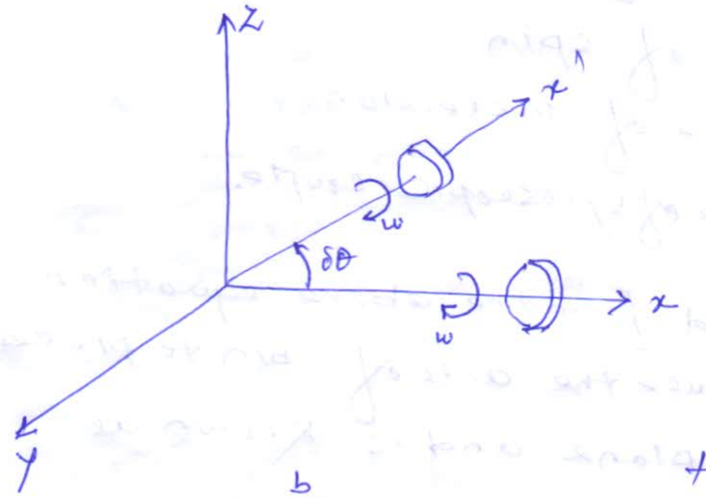
$$\boxed{\alpha = \frac{d\omega}{dt} + \omega\frac{d\theta}{dt}} \quad —— (3)$$

Equation (3) shows that, the total angular acceleration of the rotor is the sum of

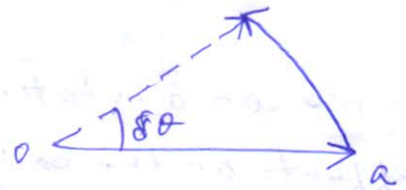
(i) $\dfrac{d\omega}{dt}$, the change in angular velocity of rotor

(ii) $\omega \frac{d\theta}{dt}$, the change in direction of axis of spin

Gyroscopic Couple :-



Let I = moment of inertia of a rotor and ω is it's angular velocity about horizontal axis of spin ox. Let's this axis of spin turn through a small angle δθ in the horizontal plane xy, to the position ox' in δt time.



The vectorial representation of the same is as shown, oa represents angular velocity vector when the axis is ox and ob when the axis is changed to ox'. Then ab represents the change in angular velocity due to change in direction of axis of spin,

- This change in angular velocity is clockwise when viewed from a towards b and is in the vertical plane xz. This change results in an angular acceleration

Now change in angular velocity ab = $\omega \times \delta\theta$

angular acceleration, $\alpha = \omega \frac{\delta\theta}{\delta t}$

Usually $\frac{\delta\theta}{\delta t}$, the angular velocity of axis of spin is called angular velocity of precession and is denoted by ω_p.

So angular acceleration = $\omega \cdot \omega_p$.

And ~~angular~~ torque required to ~~produce~~ this acceleration is called gyroscopic couple or torque.

Gyroscopic torque = $I\alpha$

$= \boxed{I \omega \omega_p.}$

where

$Ox \rightarrow$ axis of spin

$Oz \rightarrow$ axis of precession

$Oy \rightarrow$ axis of gyroscopic couple.

$yz \rightarrow$ plane of spin

$xy \rightarrow$ plane of precession

$xz \rightarrow$ plane of gyroscopic couple

- The torque obtained from the above equation is required to cause the axis of spin to precess in the horizontal plane and is known as active gyroscopic torque.
- The effect of gyroscopic couple on a rotating body is known as the gyroscopic effect on the body.

Q.1 A uniform disc having a mass of 8 kg and radius of gyration of 150 mm is mounted on one end of a horizontal arm of length 200 mm. The other end of the arm can rotate freely in a bearing. The disc is given a clockwise spin of 250 rpm when seen from the end. Determine the motion of disc if the arm remains horizontal.

$m = 8\,kg$, $l = 200\,mm = 0.2\,m$, $k = 0.15\,m$, $N = 240\,rpm$,

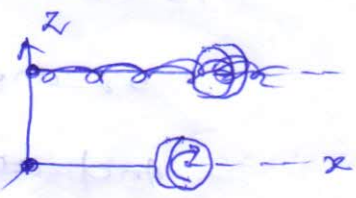
$I = mk^2 = 8 \times (0.15)^2 = 0.18\,kg\,m^2$

$\omega = \frac{2\pi \times 240}{60} = 25.13\,rad/s.$

$C = I\omega\omega_p$

$= 0.18 \times 25.13 \times \omega_p$

$= mg \cdot l$



$\Rightarrow 0.18 \times 25.13 \times \omega_p = 8 \times 9.81 \times 0.2$

$\Rightarrow \boxed{\omega_p = 3.47\,rad/s.}$

Q.2 A disc of radius of gyration 60 mm and mass of 4 kg. is mounted centrally on a horizontal axle of 80 mm length betn the bearings. It spins about the axle at 800 rpm counterclockwise when viewed from the right hand side bearing. The axle precesses about vertical axis at 50 rpm in clockwise direction when viewed from above. Determine the resultant reaction at each bearing due to the mass and gyroscopic effect.

$K = 60$ mm $= 0.06$ m $m = 4$ kg. $l = 80$ mm $= 0.08$ m. $N = 800$ rpm $N_p = 50$ rpm.

Now $I = mk^2$

$= 4 \times 0.06^2 = 0.0144$ kg m^2

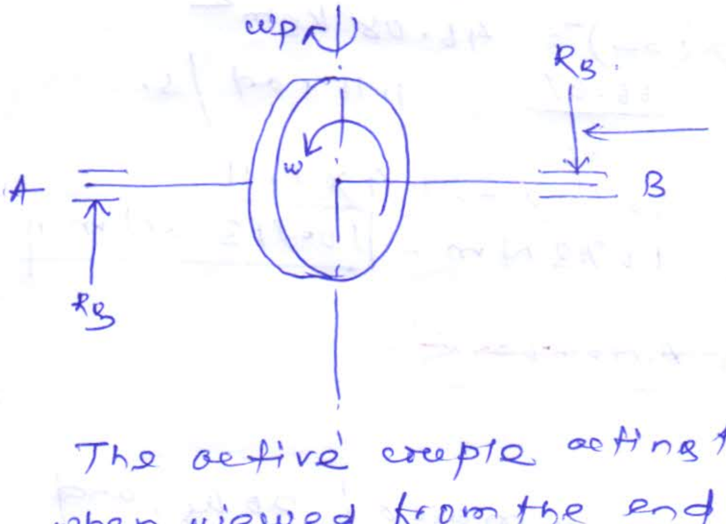
$$\omega = \frac{2\pi \times 800}{60} = 83.78 \text{ rad/s.}$$

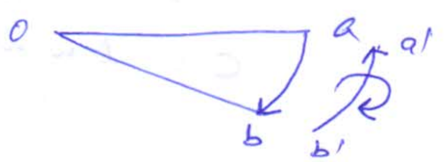
$$\omega_p = \frac{2\pi \times 50}{60} = 5.24 \text{ rad/s.}$$

$C = I \omega \omega_p$

$= 0.0144 \times 83.78 \times 5.24$

$= 6.32$ N m.





The active couple acting the disc is anticlockwise when viewed from the end and the reactive couple is clockwise when viewed from the top.

force at bearing A due to gyroscopic couple

$$= \frac{C}{l} = \frac{6.32}{0.08} = 79 \text{ N (upward)}$$

force at bearing B due to gyroscopic couple each bearing due to ωt

$= 19.6\ N$ (upward)

So reaction at A $= 79 + 19.6 = 98.6\ N$ (upward)

Reaction at B $= 79 - 19.6 = 59.4\ N$ (downward)

Q.3 An aeroplane flying at 240 km/hr turns towards left and completes a quarter circle of radius 60 m. The mass of rotary engine and propeller plane amounts to 450 kg with a radius of gyration of 320 mm. The engine speed is 2000 rpm clockwise when viewed from the rear. Determine the gyroscopic couple on the aircraft and state it's effect.

$m = 450\ kg$

$k = 0.32\ m$

$\omega = \dfrac{2\pi \times 2000}{60} = 209.4\ rad/s$

$v = \dfrac{240 \times 1000}{3600} = 66.67\ m/s$

$I = mk^2 = 450 \times (.32)^2 = 46.08\ kgm^2$

$\omega_p = \dfrac{v}{r} = \dfrac{66.67}{60} = 1.11\ rad/s$

$C = I\omega\omega_p = 46.08 \times 209.4 \times 1.11$

$= 10713\ Nm = \boxed{10.713\ kNm}$

~~y of an Automobile:-~~

nt

heel having a mass of 20 kg, and
of gyration of 300 mm is given a spin
m about it's axis which is horizontal
is suspended at a point 250 mm
plane of rotation of the flywheel.
e of precession.

force at each bearing due to the wt of the disc $= \frac{4 \times 9.81}{2} = 19.6\ N$.

Reactive force at each bearing due to wt $= 19.6\ N$ upward

So reaction at $A = 79 + 19.6 = 98.6\ N$ (upward)

Reaction at $B = 79 - 19.6 = 59.4\ N$ (downward)

Q-3 An aeroplane flying at 240 km/hr turns towards left and completes a quarter circle of radius 60 m. The mass of rotary engine and propeller plane amounts to 450 kg with a radius of gyration of 320 mm. The engine speed is 2000 rpm clockwise when viewed from the rear. Determine the gyroscopic couple on the aircraft and state it's effect.

$m = 450\ kg$ $\qquad \omega = \frac{2\pi \times 2000}{60} = 209.4\ rad/s$

$K = 0.32\ m$ $\qquad v = \frac{240 \times 1000}{3600} = 66.67\ m/s.$

$I = mK^2 = 450 \times (.32)^2 = 46.08\ kg m^2$

$\omega_p = \frac{v}{r} = \frac{66.67}{60} = 1.11\ rad/s.$

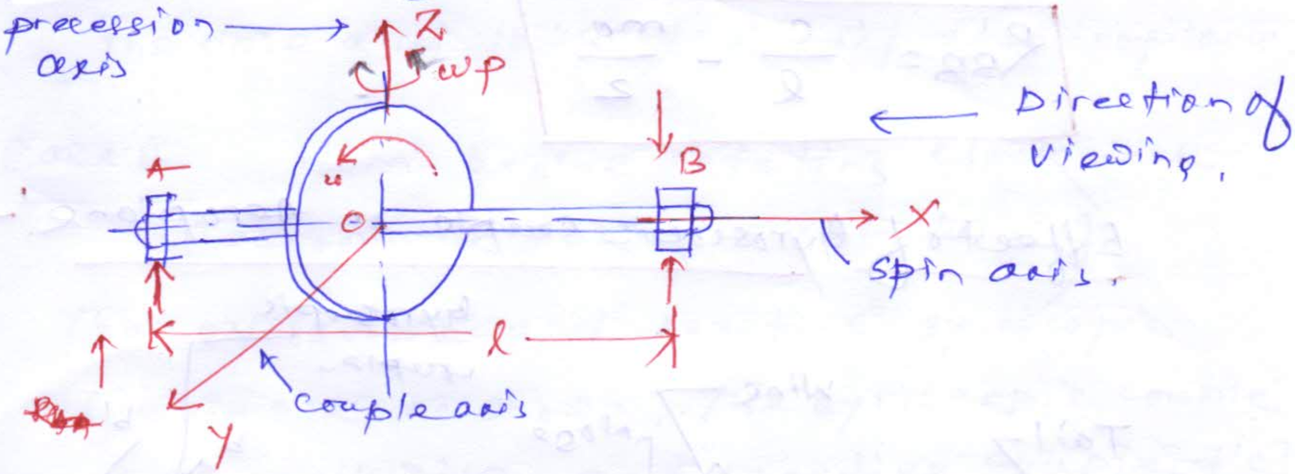
$C = I \omega \omega_p = 46.08 \times 209.4 \times 1.11$
$= 10713\ Nm = \boxed{10.713\ kN\ m}$

~~Stability of an Automobile~~ -

Assignment

Q-1 A flywheel having a mass of 20 kg, and radius of gyration of 300 mm is given a spin of 500 rpm about it's axis which is horizontal. The flywheel is suspended at a point 250 mm from the plane of rotation of the flywheel. Find the rate of precession.

Effect of Gyroscopic couple on bearing



Consider a rotating disc of mass m and radius of gyration K is supported in bearings A and B through the shaft. The disc is rotating in anticlockwise when viewed from the right hand side of bearing B. The shaft AB turns to the right by application of couple and thus causing change of angular momentum.

So, gyroscopic couple $\boxed{C = I.\omega.\omega_p}$

forces at bearings A and B

$$\boxed{F_A = F_B = \frac{C}{l}}$$

Reactions at each bearing due to wt. of disc $R_A = R_B = \frac{m.g}{2}$

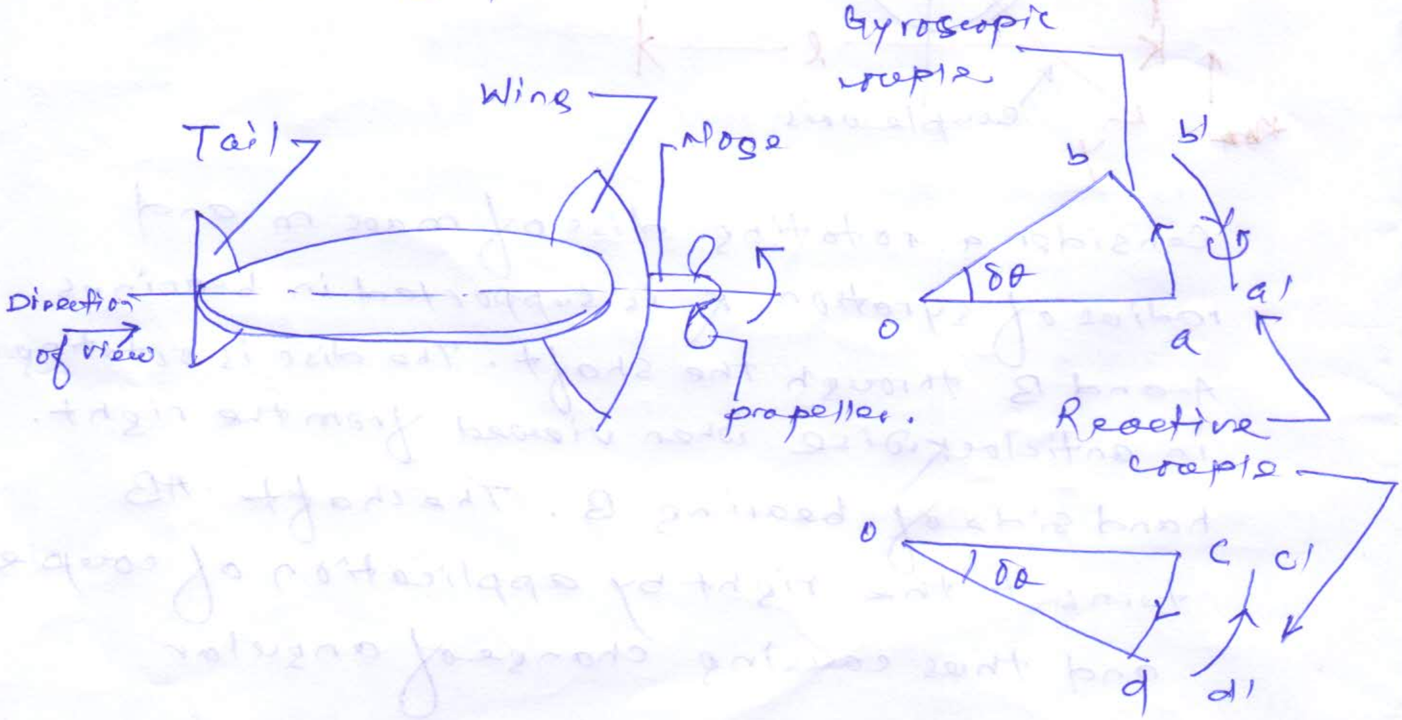
Applying right hand thumb rule in the direction of ab, the direction of thumb will indicate the change of angular momentum, secondly the direction of fingers will show the direction of couple applied by bearing which is clockwise and the reaction couple will be equal in magnitude and opposite in direction,

So Net reaction at A $\boxed{R_{AA} = \frac{C}{l} + \frac{mg}{2}}$

Net reaction at B

$$\boxed{R_{BB} = \frac{c}{l} - \frac{mg}{2}}$$

Effect of Gyroscopic couple on Aeroplane :-



An ~~car~~ aeroplane's engine rotates clockwise when viewed from the rear end. Aeroplane takes turn to the left.

From the angular momentum diagram oa is the angular momentum vector before turning.

ob = angular momentum vector after turning

ab = active gyroscopic couple.

b'a' = reactive "

Now applying right hand thumb rule in and ~~thumb being is right~~ looking the direction of ~~ab~~ ab. The direction of active gyroscopic couple will be clockwise, so reactive gyroscopic couple will be anticlockwise.

Effect :- reactive couple tends to raise the nose and lower the tail of aeroplane.

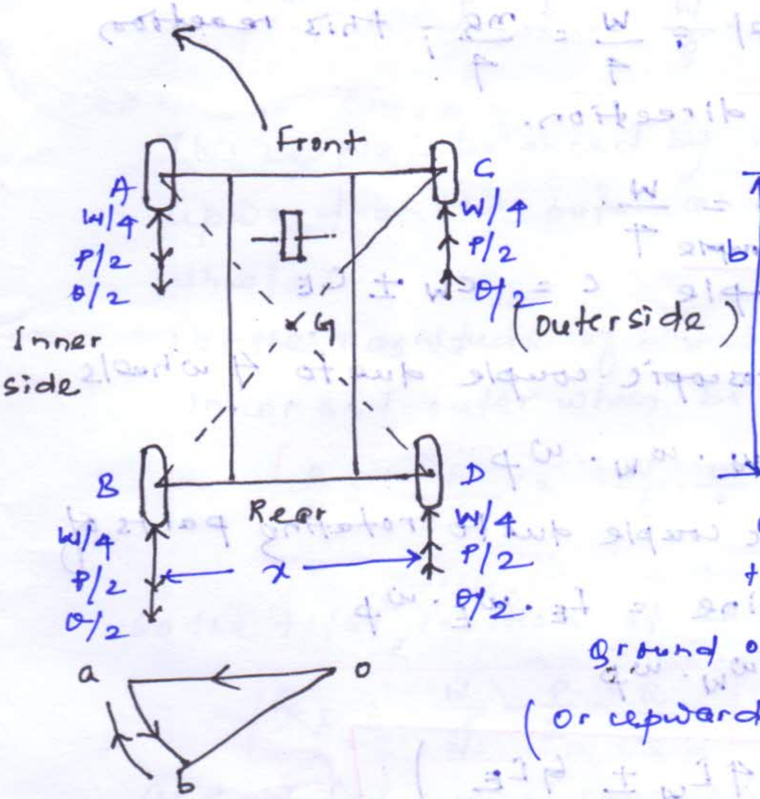
Case II ~~When~~ Engine rotating clockwise (aeroplane taking right turn)

The gyroscopic and reactive gyroscopic couple are as shown, The gyroscopic couple is anticlockwise and reactive couple d'c' will be in clockwise direction

Effect :- reactive gyroscopic couple tends to raise the tail and lower the nose.

Case - III When viewing from the nose end

Stability of an Automobile (while moving in a curved path):-



Consider a four wheel automobile taking turn towards leftside. A and B are the inner and C and D are the outer wheels while taking the turn it is essential that no wheel is lifted off the ground. The condition is fulfilled as long as the vertical reaction of the ground on any wheel is possitive (or upward).

Let W = wt. of the vehicle in N = m.g

m = mass of the vehicle in kg.

x = width of the track in m.

b = wheel base in m.

R = Radius of curvature.

h = distance of centre of gravity above the road surface in m.

r_w = radius of each wheel in m.

I_W, I_E = mass moment of inertia of each wheel and rotating part of engine in kg-m^2 respectively.

ω_P = angular velocity of precession in rad/s

$= \frac{v}{R}$

v = linear velocity of vehicle in m/s

ω_W = angular velocity of wheels = $\frac{v}{r}$

ω_E = angular velocity of rotating parts of engine

G = gear ratio = $\frac{\omega_E}{\omega_W}$

Reaction due to weight of the vehicle:-

wt of the vehicle is distributed over four wheels equally.

so load on each wheel $= \frac{W}{4} = \frac{mg}{4}$; this reaction will be in upward direction.

thus road reaction $= \frac{W}{4}$

Reaction due to Gyroscopic couple

Total gyroscopic couple $C = C_W \pm C_E$

where C_W = gyroscopic couple due to 4 wheels

$= 4 I_W \cdot \omega_W \cdot \omega_p$

C_E = gyroscopic couple due to rotating parts of the engine $= I_E \cdot \omega_E \cdot \omega_p$

$= I_E \cdot G \cdot \omega_W \cdot \omega_p$

so $$\boxed{C = \omega_W \cdot \omega_p \left(4 I_W \pm G I_E\right)}$$

+ve sign is taken when the engine and the vehicle wheel rotate in same direction

This gyroscopic couple produces reaction on the road surface. This reaction is +ve on the outer and -ve on the inner wheels. Let the magnitude of this reaction on each two outer and inner wheels be $P/2$.

so $$\boxed{\frac{P}{2} = \frac{C}{2x}}$$

if C value is -ve, the direction of reactions on wheels will be reversed i.e it will be -ve on the outer wheel and +ve on the inner wheel.

Reaction due to centrifugal effect:-

vehicle while taking the turn will try to over turn because of the centrifugal effect. The magnitude of centrifugal force is

$$\boxed{F_c = \frac{W}{g} \cdot \frac{v^2}{R}}$$

This force produces a couple while tend to overturn the vehicle. The magnitude of the couple is

$$\boxed{C_F = F_c \cdot h = \frac{W}{g} \cdot \frac{v^2}{R} \cdot h}$$

This couple is balanced by vertical reactions which are upward on the outer and downward on the inner wheels.

Let the magnitude of this reaction at each of the two inner and outer wheel be $Q/2$ N.

$$\boxed{\frac{Q}{2} = \frac{C_F}{2x} = \frac{W}{g} \cdot \frac{v^2}{R} \cdot \frac{h}{2x}}$$

So the total vertical reaction at each inner wheel

$$\boxed{R_I = \frac{W}{4} - \frac{P}{2} - \frac{Q}{2}}$$

and total vertical reaction at each outer wheel

$$\boxed{R_O = \frac{W}{4} + \frac{P}{2} + \frac{Q}{2}}$$

In case of $\frac{W}{4}$ is equal to or less than $\left(\frac{P}{2} + \frac{Q}{2}\right)$ then the value of R_I is zero or negative the inner wheel will leave the ground. For stability this situation should be avoided.

Q.1 Each wheel of a 4-wheeled ~~rear engine~~ automobile has a moment of inertia of 2.4 kgm² and effective diameter of 660 mm. The rotating parts of the engine have a mi of 1.2 kgm². The gear ratio of engine to back wheel is 3:1. The engine axle is parallel to the rear axle. The mass of vehicle is 2200 kg and centre of mass is 550 mm above the ground level. The track width of the vehicle is 1.5 m. Determine the limiting speed of the vehicle around a curve with 80 m radius so that all the 4 wheels maintain contact with road.

Given data:

mass of vehicle $m = 2200$ Kg.

$h = 0.55$ m. $I_w = 2.4$ Kg m^2 $I_E = 1.2$ Kg m^2

$x = 1.5$ m. $r = 0.33$ m. $R = 80$ m.

(i) Reaction due to wt

$$R_w = \frac{mg}{4} = \frac{2200 \times 9.81}{4} = 5395.5 \text{ N (upward)}$$

(ii) Reaction due to gyroscopic couple

Total gyroscopic couple $C = C_W + C_E$

$$= \omega_W \cdot \omega_P (4 I_W + G I_E)$$

$$= \frac{v}{r} \cdot \frac{v}{R} (4 \times 2.4 + 3 \times 1.2)$$

$$= \frac{v^2}{rR}(9.6 + 3.6) = \frac{v^2}{0.33 \times 80} \times 13.2$$

$$= 0.5 v^2$$

reaction due to gyroscopic couple $\frac{P}{2} = \frac{C}{2x}$

$$\Rightarrow \frac{P}{2} = \frac{0.5 v^2}{2 \times 1.5} = \boxed{0.167 v^2}$$

this reaction on inner wheel is upward and on the outer wheel on downward direction.

(iii) Reaction due to centrifugal couple

centrifugal force acting on the vehicle

$$F_C = \frac{W}{g} \times \frac{v^2}{R} = 2200 \times \frac{v^2}{80} = 27.5 v^2$$

couple due to centrifugal force

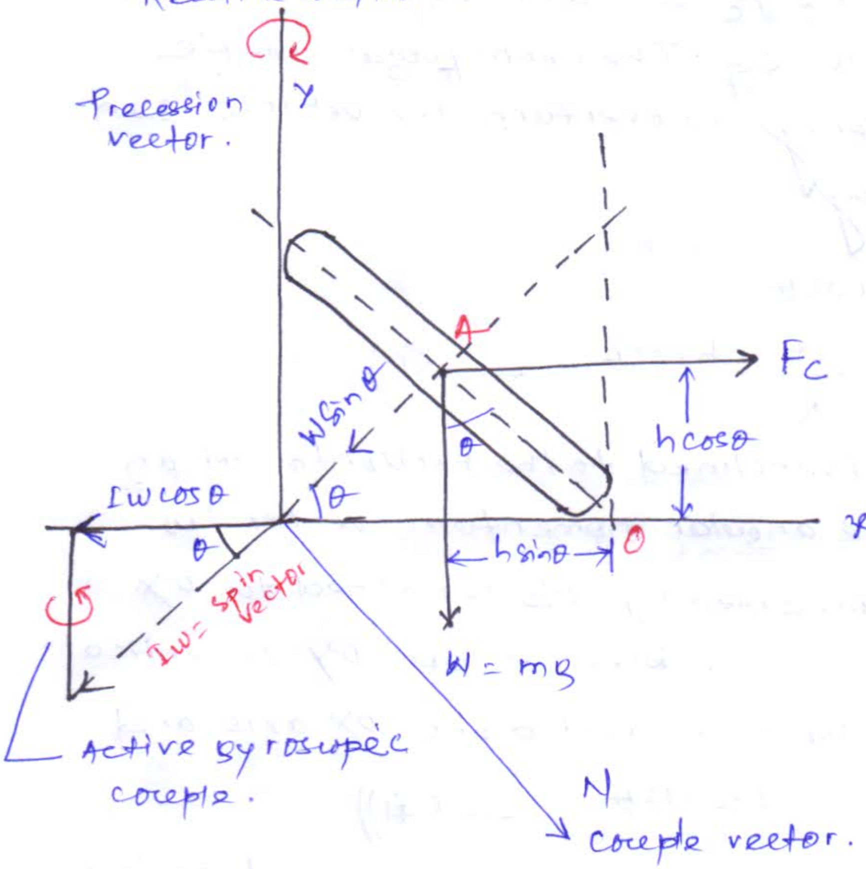
$$C_F = F_C \times h = 27.5 v^2 \times 0.55 = 15.125 v^2$$

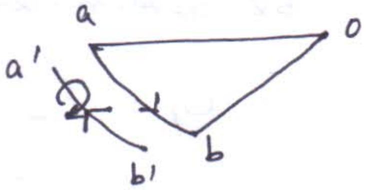
reaction due to the couple

$$R\frac{Q}{2} = \frac{C_F}{2x} = \frac{15.125 v^2}{2 \times 1.5} = 5.041 v^2$$

this reaction on outer wheel is upward and on the inner wheel is downward.

Stability of two-wheeled vehicle :-





Consider a two wheel vehicle as shown in the figure, taking a left turn.

Let I_W, I_E = mass moment of inertia of each wheel and rotating part of engine respectively in kg m^2

ω_W, ω_E = angular velocity of wheel and engine in rad/sec respectively.

G = Gear ratio = $\frac{\omega_E}{\omega_W}$

v = linear velocity of the vehicle in m/s.

h = height of centre of gravity of the vehicle and rider in N

r_W = radius of each wheel in m.

R = radius of curvature in m.

θ = angle of inclination of wheel to the vertical

$\omega_p = \frac{v}{R}$

Centrifugal force:-

The centrifugal force F_c will act outward on the wheel through it's CG. The centrifugal couple which has a tendency to overturn the vehicle can be determined by

$$C_1 = F_c \cdot h\cos\theta$$

$$= \frac{W}{g} \cdot \frac{v^2}{R} \cdot h\cos\theta \quad \text{—(i)}$$

The axis of spin is inclined to the horizontal at an angle θ. Thus the angular momentum vector $I\omega$ due to spin is represented by OB is inclined to OX at an angle θ. The precession axis OY is vertical so the spin vector is resolved along OX axis and it's value is $= I\omega\cos\theta$ —(ii)

Total angular momentum of two wheels and engine is given by

$$I.\omega = 2I_W \cdot \omega_W \pm I_E \cdot \omega_E$$

$$= 2I_W \cdot \frac{v}{r_W} \pm I_E \cdot G \cdot \omega_W$$

$$= \frac{v}{r_W}\left(2I_W \pm G I_E\right) \quad \left(\because \omega_W = \frac{v}{r_W}\right)$$

Using equation (2) total angular momentum can be written as:

$$I.\omega = \boxed{\frac{v}{r_W}\left(2I_W \pm G I_E\right)\cos\theta}$$

Total gyroscopic couple may be written as

$$C_2 = \frac{v}{r_W}\left(2I_W \pm G I_E\right)\cos\theta \cdot \omega_P$$

$$= \frac{v}{r_W}\left(2I_W \pm G I_E\right)\cos\theta \cdot \frac{v}{R}$$

$$\Rightarrow \boxed{C_2 = \frac{v^2}{R \cdot r_W}\left(2I_W \pm G I_E\right)\cos\theta} \quad \text{—(iii)}$$

- The reactive gyroscopic couple can be obtained by adding equations (ii) and (iii). This couple act in clockwise direction when viewed from the back of the vehicle and tend to overturn the vehicle in outward direction.

So total overturning couple

$$= \frac{W}{g} \cdot \frac{v^2}{R} h\cos\theta + \frac{v^2}{R \cdot r_W} (2I_W \pm G \cdot I_E) \cos\theta \quad \text{—(iv)}$$

(+ve sign is taken when the engine rotor and wheel rotate in same sense)

- For the vehicle to be in equilibrium, overturning couple should be equal to the restoring couple.

$$\text{Restoring couple} = W \cdot h \sin\theta \quad \text{—(v)}$$

So

$$\boxed{\frac{W}{g} \cdot \frac{v^2}{R} \cdot h\cos\theta + \frac{v^2}{R \cdot r_W} (2I_W + G \cdot I_E) \cos\theta = W \cdot h \sin\theta} \quad \text{—(vi)}$$

From equation (vi) angle θ can be obtained which is necessary to maintain in order to avoid skiding.

Q.1 The wheel of motor cycle have a total moment of inertia $2.5\ Kg\,m^2$ and the engine parts have a moment of inertia of $0.14\ Kg\,m^2$. Gear ratio is 5:1 and the axis of rotation of the engine crankshaft is ~~same~~ parallel to that of the rear wheel, which have a diameter of 65 cm. Determine the magnitude and direction of the gyroscopic couple when motor cycle rounds a curve of 25 m radius at a speed of 50 Km/hr. Total mass of the system is 180 Kg and h = 0.6 m.

Given data:-

$I_W = 2.5\ Kg\,m^2$ $\quad I_E = 0.14\ Kg\,m^2$ $\quad G = \frac{\omega_E}{\omega_W} = 5$

$m = 180\ Kg$ $\quad h = 0.6\ m$ $\quad d_W = 0.65\ m$ $\quad r_W = \frac{0.65}{2} = 0.325\ m$

$R = 25\ m$, $\quad v = 50\ Km/hr = \frac{50 \times 1000}{3600} = 13.89\ m/s$

$\omega_p = \frac{v}{R} = \frac{13.89}{25} = 0.555\ rad/sec.$

Centrifugal couple

$C_1 = \frac{m v^2}{R} \times h \cos\theta$

$= \frac{180 \times 13.89^2}{25} \times 0.6 \cos\theta$

$= \boxed{833.226 \cos\theta} \quad -(1)$

Gyroscopic couple $C_2 = \frac{v^2}{R \cdot r_W} (2I_W + GI_E) \cos\theta$

$= \frac{13.89^2}{25 \times 0.325} (2 \times 2.5 + 5 \times 0.14) \cos\theta$

$= \cancel{833}. \boxed{135.31 \cos\theta}$

Total overturning couple $C = C_1 + C_2$

$= (833.226 + 135.31) \cos\theta$

$= 968.536 \cos\theta \quad -(2)$

Now restoring or balancing couple

$= m \cdot g \cdot h \sin\theta$

$= 180 \times 9.81 \times 0.6 \sin\theta = 1059.48 \sin\theta \quad -(3)$

Equating eq. (1) and (2)

$968.536 \cos\theta = 1059.48 \sin\theta$

$\Rightarrow \tan\theta = 0.914 \Rightarrow \boxed{\theta = 42.43^\circ}$

so total over turning couple

$C = 968.536 \times \cos(42.43) = \boxed{714.88 \text{ N.m}}$

Gyroscopic couple $= 135.31 \cos\theta$

$= 135.31 \times \cos(42.43) = \boxed{99.87 \text{ Nm}}$

Q.2 Find the angle of inclination wrt the vertical of a two wheeler negotiating a turn. Given combined mass of the vehicle with rider is 250kg. moment of inertia of engine flywheel 0.3 kg-m^2. MI of each wheel is 1 kg-m^2. speed of engine flywheel is 5 times that of the road wheels and in the same ~~time~~ direction, Ht of C.G. of the rider with vehicle is 0.6 m, vehicle speed 90 Km/hr and wheel radius 0.3 m and radius of turn 50 m.

We have $m = 250\,Kg$ $I_E = 0.3\,Kg\,m^2$

$I_W = 1\,Kg\,m^2$ $G = 5 = \frac{\omega_E}{\omega_W}$

$h = 0.6\,m$ $r_W = 0.3\,m$ $R = 50\,m$ $v = \frac{90 \times 1000}{3600}$

$= 25\,m/s.$

Let θ = angle of inclination

Now centrifugal couple $C_1 = \frac{mv^2}{R} h\cos\theta$

$= \frac{250 \times 25^2}{50} \times 0.6\cos\theta$

$= 1875\cos\theta$ Nm.

Gyroscopic couple $C_2 = \frac{v^2}{R \cdot r_W}(2I_W + GI_E)\cos\theta$

$= \frac{25^2}{50 \times 0.3}(2 \times 1 + 5 \times 0.3)\cos\theta$

$= 145.83\cos\theta$ N.m.

Total overturning couple $C = C_1 + C_2$

$= \boxed{2020.8\cos\theta \text{ Nm.}}$

Restoring or balancing couple $= mgh\sin\theta$

$= 250 \times 9.81 \times 0.6 \times \sin\theta$

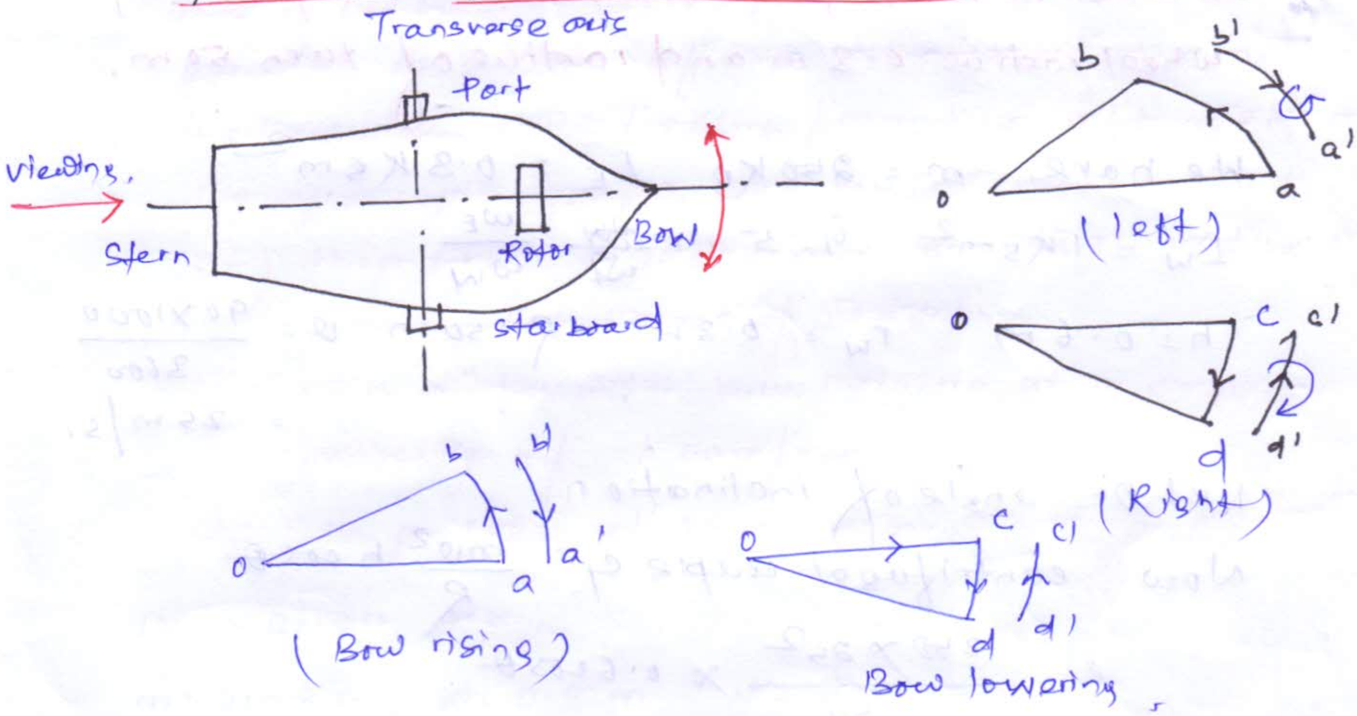
$= 1471.5\sin\theta$ N.m

Balancing both the equations

$1471.5\sin\theta = 2020.8\cos\theta$

or $\boxed{\theta = 53.94°}$

Gyroscopic Effect on Naval ships



Terms :-

(a) Bow — Front end of ship
(b) stern or aft = rear end of ship
(c) starboard = righthand side while looking from stern
(d) port = left hand side when looking from the stern.
(e) steering = turning on one side when viewing from top
(f) pitching = limited angular motion of ship about transverse axis
(g) Rolling = limited angular motion of ship about longitudinal axis.

Gyroscopic effect during steering :-

— angular momentum vector changes from oa to ob. Reaction couple is found to b'a' which tends to raise the bow and lower the stern.

— on turning right, the reaction couple is revered and during this bow is lowered and stern is

Gyroscopic effect on Pitching:-

Pitching of ship takes place in simple harmonic motion in mathematical form $x = X \sin \omega_0 t$

where x = displacement from the mean position

X = rotating vector

~~ω_0 = angular velocity = $\frac{2\pi}{\text{time period}}$~~

angular displacement θ can be expressed in the same way. This is equal to

$$\boxed{\theta = \phi \sin \omega_0 t}$$

where ϕ = amplitude of angular swing.

ω_0 = angular velocity = $\frac{2\pi}{\text{time period}}$

angular velocity of precession $\boxed{\frac{d\theta}{dt} = \phi \omega_0 \cos \omega_0 t}$

The value is max^m when ~~cos φ~~ $\cos \omega_0 t = 1$

So, max^m angular velocity of precession $\omega_p = \phi \omega_0$

$$\boxed{\begin{aligned} \text{Gyroscopic couple} &= I.\omega.\omega_p \\ &= I.\omega \left(\phi . \frac{2\pi}{\text{time period}} \right) \end{aligned}}$$

$$\boxed{\text{Angular acceleration} = -\phi \omega_0^2 \sin \omega_0 t}$$

$$\boxed{\text{Max}^m \text{ angular acceleration} = \phi \omega_0^2}$$

Gyroscopic effect on Rolling:-

As the axes of Rolling of ship and that of the rotor are parallel, there is no precession of the axis of spin and so there is no gyroscopic effect.

Q. The turbine rotor of a ship have a mass of 200kg and it rotates at 2000 rpm and it's radius of gyration is 0.3m. If the rotation of the rotor is clockwise while looking from the aft; determine the gyroscopic couple set by the rotor when

(i) ship takes a left hand turn with radius of 300m at a speed of 30km/hr.

(ii) ship pitches with bow raising at an angular velocity of 1 rad/sec.

(iii) ship rolls at an angular velocity 0.1 rad/s

Data given are:

$m = 200kg$, $K = 0.3m$ $N = 2000 rpm$

so $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 2000}{60} = 209.43$ rad/s

$I = mK^2 = 200 \times 0.3^2 = 18 kg m^2$

(i) when ship takes a left turn:

$R = 300m$ $v = 30 km/hr$

$= \frac{30 \times 1000}{3600} = 8.33 m/s.$

$\omega_p = \frac{v}{R} = \frac{8.33}{300} = 0.027$ rad/s.

Gyroscopic couple $C = I.\omega.\omega_p = 18 \times 209.43 \times .027$

$= 104.42$ Nm.

(ii) ship pitches with bow raising:

$\omega_p = 1$ rad/s

$C = I\omega\omega_p = 18 \times 209.43 \times 1 = 3769.74$ Nm.

effect: Bow rising moves the ship towards starboard

(iii) ship rolls $\omega_p = 0.1$ rad/s

$C = I.\omega.\omega_p = 18 \times 209.43 \times 0.1 = 376.974$ Nm

There is no ~~The effect of rolling~~. ~~as~~ During rolling the axis of precession is parallel to the axis of spin, so there is no gyroscopic couple. Hence there is no effect of rolling.

GOVERNORS:-

Introduction:-

speed variation in an engine occurs in two ways:

- cyclic variation
- variation of speed over a number of revolutions

Cyclic variations:-

Cyclic variation occurs because of variation in the turning moment of the engine. These variations can be reduced by mounting a suitable flywheel on the shaft.

Variation of speed over a number of revolutions:-

Variation of speed over a number of revolutions is because of variation of load on the engine.

- In this case a governor is mounted which controls the mean speed of the engine by regulating fuel supply to it.
- When the load increases, speed decreases and it is necessary to increase the fuel supply by opening the throttle valve to maintain mean speed of the engine, and vice versa.

Difference betn Governor and Flywheel:-

Governor	Flywheel
1. Maintains the variation of mean speed within prescribed limit	1. Controls the speed variations in an engine caused due to fluctuations of turning moment.
2. It regulates the speed over a period of time.	2. It regulates the speed during a cycle only.
3. It regulates speed by	

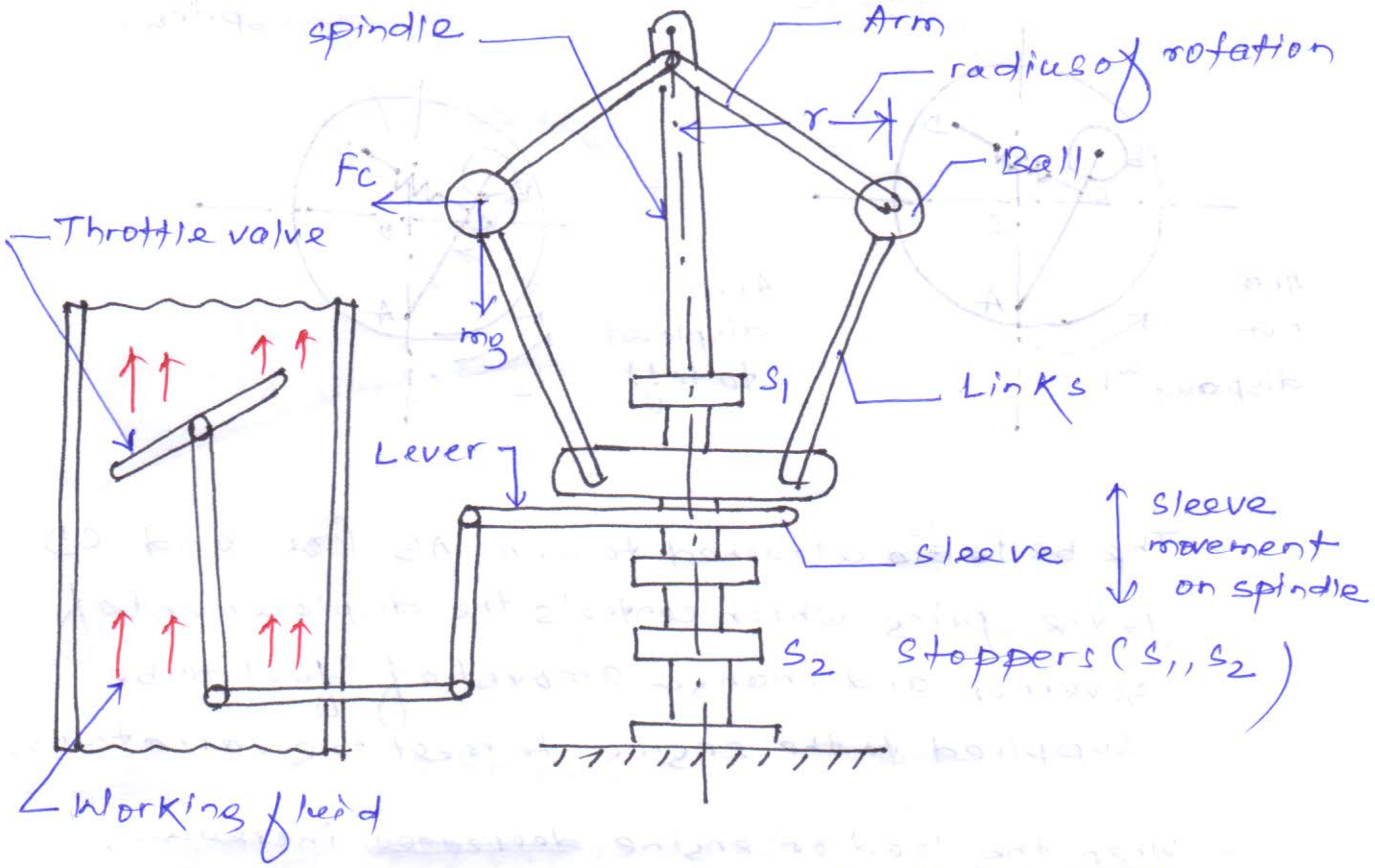
Governor	Flywheel
charge of the engine or prime mover ~~So it controls the~~	It stores energy and give it up whenever required in a cycle,

Types of Governor:-

- Governor
  - Centrifugal governor
    - Pendulum type
      - Watt Governor
    - Loaded governor.
      - Dead wt.
        - Porter governor
        - Proell governor.
      - Spring controlled
        - Hartnell governor
        - Wilson Hartnell governor
        - Hartung governor
        - Pickering governor.
  - Inertia governor.

classification of different governors are explained in the above figure,

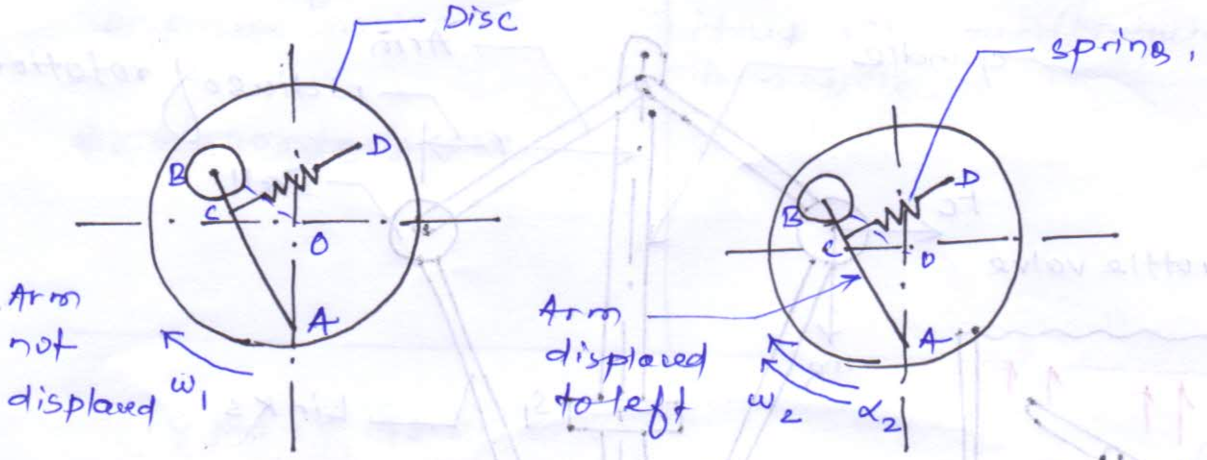
Centrifugal Governor:-



Centrifugal governor consists of two balls connected to spindle through arms. The upper arms are keyed to the spindle and lower arms (links) are connected to the sleeve. The sleeve is free to slide on the spindle. The balls rotate with spindle (shaft), giving rise to the centrifugal force which radially ~~acts~~ acts outwards. When the speed increases, the balls rotate at a larger radius and the sleeve slides upwards on the spindle, ~~this~~ and with the helps ~~the~~ lever the throttle is closed to the required extent.

- With the decrease in speed the governor ball rotate at smaller radius of rotation, compelling the sleeve to move down on the spindle. The downward movement of sleeve opens the throttle to the required extent to admit the required fuel into

Inertia Governor:-



- The ball is attached to arm AB and CD is the spring which controls the displacement of governor and changes amount of fuel to be supplied to the engine to meet the variations.

- When the load on engine ~~decreases~~ increases, the speed of the disc increases to ω_2 and is subjected to an angular acceleration α_2 also.

  and $\boxed{\omega_2 = \omega_1 + \alpha_2 t}$

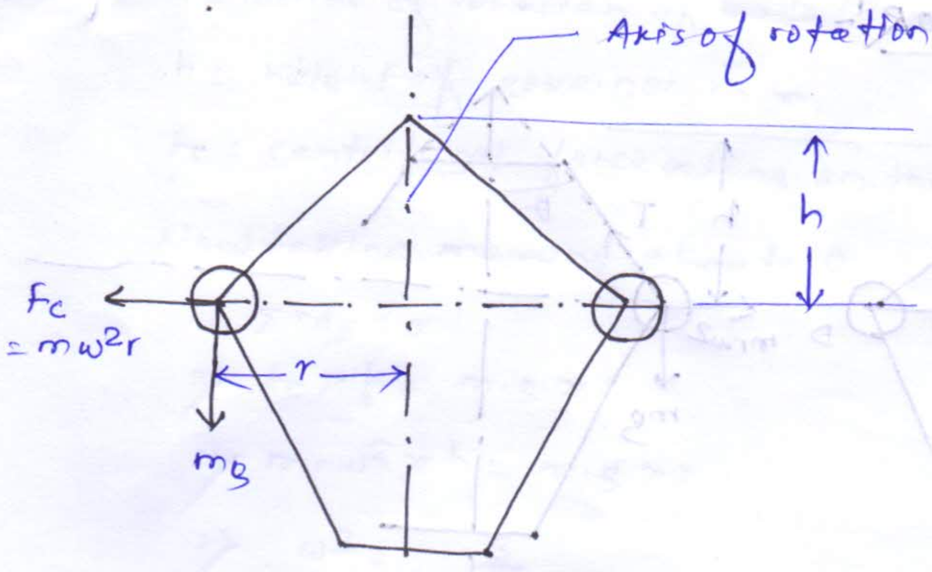
  The arm is displaced to left due to centrifugal force on the governor ball and the energy supplied to the engine is cut-off till new equilibrium position is gained.

Terminology:-

following terms are used in governor:-

(a) Height of governor:- vertical distance from centre of ball to point on the spindle axis where the axes of arms intersect.

- It is denoted by 'h'.



(b) centrifugal force:-

centrifugal force $\boxed{F_C = m r \omega^2}$

Where m = mass of the ball in Kg.

r = radius of rotation in m,

ω = angular speed rad/s

(c) controlling force:-

It is an equal and opposite force to that of the centrifugal force.

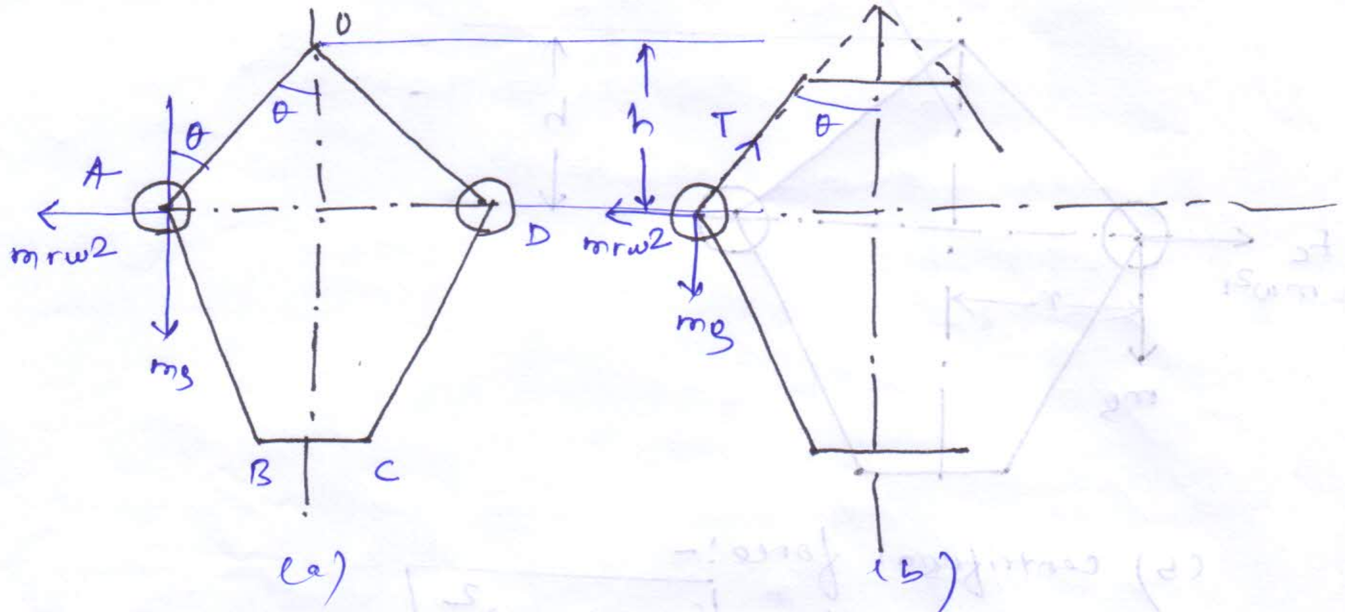
(d) Equilibrium speed:- speed at which governor balls, arms, sleeve etc are in equilibrium and there is no upward or downward movement of sleeve.

(e) Radius of rotation:- It is the horizontal distance between centre of ball and the axis of rotation, denoted by 'r'

(f) Mean equilibrium speed:- It is the speed at ~~of~~ the mean position of the ball or sleeve.

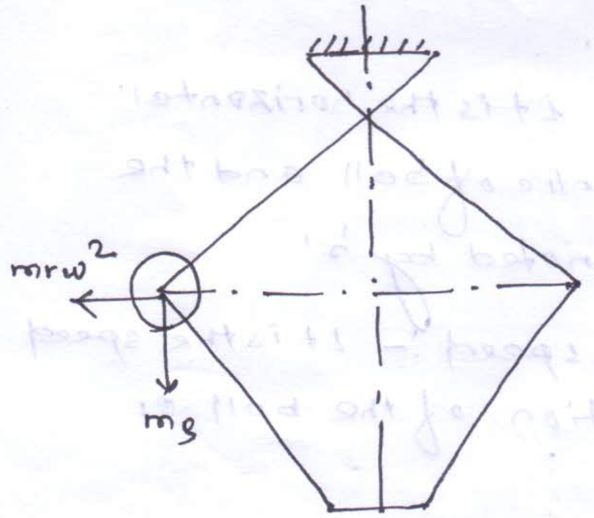
(g) sleeve lift:- It is the vertical distance travelled by sleeve on the spindle in equilibrium speed.

WATT GOVERNOR:-



Watt governor is the simplest form of centrifugal governor. This governor is named after Watt who used it for steam engines.

- It is basically of three types depending upon the position of upper arms.
- When arms intersect at spindle axis, it is known as pinned arm type watt governor. (fig-a)
- The open-arm and cross-arm type watt governors are shown in fig. (b) and (c)



- In each of these three cases the lower arms i.e the links are fixed to the sleeve.
- When speed increases the ball moves outwards due to centrifugal force and pull the sleeve upwards on the spindle through the links and the vice versa.

Let m = mass of each ball in Kg.

w = angular velocity of the balls, about the spindle axis in rad/s.

r = radius of rotation of balls in m

h = height of governor in m.

F_c = centrifugal force acting on the balls, in N.

Now taking moment about O

$\Sigma M_O = 0$

$\Rightarrow F_c \times h = m \cdot g \times r$

$\Rightarrow m r \omega^2 \times h = m \cdot g \times r$

$\Rightarrow \omega^2 = \frac{g}{h}$

$\Rightarrow \left(\frac{2\pi N}{60}\right)^2 = \frac{g}{h}$

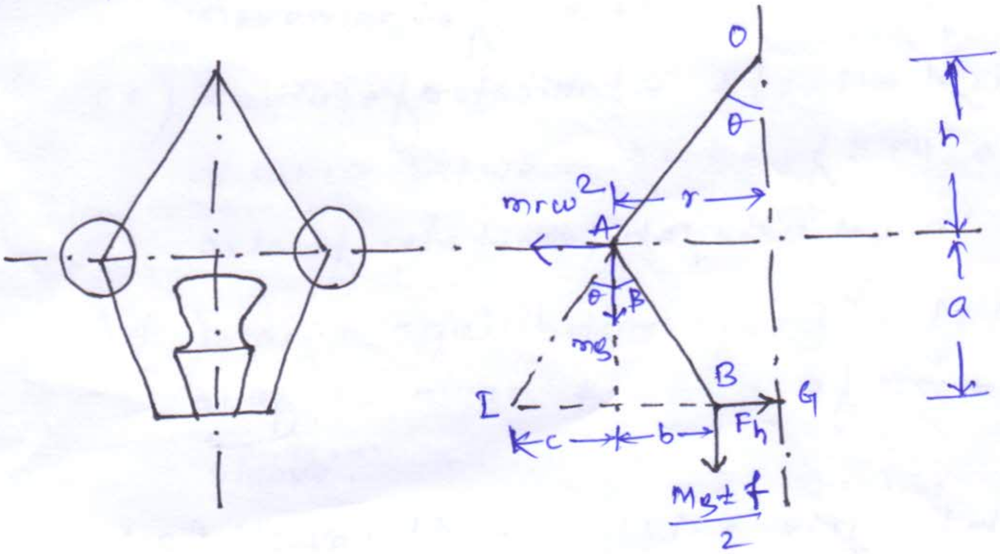
$\Rightarrow N^2 = \frac{9.81 \times 60^2}{(2\pi)^2 \times h} = \frac{895}{h}$

$\Rightarrow \boxed{h = \frac{895}{N^2}}$ m.

from the above equation it's clear that 'h' is inversly proportional to speed 'N' of governor.

PORTER GOVERNOR :-

The modification of Watt's governor with a central load attached to the sleeve is known as porter Governor.



Let M = mass of sleeve

m = mass of each ball

f = force of friction of ~~each ball~~ at the sleeve

The force of friction always acts in a direction opposite to that of the motion. When the sleeve moves up, the force of friction acts in downward direction and total downward force acting on the sleeve is $(Mg+f)$. Similarly when the sleeve moves down, the force on the sleeve will be $(Mg-f)$. In general the net force acting on sleeve is $(Mg \pm f)$

Let h = ht. of the governor

r = distance of center of each ball from the spindle axis.

- Now from the geometry $\triangle BAO$ is a kinematically equivalent to a slider crank mechanism with B as slider (vertical motion), the instantaneous centre of rotation of the link AB is at I for the given configuration of the governor.

- considering equilibrium of left hand half of the governor and taking moment about I

$$\Sigma M_I = 0$$

$$mr\omega^2 \times a = mg \times c + \frac{Mg \pm f}{2}(c+b)$$

$$\text{or } mr\omega^2 = mg \times \frac{c}{a} + \frac{Mg \pm f}{2}\left(\frac{c}{a}+\frac{b}{a}\right)$$

$$= mg\tan\theta + \frac{Mg \pm f}{2}(\tan\theta + \tan\beta)$$

$$= \tan\theta\left[mg + \frac{Mg \pm f}{2}(1+K)\right]$$

where $K = \dfrac{\tan\beta}{\tan\theta}$

From the above equation

$$mr\omega^2 = \frac{r}{h}\left[mg + \frac{Mg \pm f}{2}(1+K)\right]$$

$$\Rightarrow \boxed{h = \frac{g}{\omega^2} + \frac{(Mg \pm f)(1+K)}{2m\omega^2}}$$

Q.1 A Watt governor runs at 100 rpm. Determine the height of the governor. If the speed of governor increases to 102 rpm find the change in vertical height.

Given: $N_1 = 100$ rpm $\quad N_2 = 102$ rpm,

initial height $h_1 = \frac{895}{N_1^2} = \frac{895}{(100)^2} = 0.0895$ m

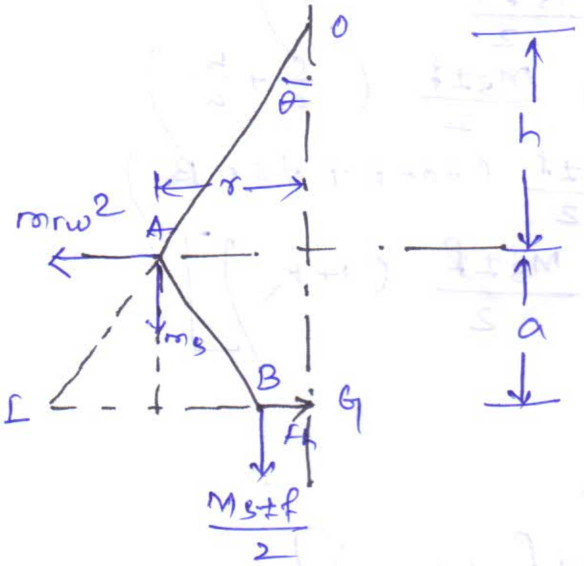
final height $h_2 = \frac{895}{(102)^2} = 0.086$ m

change in vertical height $= h_1 - h_2$

$= 0.0895 - 0.0860$

$= 0.0035$ m $= \boxed{3.5 \text{ mm}}$

Q.2 In a porter governor, each arm is 400 mm long. The lower arms are attached to the sleeve at a distance 45 mm from the axis. Each ball has a mass of 8 kg and the load on the sleeve is 60 kg. What will be the equilibrium speeds for two extreme radii of 250 mm and 300 mm of rotation of governor.



Given data:-

$m = 8$ kg $\quad M = 60$ kg.

$BG = 45$ mm $\quad OA = 400$ mm

Now we have

$$mr\omega^2 = \tan\theta \left[mg + \frac{Mg}{2}(1+k) \right)$$

(∵ force of friction neglected)

(i) when $r = 250$ mm

$\tan\theta = \frac{r}{h} = \frac{250}{\sqrt{400^2 - 250^2}} = 0.8$

$K = \frac{\tan\beta}{\tan\theta} = \frac{b/a}{0.8} = \frac{(250-45)/a}{0.8}$

Now $a = \sqrt{(AB)^2 - b^2}$

$= \sqrt{(400)^2 - (205)^2} = 343.4\text{ mm}$

so $K = \dfrac{b/a}{0.8} = \dfrac{205/343.4}{0.8} = \boxed{0.746}$

So we have

$8 \times 0.25 \times w^2 = 0.8\left[8 \times 9.81 + \dfrac{60 \times 9.81}{2}(1 + 0.746)\right]$

$\Rightarrow 2w^2 = 0.8(78.48 + 513.85)$

$\Rightarrow w^2 = 237$

or $w = 15.39$

Now $w = \dfrac{2\pi N}{60} = 15.39$

$\Rightarrow N = \dfrac{15.39 \times 60}{2\pi} = \boxed{147\text{ rpm}}$

(ii) when radius r = 300 mm

$\tan\theta = \dfrac{r}{h} = \dfrac{300}{\sqrt{400^2 - 300^2}} = 1.134$

$b = 300 - 45 = 255\text{ mm}$

$a = \sqrt{400^2 - 255^2} = 308.2\text{ mm}$

$K = \dfrac{\tan\beta}{\tan\theta} = \dfrac{b/a}{1.134} = \dfrac{(255/308.2)}{1.134} = 0.73$

so,

$8 \times 0.3 \times w^2 = 1.134\left[8 \times 9.81 + \dfrac{60 \times 9.81}{2}(1 + 0.73)\right]$

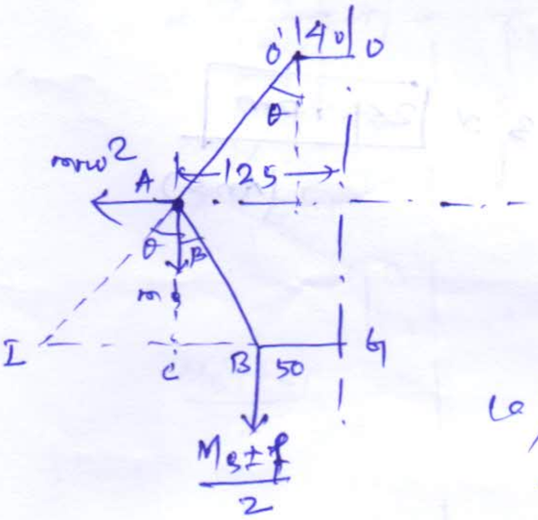
$\Rightarrow 2.4w^2 = 1.134(78.48 + 509.139)$

$\Rightarrow w^2 = 277.6$

$\Rightarrow w = 16.66 = \dfrac{2\pi N}{60}$

$\Rightarrow \boxed{N = 159.1\text{ rpm}}$

Q.2 Each arm of a porter governor is 250 mm long. The upper ~~arm~~ and lower arm are pivoted to links of 40 mm and 50 mm rerspectively from axis of rotation. Each ball has a mass of 5 kg and sleeve mass is 50 kg. The force of friction on sleeve mechanism is 40N. Determine the range of speed of the governor for extreme radii of rotation of 125 mm and 150 mm.



Given data:-

arm length = 250 mm

$OO' = 40$ mm, $BG = 50$ mm

$m = 5$ kg, $M = 50$ kg

$f = 40N$, $r_1 = 125$ mm

$r_2 = 150$ mm.

(a) when $r_1 = 125$ mm

$$\tan\theta = \frac{(125-40)}{\sqrt{250^2-85^2}} = \frac{85}{235.1} = .361$$

$$\Rightarrow \boxed{\theta = 19.87^\circ}$$

$$\tan\beta = \frac{125-50}{\sqrt{250^2-75^2}} = \frac{75}{238.48} \Rightarrow \boxed{\beta = 17.47^\circ}$$

$\Rightarrow \tan\beta = .315$

$$K = \tan\beta/\tan\theta = \boxed{.872}$$

Using the relation

$$mr\omega^2 = \tan\theta\left[mg + \frac{Mg-f}{2}(1+K)\right]$$ (∵ as the sleeve moves down, force of friction act upward)

$$\Rightarrow 5\times0.125\times\omega^2 = 0.361\left[5\times9.81 + \frac{50\times9.81-40}{2}(1+.872)\right]$$

$\Rightarrow 0.625\omega^2 = .169.929$

$\Rightarrow \omega = 16.489$

$$\Rightarrow N = \frac{16.489\times60}{2\pi} = \boxed{157.45 \text{ rpm}}$$

(b) when $r = 150$ mm

$$\tan\theta = \frac{150-40}{\ldots} = \frac{110}{\ldots} = 0.49$$

$$\tan\beta = \frac{150-50}{\sqrt{250^2-100^2}} = \frac{100}{229.128} = 0.44$$

$$K = \frac{\tan\beta}{\tan\theta} = \frac{0.44}{0.49} = \boxed{0.897}$$

Now, we have

$$mr\omega^2 = \tan\theta\left[mg + \frac{Mg+f}{2}(1+K)\right]$$

$$\Rightarrow 5\times0.15\times\omega_2^2 = 0.49\left[5\times9.81 + \frac{50\times9.81+40}{2}(1+0.897)\right]$$

$$\Rightarrow 0.75\omega_2^2 = 270.592$$

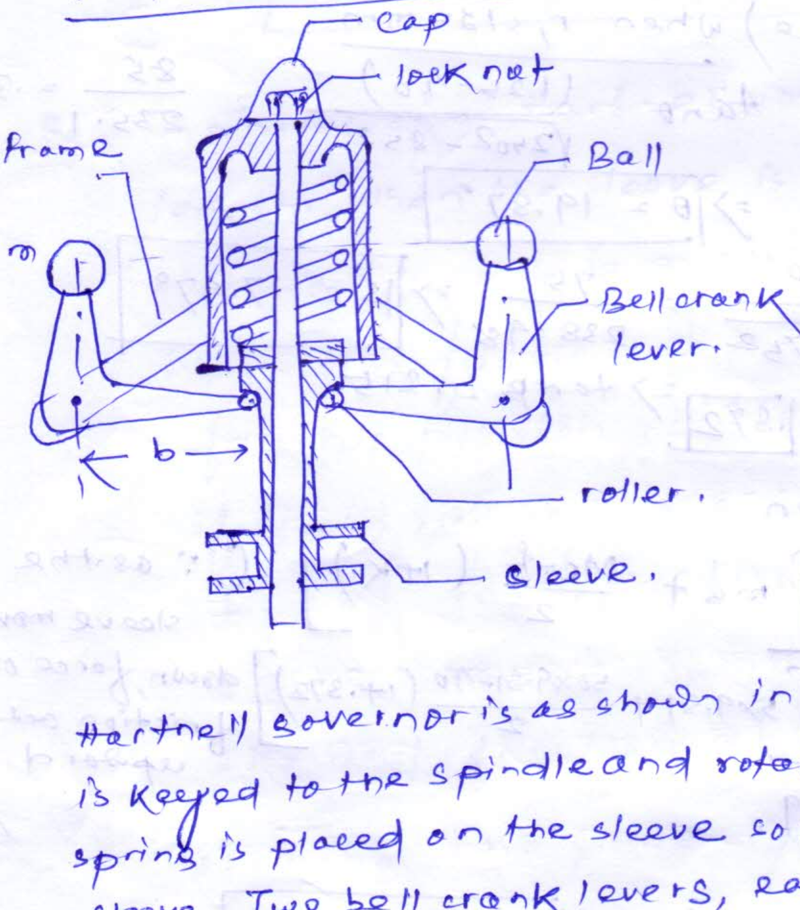
$$\Rightarrow \omega_2 = 18.99 \simeq 19$$

$$\Rightarrow N_2 = 181.38 \text{ rpm},$$

Range of speed $= N_2 - N_1 = 23.93 \simeq \boxed{24 \text{ rpm}}$

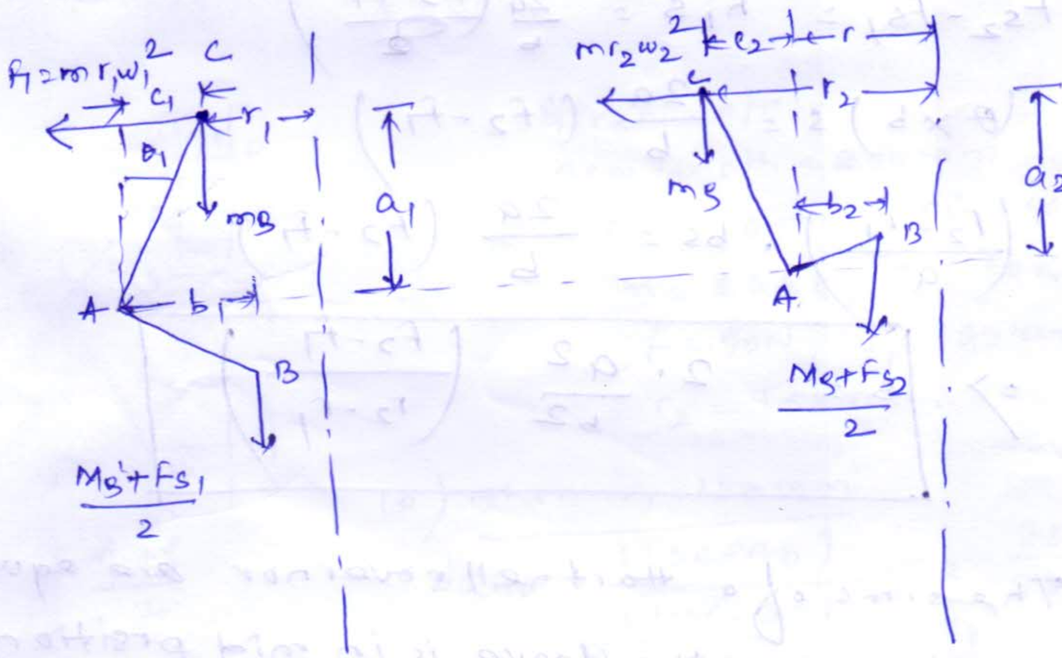
(Ans)

Hartnell Governor:-



Hartnell governor is as shown in the figure. The frame is keyed to the spindle and rotates with it. A compressed spring is placed on the sleeve so that it can exert on sleeve. Two bell crank levers, each carrying a ball at one end and a roller at the other end, are pivoted to a pair of arms. The rollers are fitted into the groove in the sleeve. When speed increases balls move outward compelling the sleeve to slide on the spindle upward against the spring force. If the force decreases, the sleeve moves downward. The spring force is adjusted with the help of locknut. The movement of sleeve is communicated to the throttle to perform necessary task.

The ~~three~~ positions of bell crank levers are shown in the figure.



Let F = centrifugal force $= mr\omega^2$

F_S = spring force,

Now taking moment about fulcrum A, $\Sigma M_A = 0$.

$$F_1 a_1 = mg\, c_1 + \left(\frac{Mg + Fs_1}{2}\right) b_1$$

$$F_2 a_2 = mg\, c_2 + \left(\frac{Mg + Fs_2}{2}\right) b_2$$

[Neglecting friction force f] — (i)

In the working range of governors, θ is very small and so the obliquity effect may be neglected and we have.

$$a_1 = a_2 = a, \quad b_1 = b_2 = b \quad c_1 = c_2 = 0.$$

$$\text{so,} \quad F_1 . a = \frac{Mg + Fs_1}{2} . b \quad \text{— (ii)}$$

subtracting equation (iii) from (ii)

$$(F_2 - F_1)\,a = \left(\frac{F_{S_2} - F_{S_1}}{2}\right) b.$$

$$\text{or} \quad \boxed{F_{S_2} - F_{S_1} = \frac{2a}{b} \cdot (F_2 - F_1)}$$

Now let s = stiffness of spring

h_1 = movement of sleeve.

$$F_{S_2} - F_{S_1} = h_1 s = \frac{2a}{b}\left(F_2 - F_1\right)$$

$$(\theta \times b)\, s = \frac{2a}{b}\,(F_2 - F_1)$$

$$\left(\frac{r_2 - r_1}{a}\right) \cdot bs = \frac{2a}{b}\,(F_2 - F_1)$$

$$\Rightarrow \boxed{s = 2 \cdot \frac{a^2}{b^2}\left(\frac{F_2 - F_1}{r_2 - r_1}\right)}$$

Q. The arms of a Hartnell governor are equal length. When the sleeve is in mid position, the masses rotate a circle of diameter 150 mm. Neglecting friction the equilibrium speed for this position is 360 rpm. Maxm variation of speed taking into account friction, is ±6% of mid-position speed for a maxm sleeve moment 30 mm. Sleeve mass is 5 kg and friction at the sleeve is 35 N.

- Assuming the power of governor is sufficient to overcome the friction by 1% of change of speed on each side of mid-position find

(i) mass of rotating mass of ball, (ii) spring

21/02/2015

Q.3 For a Hartnell governor, following data are provided:

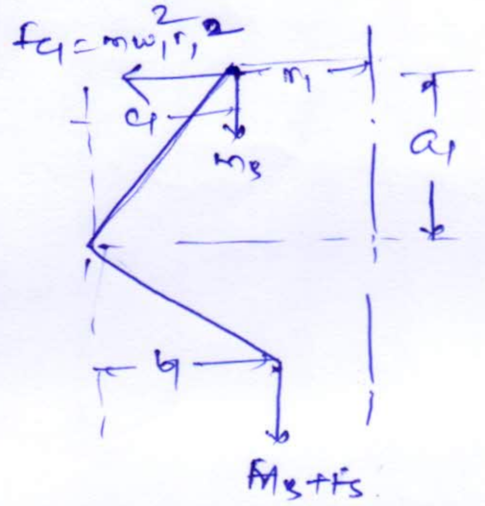
mass of each ball = 1.8 Kg., M = 50 Kg.

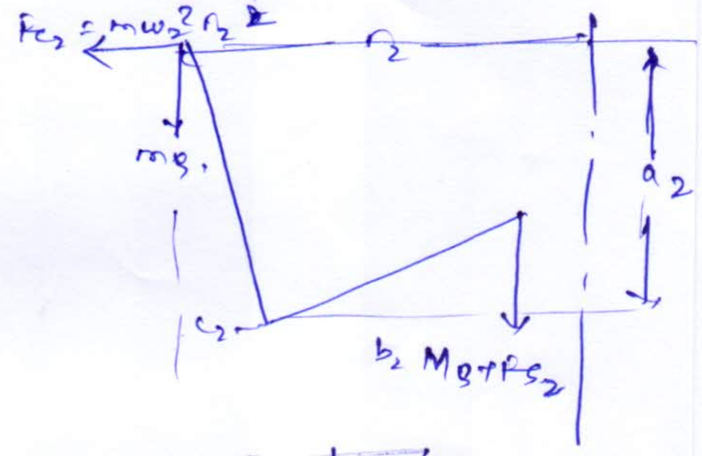
length of vertical arm of bell crank lever = 8.75 cm.

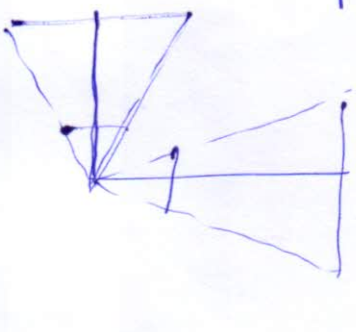
length of other arm of lever = 10 cm.

Speed corresponding to radii of rotations of 12 cm and 13 cm are 296 rpm and 304 rpm respectively. Determine spring stiffness.

Ans :- m = 1.8 Kg, M = 50 Kg.







$r_1 = 12$ cm $\quad r_2 = 13$ cm,

$\omega_1 = \dfrac{2\pi \times 296}{60} = 30.99$ rad/s

$\omega_2 = \dfrac{2\pi \times 304}{60} = 31.83$ rad/s.

$f_{c1} = m\omega_1 r_1^2 = 1.8 \times 30.99^2 \times (.12)^{\cancel{2}} = \cancel{0.803}$

$= 207.44$ N

$f_{c2} = 1.8 \times 31.83^2 \times .13 = 237.07$ N.

spring stiffness $s = 2 \cdot \dfrac{a^2}{b^2}\left(\dfrac{f_{c2} - f_{c1}}{r_2 - r_1}\right)$

$= 2 \times \dfrac{0.0875^2}{0.1^2} \times \left(\dfrac{237.07 - 207.44}{0.13 - 0.12}\right)$

Controlling Force Diagram :-

The governor balls rotating in a circular path experience a force which acts radially outwards. This force is known as centrifugal force. This force is opposed by an equal and opposite force, acting radially inward. This inward force is called controlling force.

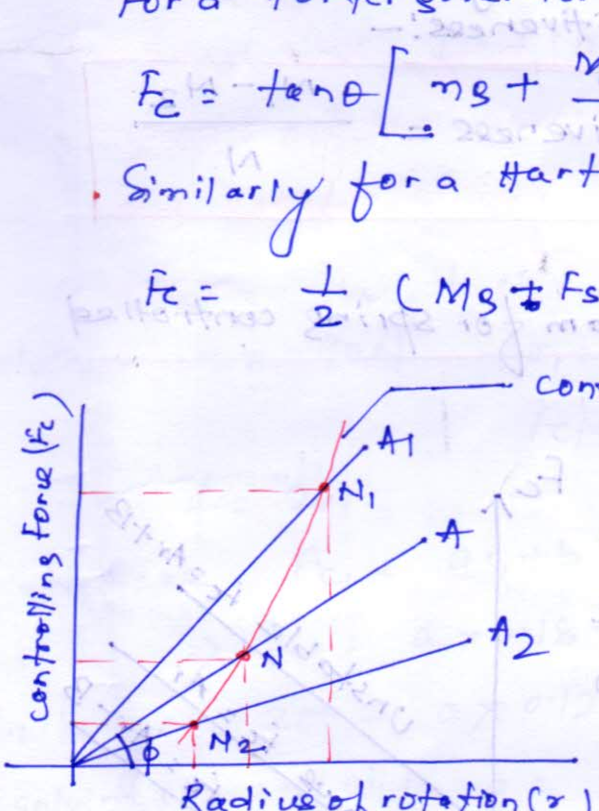
The magnitude of centrifugal force $F_c = mr\omega^2$. When a graph is plotted with the controlling force (F_c) as ordinate and radius of rotation (r) of the ball as abscissa, it's called controlling force diagram. This graph is useful for finding the stability of a governor.

For a Porter governor, controlling force is given by,

$$F_c = \tan\theta\left[mg + \frac{Mg \pm f}{2}(1+K)\right]$$

Similarly for a Hartnell governor, it is given by

$$F_c = \frac{1}{2}(Mg \pm F_s \pm f)\frac{b}{a}$$



Let's consider the controlling force diagram of a Porter governor, neglecting the frictional force.

Controlling force $F_c = m\omega^2 r$

$$= m\left(\frac{2\pi N}{60}\right)^2 r$$

$$\Rightarrow \frac{F_c}{r} = m\left(\frac{2\pi}{60}\right)^2 \times N^2$$

$$\Rightarrow \boxed{\frac{F_c}{r} = KN^2}$$

$\left[\text{where } K = m\left(\frac{2\pi}{60}\right)^2 = \text{a constant}\right]$

from the diagram we can say $\tan\phi = \frac{F_c}{r}$, so substituting the value of $\frac{F_c}{r}$ we have

$$\boxed{\tan\phi = KN^2} \quad -(1)$$

Using the above relation in Eq.(1), value of ϕ may be obtained for different values of N and number of lines or curves, like OA, OA_1 and OA_2 may be obtained.

- For the particular curve it can be seen that when the radius of rotation increases, centrifugal force also increases and vice versa. This type of governor is said to be stable.
- For an unstable governor, the radius of rotation of balls does not increase by increase of speed.

Coefficient of Insensitiveness:-

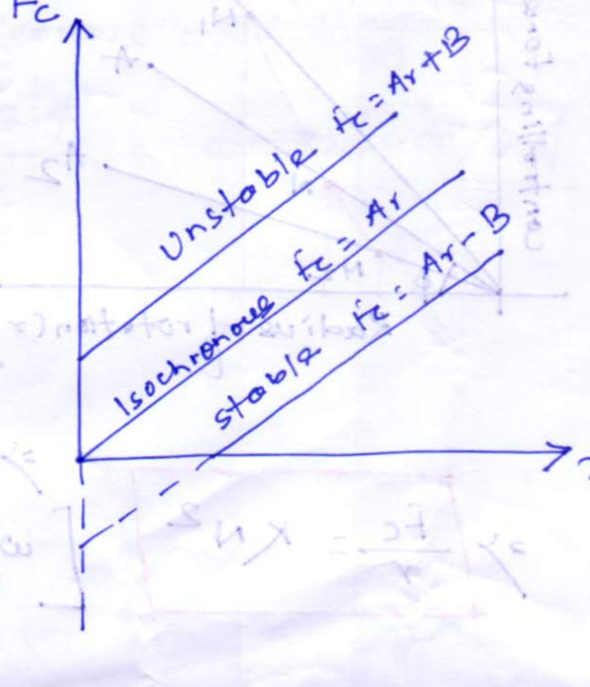
$$\boxed{\text{coefficient of insensitiveness} = \frac{N_1 - N_2}{N}}$$

Controlling Force Diagram for spring controlled governors:-

For the spring controlled governor the relation betn the centrifugal ~~and~~ force and radius of rotation can be expressed as

$F_c = A \cdot r + B$

Where A and B are constants.



We have $\tan\phi = \frac{F_c}{r} = A + \frac{B}{r}$

when $B = 0$, $\tan\phi = \frac{F_C}{r} = \frac{m\omega^2 r}{r} = m\omega^2$.

m is a constant. And the curve indicates, ~~at same~~ the governor rotates at same speed for different radii of rotation. So, the governor is isochronous.

- If B is possitive, $\tan\phi$ decreases with increase in r and the governor will be unstable.
- If B is negative, $\tan\phi$ increases with r and the governor is stable.

Q The controlling force curve of a spring controlled governor is a straight line. The wt. of each ball is 40N and the extreme radii of rotation are 12cm and 18cm. If the values of the controlling force at the above radii be respectively 200N and 360N and the friction mechanism is equivalent to 2N at each ball, find (a) extreme equilibrium speeds of the governor, (b) equilibrium speed and coefficient of insensitiveness at a radius of 15cm.

Soln:- $mg = 40N$ $\quad r_1 = 18cm \quad r_2 = 12cm$,

$F_{C_1} = 360N \quad F_{C_2} = 200N$

$f = 2N$

We have $F_{C_1} = ar_1 + b$

$\Rightarrow 360 = a \times 0.18 + b$ ——— (1)

Similarly, $200 = a \times 0.12 + b$ ——— (2)

solving (i) and (ii) $a = 2666.67$

$b = -120$

(a) Extreme equilibrium speeds

Highest equilibrium speed

$\frac{40}{\dots} \times 0.18 \times \left(\frac{2\pi}{\dots}\right)^2 \times N^2 \dots \boxed{N = \dots}$

Lowest speed

$$200 = \frac{40}{9.81} \times 0.12 \times \left(\frac{2\pi}{60}\right)^2 \times N_2^2$$

$$\Rightarrow \boxed{N_2 = 193 \text{ rpm}}$$

(b) Equilibrium speed:-

We have $F_C = Ar + B$

$$= 2666.67 \times 0.15 + (-120) = 280\,N$$

so equilibrium speed

$$280 = \frac{40}{9.81} \times 0.15 \times \left(\frac{2\pi}{60}\right)^2 \times N^2$$

$$\Rightarrow \boxed{N = 204.22 \text{ rpm}}$$

Coefficient of insensitiveness! $= \frac{N_1 - N_2}{N}$

Now for max^m speed

$$F_C + f = \frac{W}{g} \omega^2 r$$

$$\Rightarrow 280 + 2 = \frac{40}{9.81} \times 0.15 \times \left(\frac{2\pi}{60}\right)^2 \times N_{max}^2$$

$$\Rightarrow \boxed{N_{max} = 204.94 \text{ rpm.}}$$

Min^m speed

$$F_C - f = \frac{W}{g} \omega^2 r$$

$$\Rightarrow 280 - 2 = \frac{40}{9.81} \times 0.15 \times \left(\frac{2\pi}{60}\right)^2 \times N_{min}^2$$

$$\Rightarrow \boxed{N_{min} = 203.4 \text{ rpm.}}$$

So coefficient of insensitiveness $= \frac{N_{max} - N_{min}}{N}$

$$= \frac{204.94 - 203.4}{204.22} = \cancel{0.0072} \; \boxed{0.0072}$$

$$mr(c\omega)^2 \times a = \frac{1}{2}(Mg+E+fs)\,b \quad \text{--- (7)}$$

Dividing equation (6) by (7)

$$\frac{1}{c^2} = \frac{Mg+fs}{Mg+E+fs}$$

or, $$\frac{Mg+E+fs}{Mg+fs} = c^2$$

$$\Rightarrow \frac{E}{Mg+fs} = (c^2-1)$$

$$\Rightarrow \text{Effort, } \boxed{\frac{E}{2} = \frac{c^2-1}{2}(Mg+fs)} \quad \text{--- (8)}$$

Power of a Governor:-

Power of a governor is the work done at the sleeve for a given percentage change of speed i.e., it is the product of effort and displacement of the sleeve.

- for a Porter governor, having all equal arms which intersects on the axis.

$$\boxed{\text{Power } P = \frac{E}{2} \times (2 \times \text{ht. of governor})}$$

If the height of the governor changes from h to h_1 when the speed changes from ω to $c\omega$

$$h = \frac{2m + Mg(1+K)}{2m\omega^2} \text{ and}$$

$$h_1 = \frac{2m + Mg(1+K)}{2m(c\omega)^2}$$

or $$\frac{h_1}{h} = \frac{1}{c^2}$$

Displacement of sleeve $= 2(h - h_1)$

$$= 2h\left(1-\frac{h_1}{h_0}\right)$$

$$= 2h\left(1-\frac{1}{c^2}\right)$$

$$= 2h\left(\frac{c^2-1}{c^2}\right)$$

so $\text{power} = \frac{c^2-1}{2}(m+M)g \times 2h\left(\frac{c^2-1}{c^2}\right)$

$$\boxed{= (m+M)gh\left(\frac{c^2-1}{c}\right)^2}$$

~~Controlling Force~~ :—

Q.1 The upper arm of a Porter Governor are pivoted on the axis of rotation, their lengths being 30cm. The lower arms are pivoted on the sleeve at a distance of 3cm from the axis, their lengths being 27cm. Mass of each ball is 6kg and sleeve mass is 50kg. Determine the equilibrium speed for a radius of rotation of 17cm and also the effort and power for 1% change of speed.

Q.2 In a Hartnell Governor, the lengths of ball and sleeve arms of a bell crank lever are 120mm and 100mm respectively. The distance of the fulcrum of bell crank lever from governor axis is 140mm. Each governor ball has a mass of 4kg. The governor runs at a mean speed of 300 rpm with ball arms vertical and sleeve arms horizontal.

For an increase speed of 4% , the sleeve moves 10 mm upwards. Neglecting friction find (a) min^m equilibrium speed if the total sleeve momement is limited to 20 mm, (b) spring stiffness, (c) sensitiveness of governor, (d) spring stiffness if the governor is to be isochronous at 300 rpm,

Effort of a Governor :-

The effort of a governor is the mean force acting on the sleeve to raise or lower it for a given change of speed. The governor is in equilibrium at constant speed and the resultant force acting on the sleeve is zero. In case of a speed variation, a force is required to be exerted on the sleeve which tends to move it. When the sleeve occupies a new steady position, the resultant force acting on it is zero again.

- If the force acting on the sleeve changes gradually from zero to a value E, for an increased speed, the mean effort is E/2

For a porter governor

$$h = \frac{g}{\omega^2} + \frac{Mg(1+K)}{2m\omega^2}$$

$$= \frac{2mg + Mg(1+K)}{2m\omega^2} \quad \text{---} (1)$$

Let ω be increased to c times ω, and E be the force applied on the sleeve to prevent it from moving, then force on the sleeve is increased to $(Mg+E)$

so $$h = \frac{2mg + (Mg+E)(1+K)}{2m(c.\omega)^2} \quad \text{---} (2)$$

Dividing equation (2) by (1)

$$\frac{2mg + (Mg+E)(1+K)}{2m\, c^2\omega^2} \times \frac{2m\omega^2}{2mg + Mg(1+K)}$$

$$\Rightarrow \frac{2mg + (Mg+E)(1+K)}{2mg + Mg(1+K)} = \frac{c^2}{1}$$

or,

$$\frac{[2mg + (Mg+E)(1+K)] - [2mg + Mg(1+K)]}{2mg + Mg(1+K)} = \frac{c^2-1}{1}$$

$$\Rightarrow \frac{E(1+K)}{2mg + Mg(1+K)} = c^2 - 1$$

$$\Rightarrow \boxed{E = \frac{(c^2-1)}{1+K}\left[2mg + Mg(1+K)\right]}$$

Effort

$$\boxed{\frac{E}{2} = \frac{c^2-1}{(1+K)}\left[mg + \frac{Mg}{2}(1+K)\right]} \quad -(3)$$

If $K = 1$

Effort $\boxed{\frac{E}{2} = \frac{c^2-1}{2}(m+M)g.} \quad -(4)$

For a Watt governor $M = 0$

Effort, $\boxed{\frac{E}{2} = \frac{c^2-1}{2} mg} \quad -(5)$

Thus effort of a Watt governor is less than that of a Porter governor.

For Hartnell Governor :-

$$mr\omega^2 a = \frac{1}{2}(Mg + F_s)b \quad -(6)$$

Let E is the force applied on the sleeve to prevent its movement, when speed changes from ω to $c\omega$.

Sensitiveness of Governor :-

A governor is said to be sensitive when it readily responds to a small change of speed. The movement of the sleeve for a fractional change of speed is the measure of sensitivity.

Mathematically, sensitiveness = $\dfrac{\text{Mean speed}}{\text{Range of speed}}$

$$= \frac{N}{(N_2 - N_1)}$$

where N = mean speed of governor = $\dfrac{N_1 + N_2}{2}$

N_1 = min^m speed

N_2 = max^m speed

so, $$\boxed{\text{sensitiveness} = \frac{N_1 + N_2}{2(N_2 - N_1)}}$$

Hunting :-

- A governor is said to be hunting if the speed fluctuates continously above and below the mean speed.

Isochronism :-

A governor with a range of speed zero, is known as an isochronous governor. For an isochronous governor.

$$\text{sensitiveness} = \frac{\text{Mean speed}}{\text{Range of speed}} = \infty$$

This means for all positions of the sleeve and ball, the governor has same speed.

- An isochronous governor is not practical due to friction at the sleeve.

for a Porter governor, we have

$$h_1 = \frac{g}{\omega_1^2} + \frac{Mg \pm f}{2m\omega_1^2}(1+K)$$

$$h_2 = \frac{g}{\omega_2^2} + \frac{Mg \pm f}{2m\omega_2^2}(1+K)$$

for equal arm lengths of the governors and intersecting at the spindle axis and neglecting frictional force.

$$h_1 = \frac{g}{\omega_1^2}\left(1+\frac{M}{m}\right) \quad h_2 = \frac{g}{\omega_2^2}\left(1+\frac{M}{m}\right)$$

for isochronism $\omega_1 = \omega_2$ i.e., $h_1 = h_2$

In case of Hartnell governor, neglecting friction

at ω_1,

$$m r_1 \omega_1^2 a = \frac{1}{2}(Mg + f s_1) b$$

at ω_2, $m r_2 \omega_2^2 a = \frac{1}{2}(Mg + f s_2) b$

for isochronism $\omega_1 = \omega_2$

$$\frac{m r_1 \omega^2}{m r_2 \omega^2} = \frac{Mg + f s_1}{Mg + f s_2}$$

$$\Rightarrow \boxed{\frac{r_1}{r_2} = \frac{Mg + f s_1}{Mg + f s_2}}$$

Stability :- A governor is said to be stable if it brings the speed of the engine to the required value without much hunting. The balls of the governor occupy a definite position for each speed of the engine within working range.

For mean speed (safe), the condition

$$R_w = \frac{P}{2} + \frac{Q}{2}$$

$$\Rightarrow 5395.5 = (0.167 + 5.041)\, v^2$$

$$\Rightarrow v^2 = 1036.002$$

$$\Rightarrow v = 32.18 \text{ m/s.}$$

$$= \frac{32.18 \times 3600}{1000} = \boxed{115.87 \text{ km/hr.}}$$

Q.2 A racing car weighs 20 kN. It has a wheel base of 2 m, track width 1 m and ht. of CG 0.3 m above the ground level. The engine flywheel rotates at 3000 rpm clockwise when viewed from the front. The moment of inertia of flywheel is 4 kg m² and M.I. of each wheel is 3 kg m². Find the reactions betn the wheels and the ground when the car takes a curve of radius 15 m towards right at 30 km/hr, taking into account the gyroscopic and centrifugal effect. Wheel radius is 0.4 m.

$$m r \omega_1^2 \times a = \left(\frac{Mg + f_s + f}{2}\right) b$$

$$m \times 0.075 \times (37.7 \times 1.01)^2 \times a = \left(\frac{5 \times 9.81 + f_s + 85}{2}\right)$$

$$m \times .075 \times (37.7 \times .099)^2 = \left(\frac{5 \times 9.81 + f_s - 85}{2}\right)$$

$$\boxed{m = 8.21 \text{ kg}}$$

$$m r \omega_1^2 \times a = \left(\frac{M_g + F_s + f_0}{2}\right) b,$$

$$m \times 0.075 \times (37.7 \times 1.01)^2 = \left(\frac{5 \times 9.81 + F_s + 35}{2}\right)$$

$$m \times 0.075 \times (37.7 \times .099)^2 = \frac{5 \times 9.81 + F_s - 35}{2}$$

$$m \times .075 \times 37.7^2 (1.01^2 - 0.99^2) = \frac{35 + 35}{2}$$

$$\Rightarrow \boxed{m = 8.21 \text{ k}}$$

$$m r_2 \omega_2^2 = \frac{M_g + F_s + f}{2}$$

$$8 \times 0.2 \quad 8.21 \times \left(.075 + \frac{.03}{2}\right)$$

We have $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 360}{60} = 37.7 \text{ rad/s}$.

(i) Considering friction at the mid position

$$m r_1 \omega_1^2 \times a = \left(\frac{Mg + F_s + f}{2}\right) \cdot b$$

$$\Rightarrow m \times \frac{0.15}{2} \times (37.7 \times 1.01)^2 = \left(\frac{5 \times 9.81 + F_s + 35}{2}\right) \quad \text{——(i)}$$

$$m \times \frac{.15}{2} \times (37.7 \times .99)^2 = \left(\frac{5 \times 9.81 + F_s - 35}{2}\right) \quad \text{——(ii)}$$

Substracting (ii) from (i)

$$m \times 0.075 (37.7)^2 \left[1.01^2 - 0.99^2\right] = \left(\frac{35 + 35}{2}\right)$$

$$\Rightarrow \boxed{m = 8.21 \text{ Kg.}}$$

(ii) At the extreme positions

$$m r_2 \omega_2^2 \times a = \left(\frac{Mg + F_{s_2} + f}{2}\right) \cdot b$$

$$\Rightarrow 8.21 \times \left(.075 + \frac{.03}{2}\right) \times (37.7 + 1.06)^2 = \left(\frac{5 \times 9.81 + F_{s_2} + 35}{2}\right) \quad (\because a = b)$$

$$\Rightarrow \boxed{F_{s_2} = 2275.8 \text{ N.}}$$

$$8.21 \times \left(0.075 - \frac{0.03}{2}\right) \times (37.7 \times 0.94)^2 = \frac{(5 \times 9.81) + F_{s_1} - 35}{2}$$

$$\Rightarrow \boxed{F_{s_1} = 1223.2 \text{ N.}}$$

$S = 35088 \, N/m$

or $\boxed{35.088 \, N/mm}$

(iii) initial compression $= \dfrac{F_{S1}}{S}$

$= \dfrac{1223.2}{35.088} = \boxed{34.86 \, mm}$

Introduction:-

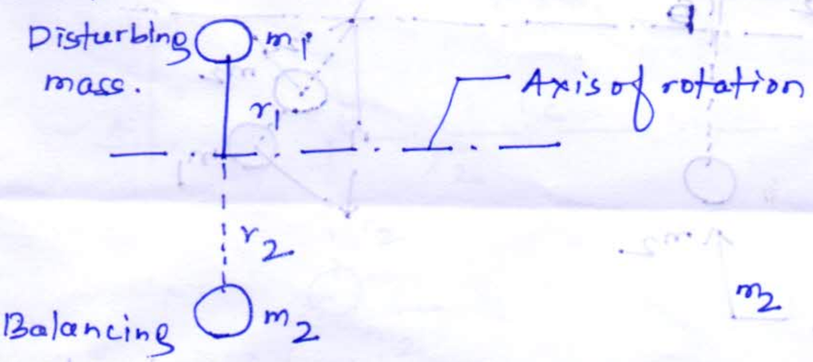
Machines have several rotating parts. Some of them have reciprocating motion e.g. piston and some of them have rotating motion e.g. crankshaft. If these moving parts are not in complete balance, inertia force generation would lead to vibration, noise, wear and tear of the parts.

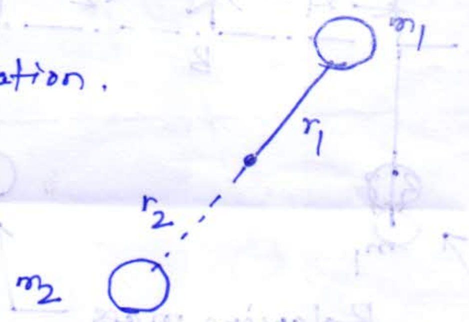
- Balancing plays a major role in designing these systems to reduce unbalance to an acceptable limit.

Balancing of single Revolving mass:-

(i) Balancing in same plane (ii) balancing in different plane.

(i) Balancing and disturbing mass revolve in same plane:-





Let m_1 = mass attached to the shaft
ω = angular velocity of the mass in rad/s.
r_1 = distance of C.G. of the mass from axis of rotation.

In order to counteract the disturbing force e.g. the centrifugal force due to m_1, a counter mass m_2 at a radius r_2 is placed in the same plane, such that the centrifugal forces due to the two masses are equal and opposite.

Mathematically, $F_{C_1} = m_1 \omega^2 r_1$

balancing force $F_{C_2} = m_2 \omega^2 r_2^2$

For balancing, $F_{C_1} = F_{C_2}$

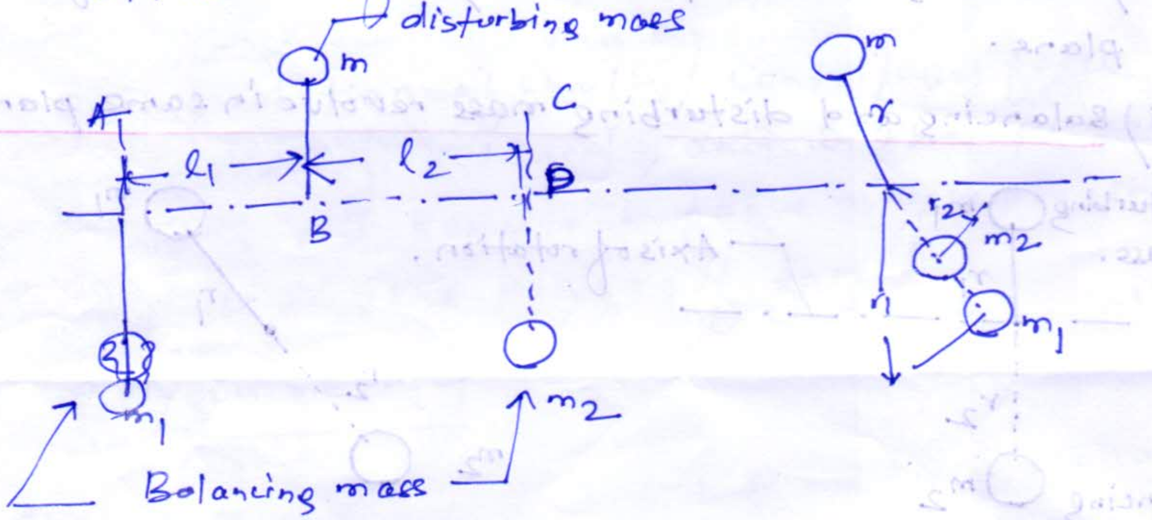
$\Rightarrow m_1 \omega^2 r_1 = m_2 \omega^2 r_2$

$\Rightarrow \boxed{m_1 r_1 = m_2 r_2}$

Generally the value of r_2 is kept is larger to reduce the value of balancing mass m_2

(ii) Balancing and Disturbing mass revolve in different Plane! —

In case the balancing and the disturbing mass lie in different planes, the disturbing mass can not be balanced by a single mass as there will be a couple left unbalanced. In such case atleast two balancing masses are required for complete balancing. The three masses are arranged in such a way that the resultant force and couple on the shaft are zero.



Let m = mass of disturbing body ~~acting~~ acting in plane B

m_1 = mass of balancing weight acting in plane A

m_2 = mass of balancing weight acting in plane B

l_1 = distance betn plane A and B

l_2 = distance betn plane B and C

$l = l_1 + l_2$

$r, r_1, r_2 \rightarrow$ distances of CG of m, m_1, m_2 respectively

Now $F_c = m\omega^2 r$

$F_{c_1} = m_1 \omega^2 r_1$

$F_{c_2} = m_2 \omega^2 r_2$.

for balancing the centrifugal force of ~~balance~~ disturbing mass must be equal to the sum of centrifugal force of balancing mass

$F_c = F_{c_1} + F_{c_2}$

or $\quad m\omega^2 r = m_1\omega^2 r_1 + m_2\omega^2 r_2$

$\Rightarrow \boxed{mr = m_1 r_1 + m_2 r_2}$

For complete balance, sum of moments should be zero.

Taking moment about ~~B~~ D

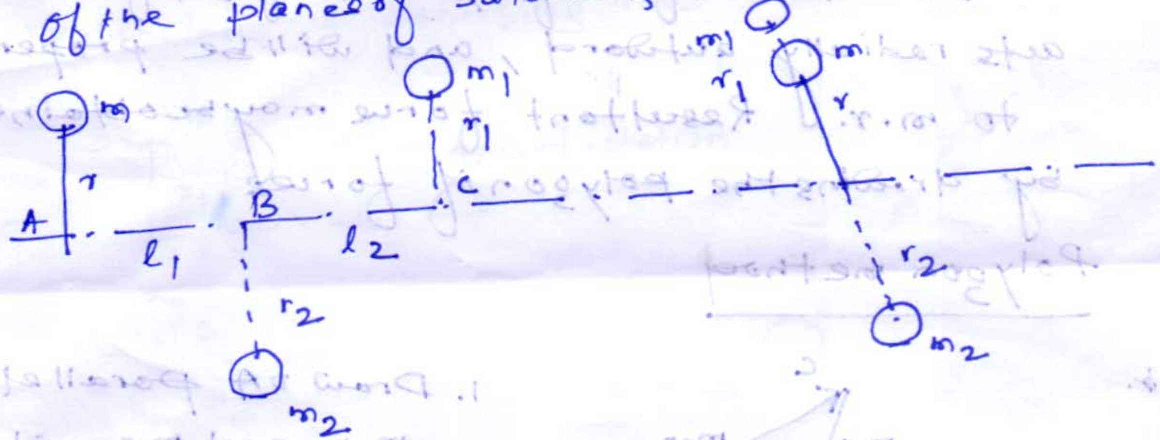
$(l_1 + l_2)\, F_{c_1} = F_c \cdot l_2$

$\Rightarrow l\, m_1 \omega^2 r_1 = m\omega^2 r \cdot l_2$

$\Rightarrow \boxed{m_1 r_1 = \dfrac{m r l_2}{l}}$

where $\boxed{l = l_1 + l_2}$

Case-II $\rightarrow$ Plane of disturbing mass lies on one side of the plane of balancing mass.



We have ~~$F_c = F_{c_1} + F_{c_2}$~~

$F_c + F_{c_1} = F_{c_2}$

or $\quad m\omega^2 r + m_1\omega^2 r_1 = m_2\omega^2 r_2$

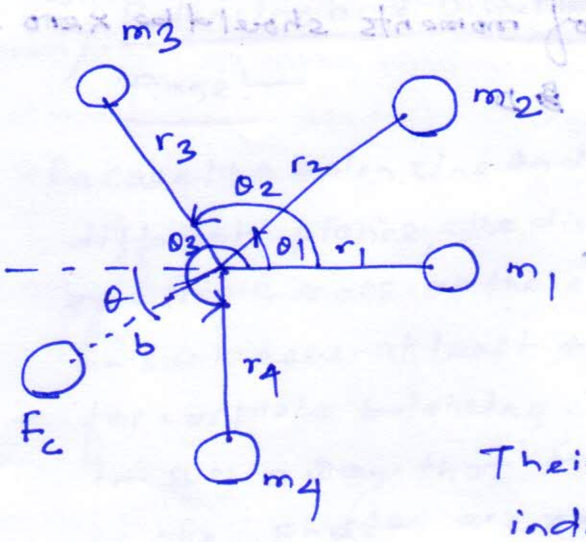
or $\quad mr + m_1 r_1 = m_2 r_2$

couple equation can be written by taking moment at B

$F_c \cdot l_1 = F_{c_1} \cdot l_2$

or $\boxed{m r l_1 = m_1 r_1 l_2}$

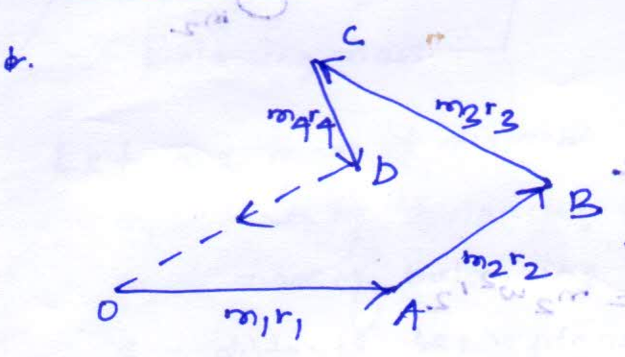
Balancing of several masses revolving in same plane :-



Consider any no of masses say four m_1, m_2, m_3 and m_4 rigidly attached to the shaft and lie in same plane. Let r_1, r_2, r_3 and r_4 be the radii of rotation of masses. Their relative positions are indicated by angles $\theta_1, \theta_2, \theta_3$ and θ_4

During rotation of shaft, centrifugal force acts radially outward, and will be proportional to m.r. Resultant force may be obtained by drawing the polygon of forces

Polygon method



1. Draw OA parallel to m_1r_1 and magnitude $OA = m_1r_1$
2. From A draw AB parallel and equal to m_2r_2.
3. From B draw BC parallel and equal to m_3r_3
4. from C draw CD parallel and equal to m_4r_4
5. Join D with O, DO represents the direction and magnitude of balanced force

Analytical method

Resolving each force horizontally and vertically

Resultant vertical component is

$$F_v = m_1r_1 \sin\theta + m_2r_2 \sin\theta_1 + m_3r_3 \sin\theta_2$$

$F_H = m_1 r_1 \cos 0 + m_2 r_2 \cos\theta_1 + m_3 r_3 \cos\theta_2 + m_4 r_4 \cos\theta_3$ —— (2)

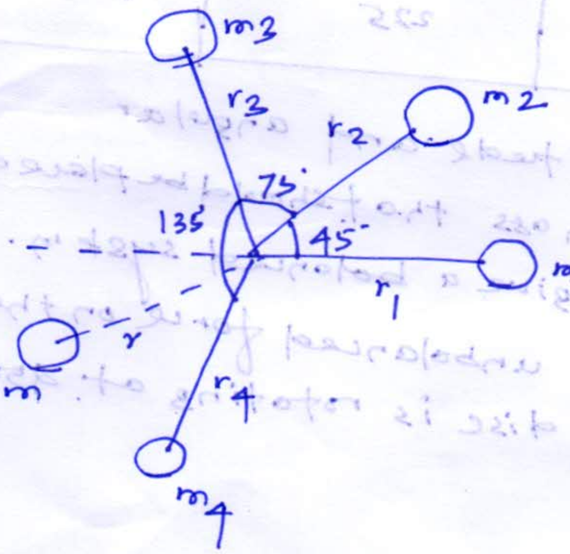
The resultant B.b may be written as

$$\boxed{B.b = \sqrt{(F_V^2 + F_H^2)}}$$

And it's direction $\tan\theta = \dfrac{F_V}{F_H}$

$$\Rightarrow \boxed{\theta = \tan^{-1}\left(\frac{F_V}{F_H}\right)}$$

Q.1 four masses m_1, m_2, m_3 and m_4 having their radii of rotations as 200 mm, 150 mm, 250 mm and 300 mm are 200 kg, 300 kg, 240 kg and 260 kg. The angle betn. the successive masses are 45°, 75° and 135°. find position and magnitude of the balance mass required if it's radius of rotation is 200 mm.



We have

$m_1 = 200\,kg$ $m_2 = 300\,kg$

$m_3 = 240\,kg$ $m_4 = 260\,kg$.

$\theta_1 = 0$ $\theta_2 = 45°$

$\theta_3 = 45 + 75 = 120°$

$\theta_4 = 120° + 135 = 255°$

$r_1 = .2\,m$ $r_2 = 0.15\,m$ $r_3 = .25\,m$

$r_4 = 0.3\,m$.

$r = 0.2\,m$

Analytical method

$\Sigma F_V = m_1 r_1 \sin\theta_1 + m_2 r_2 \sin\theta_2 + m_3 r_3 \sin\theta_3 + m_4 r_4 \sin\theta_4$

$= 40 \sin 0° + 45 \sin 45° + 60 \sin 120° + 78 \sin 255°$

$= 8.439\ kg\text{-}m.$

$\Sigma F_H = m_1 r_1 \cos\theta_1 + m_2 r_2 \cos\theta_2 + m_3 r_3 \cos\theta_3 + m_4 r_2 \cos\theta_4$

$= 40 \cos 0 + 45 \cos 45 + 60 \cos 120 + 78 \cos 255 = 21.63\ kg\,m.$

Resultant force

$$F = \sqrt{F_V^2 + F_H^2} = 23.2 \text{ Kg-m.}$$

Now $m \cdot r = 23.2$ Kg m

$$\Rightarrow m = \frac{23.2}{0.2} = \boxed{116 \text{ Kg.}}$$

Direction

$$\tan\theta = \frac{\Sigma F_V}{\Sigma F_H}$$

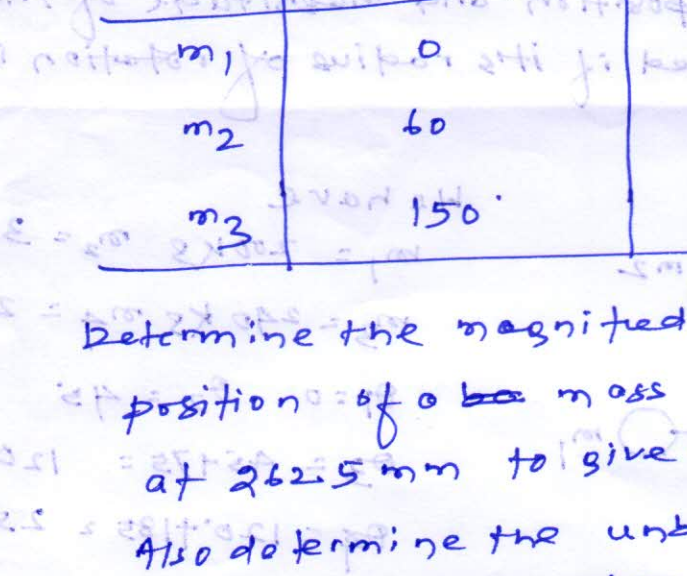
$$\Rightarrow \theta = \tan^{-1}\left(\frac{8.43}{21.63}\right) = \boxed{\cancel{201.5^\circ}} \quad 21.29^\circ$$

Direction from m_1 = $180 + 21.29 = 201.5^\circ$

Q.2 A circular disc rotating arround a vertical spindle, has the following masses placed on it.

mass	θ, wrt X-X	Distance from centre (mm)	Magnitude
m_1	0	260	2.5
m_2	60	300	3.5
m_3	150°	225	5.0

Determine the magnitude and angular position of a ~~ba~~ mass that should be placed at 262.5 mm to give a balanced system. Also determine the unbalanced force on the spindle when the disc is rotating at 250 rpm.



$$\Sigma F_V = m_1 r_1 \sin 0 + m_2 r_2 \sin \theta_2 + m_3 r_3 \sin \theta_3$$

$$= 0.65 \sin 0 + 1.05 \sin 60^\circ + 1.125 \sin 150^\circ$$

$$= 1.471 \text{ Kg m.}$$

Resultant force $F = 1.484$ kgm.

So $m \cdot r = 1.484$ kgm

$\Rightarrow \boxed{m = 5.653 \text{ kg.}}$

$\theta = \tan^{-1}\left(\dfrac{1.471}{0.2007}\right) = \boxed{82.23^\circ}$

Direction from $m_1 = \boxed{262.23^\circ}$

Magnitude of Resultant force.

$m\omega^2 r = 5.653 \times \left(\dfrac{2\pi \times 250}{60}\right)^2 \times .2625$

$= \boxed{1917 \text{ N.}}$

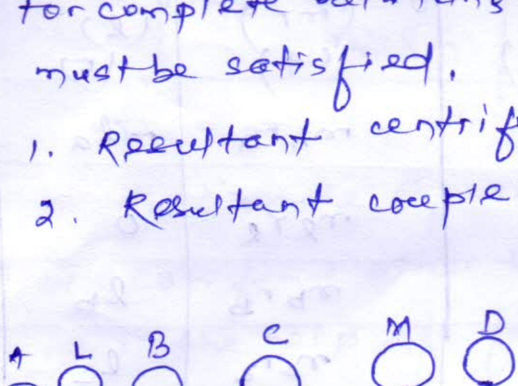
Balancing of Several masses Revolving in Different Planes :-

- Balancing of several masses revolving in different planes is done by transfer of the centrifugal force acting in different planes to a single plane, Known as reference plane, thereby masses rotating in different planes are transferred to reference plane.

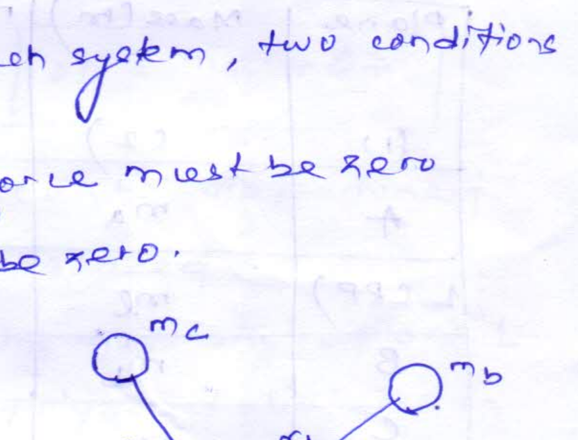
The effect of transferring the rotating mass m in the reference plane is to generate a centrifugal force $F_c = m\omega^2 r$ and a couple $C = F_c \cdot l$ in the reference plane where l = distance betn the reference plane and rotating.

For complete balancing of such system, two conditions must be satisfied.

1. Resultant centrifugal force must be zero.
2. Resultant couple must be zero.



(position of planes)



(Angular position of masses)

Let's consider several masses m_a, m_b, m_c and m_d revolving in planes A, B, C and D respectively.

Two masses for balancing are used because of the following use:

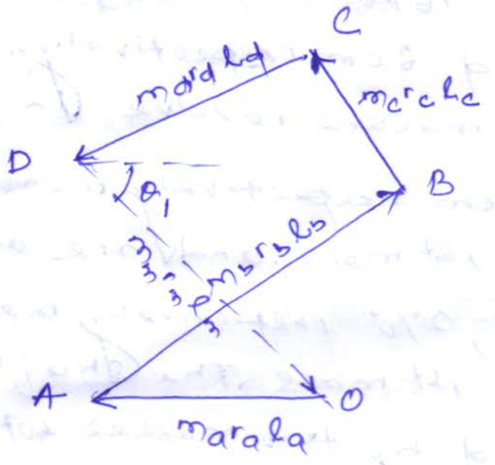
1. If a single mass is used the system will be difficult to handle.
2. If more than two masses are used, no of unknown parameters will be more than no of equations.

Procedure:-

1. Take one plane L as the reference plane. Distance to the left of this plane are taken with minus sign and those to right with +ve sign.
2. Tabulate the forces and couples as shown in the table.

Plane (1)	Mass (m) (2)	radius (r) (3)	centrifugal force $\div \omega^2$ (mr) (4)	Distance from R.P. (l) (5)	couple $\div \omega^2$ (mrl) (6)
A	m_a	r_a	$m_a r_a$	$-l_a$	$-m_a r_a l_a$
L (RP)	m_l	r_l	$m_l r_l$	0	0
B	m_b	r_b	$m_b r_b$	l_b	$m_b r_b l_b$
C	m_c	r_c	$m_c r_c$	l_c	$m_c r_c l_c$
M	m_m	r_m	$m_m r_m$	l_m	$m_m r_m l_m$
D	m_d	r_d	$m_d r_d$	l_d	$m_d r_d l_d$

3. Draw the couple polygon. Couple $m_a r_a l_a$ is -ve wrt RP. so the couple $(-m_a r_a l_a)$ is drawn radially inwards as it's in reverse direction of Om_a. couple $m_b r_b l_b$ is +ve wrt RP so it is drawn in the direction of Om_b. Similarly couples $m_c r_c l_c$ and $m_d r_d l_d$ are drawn in the directions of Om_c and Om_d respectively.



(couple polygon)

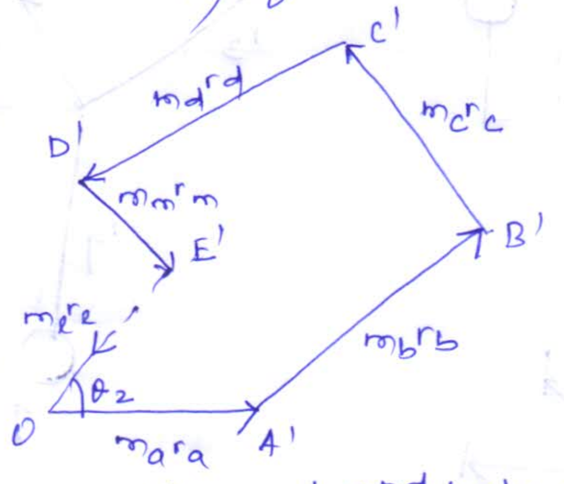
couple $m_m r_m l_m$ is the closing side. The balancing couple OD is proportional to $m_m r_m l_m$.

If the balancing radius r_m is known, balancing mass m_m can be obtained in magnitude and direction.

$$\boxed{m_m = \frac{OD}{r_m \cdot l_m}} \qquad \boxed{OD = m_m r_m l_m}$$

Thus m_m in plane M can be determined and angle θ_1 can be measured.

4. We can find other balancing mass m_l in plane L with the help of force polygon tabulated in column (4) of the table.



If the radius of 2nd balancing mass m_l is known m_l can be found in plane L and its angle of inclination θ_2 with horizontal may be measured.

Q1 A rotating shaft carries four unbalanced masses 18 kg, 14 kg, 16 kg and 12 kg at radii 5 cm, 6 cm, 7 cm and 6 cm respectively. The 2nd, 3rd and 4th masses revolve in planes 8 cm, 16 cm and 28 cm respectively measured from the plane of 1st mass and are angularly located at 60°, 135°, 270° respectively measured anticlockwise from 1st mass. The shaft is dynamically balanced by two masses both located at 5 cm radii and revolving in planes mid way betn those of 1st and 2nd masses and 3rd and 4th masses. Determine graphically the magnitude of the masses and their respective angular positions.

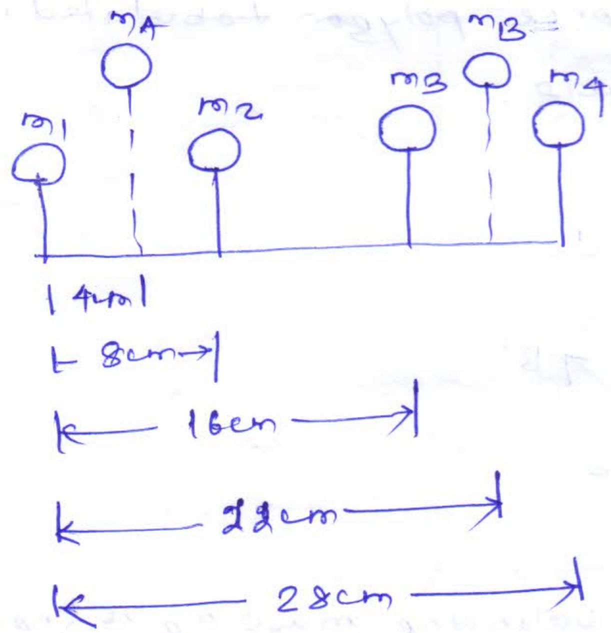
Given data :- $m_1 = 18$ kg $m_2 = 14$ kg $m_3 = 16$ kg

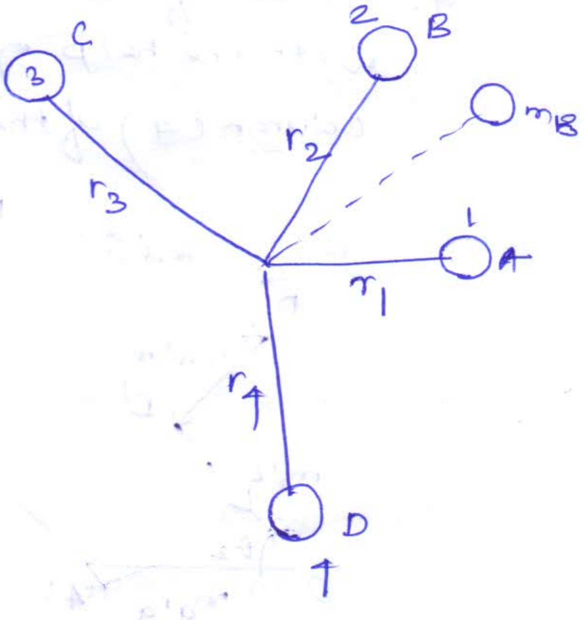
$m_4 = 12$ kg.

$r_1 = 5$ cm $r_2 = 6$ cm $r_3 = 7$ cm $r_4 = 6$ cm.

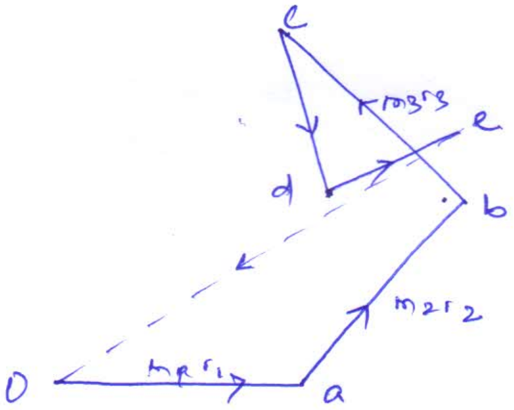
$\theta_1 = 0$ $\theta_2 = 60°$ $\theta_3 = 135°$ $\theta_4 = 270°$

Let the two balancing masses are m_A and m_B





plane	mass (m) Kg	Radius (r) m	centrifugal force ÷ ω^2 (mr)	Distance from RP (l)	couple ÷ ω^2 (mrl)
1	18	0.05	0.9	−.04	−0.036
A	m_A	0.05	$0.05m_A$	0	0
2	14	.06	0.84	.04	0.0336
3	16	.07	1.12	.12	0.1344
B	m_B	.05	$0.05m_B$	.18	$.009m_B$
4	12	.06	0.72	.24	0.1728

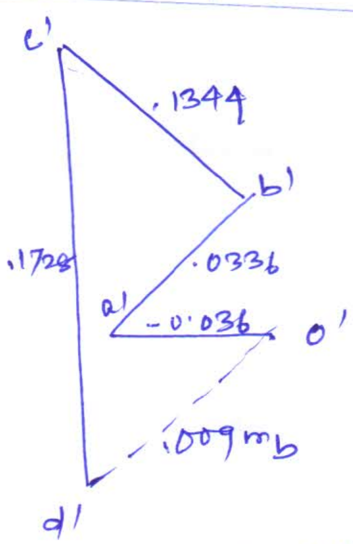


(force polygon)

$m_A = \frac{1.575}{0.05} = 31.5$ kg.

$\theta_A = 220°$ (from plane 1)

(2nd)



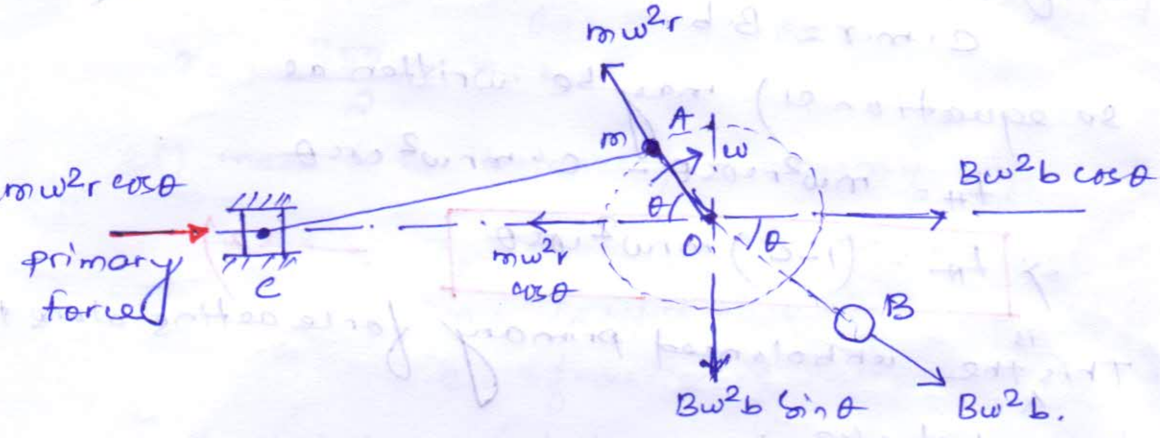
$m_B = \frac{o'd'}{.009}$

$= \frac{.12}{.009} = 13.33$ kg.

$\theta_B = 25°$

(1st)

Partial Primary Balancing:-



Consider a slider crank mechanism OAC. A primary unbalanced force $m\omega^2 r\cos\theta$ is required to accelerate the reciprocating mass, which acts along the direction from O to C. So balancing of primary force is considered equivalent to the component and parallel to the line of stroke, of the centrifugal force produced by an equal mass 'm' attached to the crank and and rotating at 'r' radius. To balance this force a rotating counter mass B is placed at a radius b, directly opposite to crank.

For complete balancing

$$B\omega^2 b\cos\theta = m\omega^2 r\cos\theta$$

$$\Rightarrow \boxed{B\cdot b = m\cdot r}$$

- However the vertical component of rotating mass B, of magnitude $B\omega^2 b\sin\theta$ remains unbalanced.

Now the resultant disturbing force parallel to the line of stroke is

$$F_H = m\omega^2 r\cos\theta - B\omega^2 b\cos\theta$$

$$\Rightarrow \boxed{F_H = (m r - B\cdot b)\,\omega^2\cos\theta} \quad \text{---- (1)}$$

If $m\cdot r = B\cdot b$, the primary disturbing force is zero and the system will be unbalanced because of the vertical component of force.

Practically, a compromise is made and only a fraction c of reciprocating mass is balanced i.e,

$c \cdot m r = B \cdot b$.

so equation (1) may be written as

$f_H = m\omega^2 r\cos\theta - c\cdot m r\omega^2\cos\theta$

$\Rightarrow \boxed{f_H = (1-c)\, m r\omega^2\cos\theta}$ — (2)

This is the unbalanced primary force acting along the line of stroke.

The unbalanced force ⊥r to the line of stroke is

$f_V = B\omega^2 b\sin\theta = c m\omega^2 r\sin\theta$ — (3)

so the resultant unbalanced force

$f = \sqrt{f_H{}^2 + f_V{}^2}$

$\Rightarrow \boxed{f = m\omega^2 r\sqrt{(1-c)^2\cos^2\theta + c^2\sin^2\theta}}$ — (4)

The value of c is kept betn 1/2 to 3/4.

The value of unbalanced force is minm when

$c = \frac{1}{2}$

$f_{min} = m\omega^2 r\sqrt{\left(\frac{1}{2}\right)^2\cos^2\theta + \left(\frac{1}{2}\right)^2\sin^2\theta}$

$\Rightarrow \boxed{f_{min} = \frac{m\omega^2 r}{2}}$

Q1 The following data relate to a single-cylinder reciprocating engine:

mass of reciprocating parts = 40 kg.

mass of revolving part = 30 kg at 180 mm radius.

speed = 150 rpm

stroke = 350 mm.

If 60% of reciprocating parts and all revolving parts are to be balanced, determine

(i) balance mass required at 320 mm radius.

turned 45° from the TDC.

We have $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 150}{60} = 15.7$ rad/s.

$r = \frac{350}{2} = 175$ mm,

(i) mass to be balanced $= c \cdot m + m_p$

where m_p = mass of crankpin

m = reciprocating mass

c = fraction of reciprocating mass

So total mass to be balanced $= 0.6 \times 40 + 30$
$= 54$ kg,

Now $B \cdot b = m \cdot r$

$B \times 320 = 54 \times 180$

Complete Balancing of Reciprocating parts of an engine:-

For complete balancing of reciprocating parts of an engine, the following conditions must be satisfied:

- Primary force polygon must close
- Primary couple polygon must close
- secondary force polygon must close
- Secondary couple polygon must close,

Partial Balancing of Locomotives:-

Most of the locomotives have two cylinders of same dimension and placed symmetrically, either inside or outside the wheels.

The ratio of length of connecting rod to crank radius ($\frac{l}{r} = n$) is generally large, so the effect of secondary unbalanced force ... $\cos 2\theta$ are neglected

In the partial balancing, two sets of unbalanced force exists

(i) an unbalanced force along the line of stroke

(ii) an unbalanced force $\perp r$ to the line of stroke.

The effect of (i) is to produce variation of tractive force along the line of stroke and unbalanced couple which is known as swaying couple.

- The effect of (ii) is to produce the variation of pressure on the rails which causes hammering action on rails. The max^m magnitude of unbalanced force $\perp r$ to the line of stroke is called Hammer blow.

Variation of Tractive force:-

The resultant unbalanced primary force due to two cylinders along line of stroke is called tractive force.

Let the crank of 1st cylinder be inclined at an angle θ with the line of stroke. crank of 2nd cylinder will be inclined at angle $(90+\theta)$ with the line of stroke.

The unbalanced force along line of stroke for cylinder 1 is

$$F_1 = (1-c)\, m\omega^2 r \cos\theta$$

Unbalanced force along the line of stroke for cylinder 2 is

$$F_2 = (1-c)\, m\omega^2 r \cos(90+\theta)$$

$$= -(1-c)\, m\omega^2 r \sin\theta$$

Tractive force $F_T = F_1 + F_2$

$$= (1-c)\, m\omega^2 r \cos\theta - (1-c)\, m\omega^2 r \sin\theta$$

$$(1-c)\, m\omega^2 r\, (\cos\theta - \sin\theta)$$

for tractive force F_T to be max^m and min^m, depends upon the value of $(\cos\theta - \sin\theta)$

$$\frac{d}{d\theta}(1-c)m\omega^2 r(\cos\theta - \sin\theta) = 0$$

$$-\sin\theta - \cos\theta = 0$$

or $\tan\theta = -1$

or $\theta = 135°$ or $315°$

so the tractive force is max^m or min^m when θ is $135°$ or $315°$.

so $F_{Tmax} = (1-c)m\omega^2 r(\cos 315 - \cancel{\cos}\sin 315)$

$$F_{Tmax} = \sqrt{2}(1-c)m\omega^2 r$$

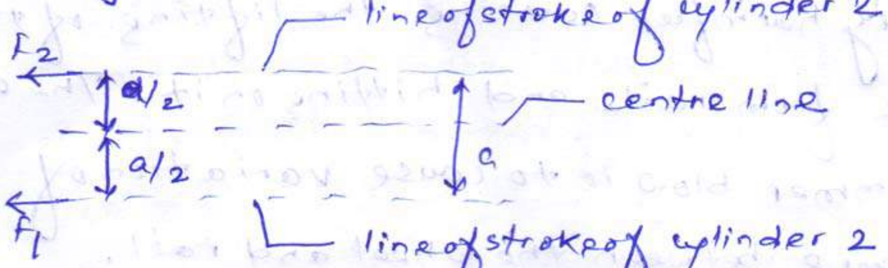
$F_{Tmin} = (1-c)m\omega^2 r(\cos 135 - \sin 135)$

$\Rightarrow F_{Tmin} = -\sqrt{2}(1-c)m\omega^2 r$

Thus $\boxed{F_T = \pm\sqrt{2}(1-c)m\omega^2 r}$

Swaying couple :-

The unbalanced force acting at a distance betn the line of stroke of two cylinders, constitute a couple in the horizontal direction. This couple is called as swaying couple.



Let a = distance betn the centre lines of the two cylinders.

F_1, F_2 = unbalanced forces for cylinder 1 and 2 respectively

$$F_1 = (1-c)\, m\omega^2 r \cos\theta$$

$$F_2 = (1-c)\, m\omega^2 r \cos(90+\theta)$$

The forces differ in phase by 90°.

swaying couple

$$T = F_1 \cdot \frac{a}{2} - F_2 \times \frac{a}{2}$$

$$= (1-c)\, m\omega^2 r \cos\theta \cdot \frac{a}{2} - (1-c)\, m\omega^2 r \cos(90+\theta) \cdot \frac{a}{2}$$

$$= (1-c)\, m\omega^2 r \cos\theta\, \frac{a}{2} + (1-c)\, m\omega^2 r \sin\theta \cdot \frac{a}{2}$$

$$= (1-c)\, m\omega^2 r \,(\cos\theta + \sin\theta) \cdot \frac{a}{2}$$

couple will be maximum or minimum when $(\cos\theta + \sin\theta)$ is maxm or minm.

Thus $\frac{d}{d\theta}(\cos\theta + \sin\theta) = 0$

$$\Rightarrow -\sin\theta + \cos\theta = 0$$

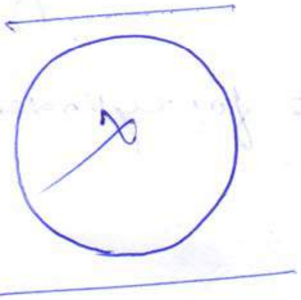
$$\Rightarrow \tan\theta = 1$$

or $\theta = 45^\circ$ or 225°

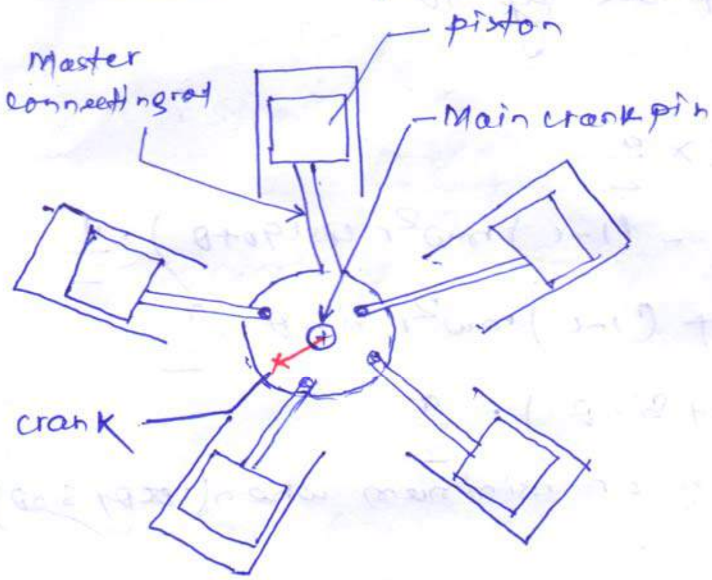
so the maxm and minm values of swaying couple

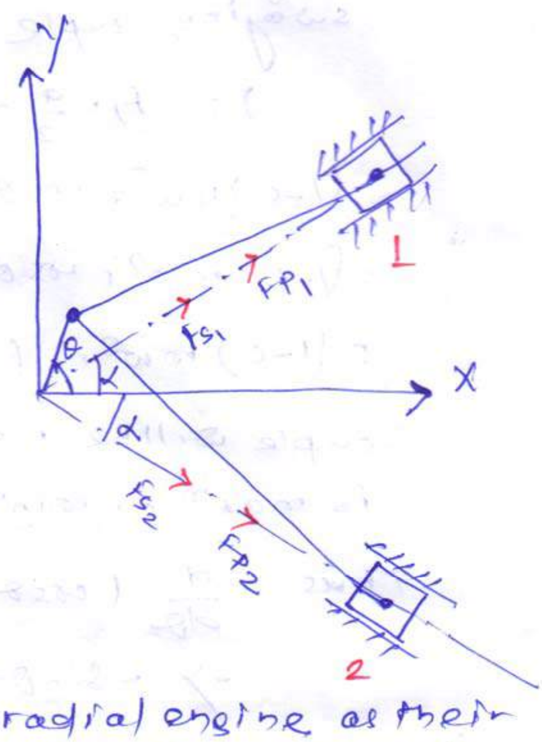
Hammer Blow:-

The max^m value of unbalanced force perpendicular to the line of stroke is called hammer blow. With very high speed this unbalance force may be very harmful causing the lifting of the wheels from rails and hitting on it. The effect of hammer blow is to cause variation of pressure between the wheel and rail.



Balancing of V- Engines:-





V- engines are also known as radial engine as their cylinders are arranged along radial lines. The centre lines of the cylinder form the shape of letter V. Three cylinder have a common crank.

on secondary couple is not required in radial engine.

Consider two cylinders symmetrically arranged as shown in the fig. common crank OA is connected by two connecting rods AB and AC. The lines of stroke OB and OC are inclined to the horizontal line OX at an angle α. The crank is moved by an angle θ with horizontal line OX.

Let m = mass of reciprocating parts per cylinder

l = length of each connecting rod.

r = crank radius.

$n = \frac{l}{r}$

ω = angular velocity of crank.

Primary force of cylinder 1 along line of stroke

$OB = m\omega^2 r \cos(\theta - \alpha)$

Primary force of cylinder 1 along x-axis

$F_{H1} = m\omega^2 r \cos(\theta - \alpha)\cos\alpha$

Primary force of cylinder 2 along line of stroke OC

$= m\omega^2 r \cos(\theta + \alpha)$

Primary force of cylinder 2 along x axis

$F_{H2} = m\omega^2 r \cos(\theta + \alpha)\cos\alpha$

for radial engine containg four or more cylinders the secondary force are in complete balance, as the secondary direct and reverse crank form a balanced system in radial engines.

So total primary force along x-axis

$F_H = F_{H1} + F_{H2}$

$= m\omega^2 r \cos(\theta - \alpha)\cos\alpha + m\omega^2 r \cos(\theta + \alpha)\cos\alpha$

$= m\omega^2 r \cos\alpha \left[\cos(\theta - \alpha) + \cos(\theta + \alpha)\right]$

$= m\omega^2 r \cos\alpha \; 2\cos\theta \cdot \cos\alpha$

As the crank on the same plane, balancing of primary on secondary couple is not required in radial engine.

Consider two cylinders symmetrically arranged as shown in the fig. common crank OA is connected by two connecting rods AB and AC. The lines of stroke OB and OC are inclined to the horizontal line OX at an angle α. The crank is moved by an angle θ with horizontal line OX.

Let m = mass of reciprocating parts per cylinder

l = length of each connecting rod.

r = crank radius.

$n = \frac{l}{r}$

ω = angular velocity of crank.

Primary force of cylinder 1 along line of stroke

$OB = m\omega^2 r\cos(\theta-\alpha)$

Primary force of cylinder 1 along x-axis

$$F_{H_1} = m\omega^2 r\cos(\theta-\alpha)\cos\alpha$$

Primary force of cylinder 2 along line of stroke OC

$$= m\omega^2 r\cos(\theta+\alpha)$$

Primary force of cylinder 2 along X axis

$$F_{H_2} = m\omega^2 r\cos(\theta+\alpha)\cos\alpha$$

For radial engine containing four or more cylinders the secondary force are in complete balance, as the secondary direct and reverse crank form a balanced system in radial engines.

So total primary force along x-axis

$$F_H = F_{H_1} + F_{H_2}$$

$$= m\omega^2 r\cos(\theta-\alpha)\cos\alpha + m\omega^2 r\cos(\theta+\alpha)\cos\alpha$$

$$= m\omega^2 r\cos\alpha\,[\cos(\theta-\alpha)+\cos(\theta+\alpha)]$$

$$\boxed{F_H = 2m\omega^2 r \cos\theta \cdot \cos^2\alpha}$$

Similarly total primary force along y axis

$$= m\omega^2 r \cos(\theta-\alpha)\sin\alpha - m\omega^2 r \cos(\theta+\alpha)\sin\alpha$$

$$= m\omega^2 r \sin\alpha\left[\cos(\theta-\alpha) - \cos(\theta+\alpha)\right]$$

$$\boxed{F_V = 2m\omega^2 r \sin^2\alpha \sin\theta}$$

Resultant primary force $F_P = \sqrt{F_H^2 + F_V^2}$

$$\boxed{= 2m\omega^2 r\sqrt{(\cos\theta\cdot\cos^2\alpha)^2 + (\sin\theta\cdot\sin^2\alpha)^2}}$$

Now deriving the expression for secondary force:

secondary force for cyl 1 along line of stroke OB

$$= \frac{m\omega^2 r}{n}\cos 2(\theta-\alpha)$$

secondary force for cylinder 1 along x-axis

$$F_{H_3} = \frac{m\omega^2 r}{n}\cos 2(\theta-\alpha)\cos\alpha$$

secondary force of cylinder 2 along the line of stroke OC

$$= \frac{m\omega^2 r}{n}\cos 2(\theta+\alpha)$$

secondary force of cylinder 2 along x-axis

$$F_{H_4} = \frac{m\omega^2 r}{n}\cos 2(\theta+\alpha)\cos\alpha$$

Thus total secondary force along x-axis

$$F_{H_S} = F_{H_3} + F_{H_4}$$

$$= \frac{m\omega^2 r}{n}\cos 2(\theta-\alpha)\cos\alpha + \frac{m\omega^2 r}{n}\cos 2(\theta+\alpha)\cos\alpha$$

$$= \frac{m\omega^2 r}{n}\cos\alpha\left[\cos 2(\theta-\alpha) + \cos 2(\theta+\alpha)\right]$$

$$= \frac{m\omega^2 r}{n}\cos\alpha \cdot 2\cos 2\alpha \cdot \cos 2\theta$$

$$\boxed{2m\omega^2 r \ \cos 2\alpha \cdot \cos 2\theta \cdot \cos\alpha}$$

$$F_H = 2mw^2r\cos\theta\cos^2\alpha$$

Similarly total primary force along y axis

$= mw^2r\cos(\theta-\alpha)\sin\alpha - mw^2r\cos(\theta+\alpha)\sin\alpha$

$= mw^2r\sin\alpha\left[\cos(\theta-\alpha)-\cos(\theta+\alpha)\right]$

$$\boxed{F_V = 2mw^2r\sin^2\alpha\sin\theta}$$

Resultant primary force $F_P = \sqrt{F_H^2 + F_V^2}$

$$\boxed{= 2mw^2r\sqrt{(\cos\theta.\cos^2\alpha)^2 + (\sin\theta.\sin^2\alpha)^2}}$$

Now deriving the expression for secondary force:

secondary force for cyl 1 along line of stroke OB

$$= \frac{mw^2r}{n}\cos 2(\theta-\alpha)$$

secondary force for cylinder 1 along x-axis

$$F_{H_3} = \frac{mw^2r}{n}\cos 2(\theta-\alpha)\cos\alpha$$

secondary force of cylinder 2 along the line of stroke OC

$$= \frac{mw^2r}{n}\cos 2(\theta+\alpha)$$

secondary force of cylinder 2 along x-axis

$$F_{H_4} = \frac{mw^2r}{n}\cos 2(\theta+\alpha)\cos\alpha$$

Thus total secondary force along x-axis

$F_{H_S} = F_{H_3} + F_{H_4}$

$= \frac{mw^2r}{n}\cos 2(\theta-\alpha)\cos\alpha + \frac{mw^2r}{n}\cos 2(\theta+\alpha)\cos\alpha$

$= \frac{mw^2r}{n}\cos\alpha\left[\cos 2(\theta-\alpha) + \cos 2(\theta+\alpha)\right]$

$= \frac{mw^2r}{n}\cos\alpha . 2\cos 2\alpha . \cos 2\theta$

$$= \boxed{\frac{2mw^2r}{n}\cos 2\alpha . \cos 2\theta . \cos\alpha}$$

Similarly, total secondary force along y-axis

$$F_{VS} = \frac{2mw^2 r}{n} \sin 2\alpha \cdot \sin 2\theta \cdot \sin \alpha$$

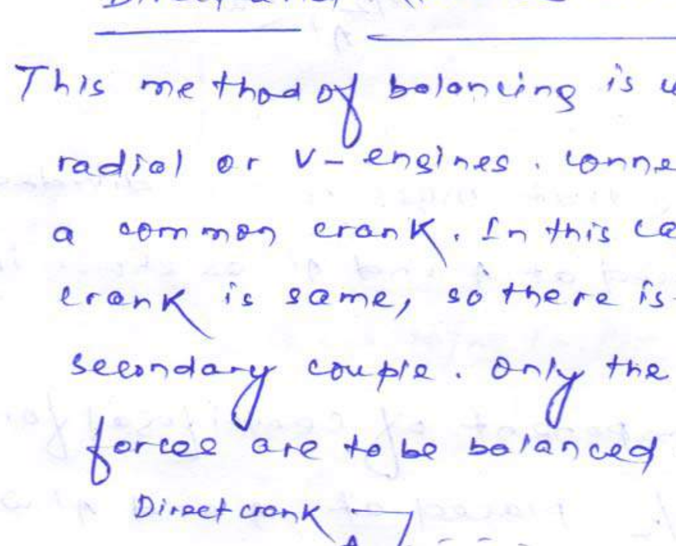
Resultant force F_S

$$F_S = \sqrt{F_{HS}^2 + F_{VS}^2}$$

$$= \boxed{\frac{2mw^2 r}{n} \sqrt{(\cos 2\alpha \cdot \cos 2\theta \cdot \cos \alpha)^2 + (\sin 2\alpha \cdot \sin 2\theta \cdot \sin \alpha)^2}}$$

Direct and Reverse crank Method of Balancing

This method of balancing is useful for balancing of radial or V-engines. connecting rods are attached to a common crank. In this case plane of rotation of crank is same, so there is no unbalanced primary or secondary couple. Only the primary and secondary forces are to be balanced.



As shown in the figure in the reciprocating engine the crank OA rotates uniformly at w rad/s in clockwise direction.

Let OA make an angle θ with OB at any instant. The reverse crank OA' is the image of direct crank OA, and it rotates in anti clockwise direction by the gear mechanism. OA and OA' are called as direct and reverse cranks respectively. The direct and reverse cranks coincide at I.D.C (inner dead centre). Let's say mass of reciprocating parts at B is 'm'.

The primary force can be obtained which is equal to $mw^2 r \cos\theta$. If a mass m is placed

Similarly total secondary force along y-axis

$$F_{VS} = \frac{2mw^2 r}{n} \sin 2\alpha \cdot \sin 2\theta \cdot \sin\alpha$$

Resultant force F_S

$$F_S = \sqrt{F_{HS}^2 + F_{VS}^2}$$

$$= \boxed{\frac{2mw^2 r}{n} \sqrt{(\cos 2\alpha \cdot \cos 2\theta \cdot \cos\alpha)^2 + (\sin 2\alpha \cdot \sin 2\theta \cdot \sin\alpha)^2}}$$

Direct and Reverse crank Method of Balancing

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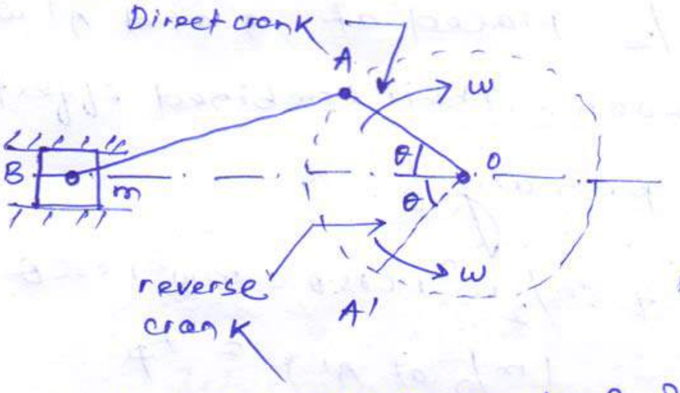

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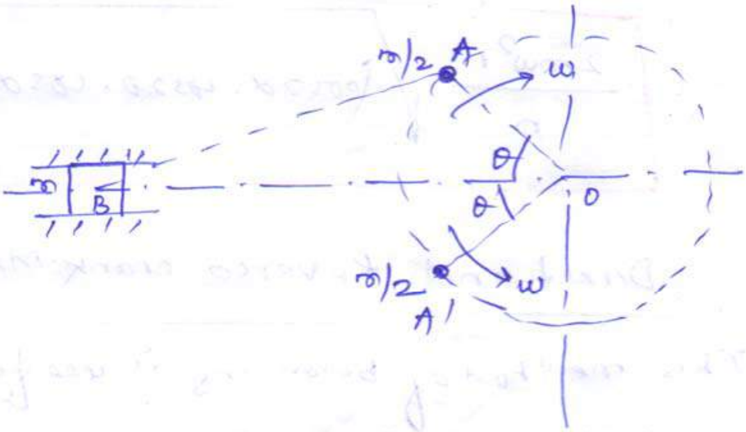
The primary force can be obtained which is equal to $mw^2 r\cos\theta$. If a mass m is placed at crank pin, it produces a centrifugal force of magnitude $m\omega^2 r$. The horizontal component of this force is $m\omega^2 r \cos\theta$ which is equal to the primary force. Thus both the forces are equal and balanced horizontally only.



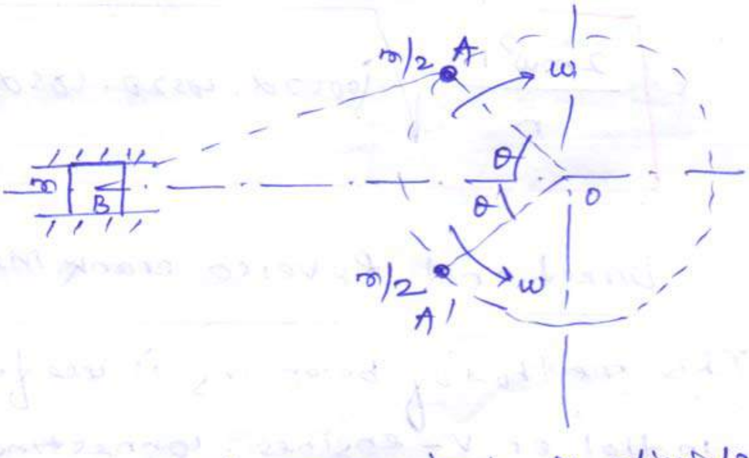
Now let's assume that mass 'm' is divided equally and placed at A and A' as shown in the figure.

The horizontal component of centrifugal forces due to masses m/2 placed at A and A' will be $\frac{m}{2}\omega^2 r\cos\theta$ each. Their combined effect will be equal to primary force

$$\frac{m}{2}\omega^2 r\cos\theta + \frac{m}{2}\omega^2 r\cos\theta = m\omega^2 r\cos\theta$$

$(m/2 \text{ at } A)$ $\quad (m/2 \text{ at } A')$ $= F_p$

In this case, we have put each revolving mass equal to one half of the reciprocating mass to determine the primary force. The components of centrifugal forces of two masses normal to the line of stroke will be equal and opposite i.e one upwards and other downwards. So vertical component magnitude $m\omega^2 r$. The horizontal component of this force is $m\omega^2 r \cos\theta$ which is equal to the primary force. Thus both the forces are equal and balanced horizontally only.



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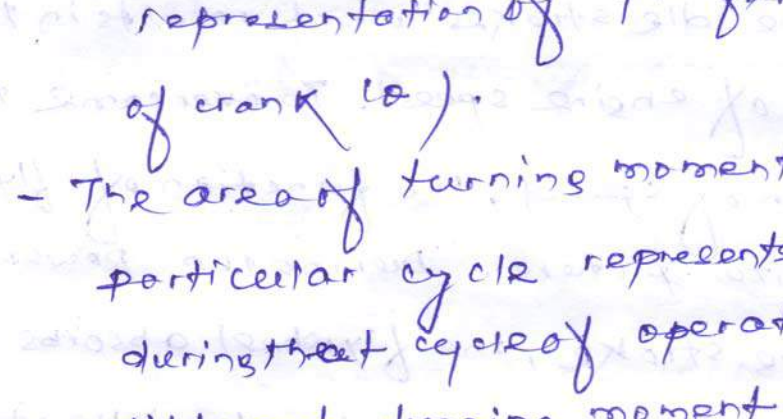
$$\underset{(m/2 \text{ at } A)}{\frac{m}{2}\omega^2 r\cos\theta} + \underset{(m/2 \text{ at } A')}{\frac{m}{2}\omega^2 r\cos\theta} = m\omega^2 r\cos\theta = F_p$$

In this case, we have put each revolving mass equal to one half of the reciprocating mass to determine the primary force. The components of centrifugal forces of two masses normal to the line of stroke will be equal and opposite i.e. one upwards and other downwards. so vertical component also balance each other.

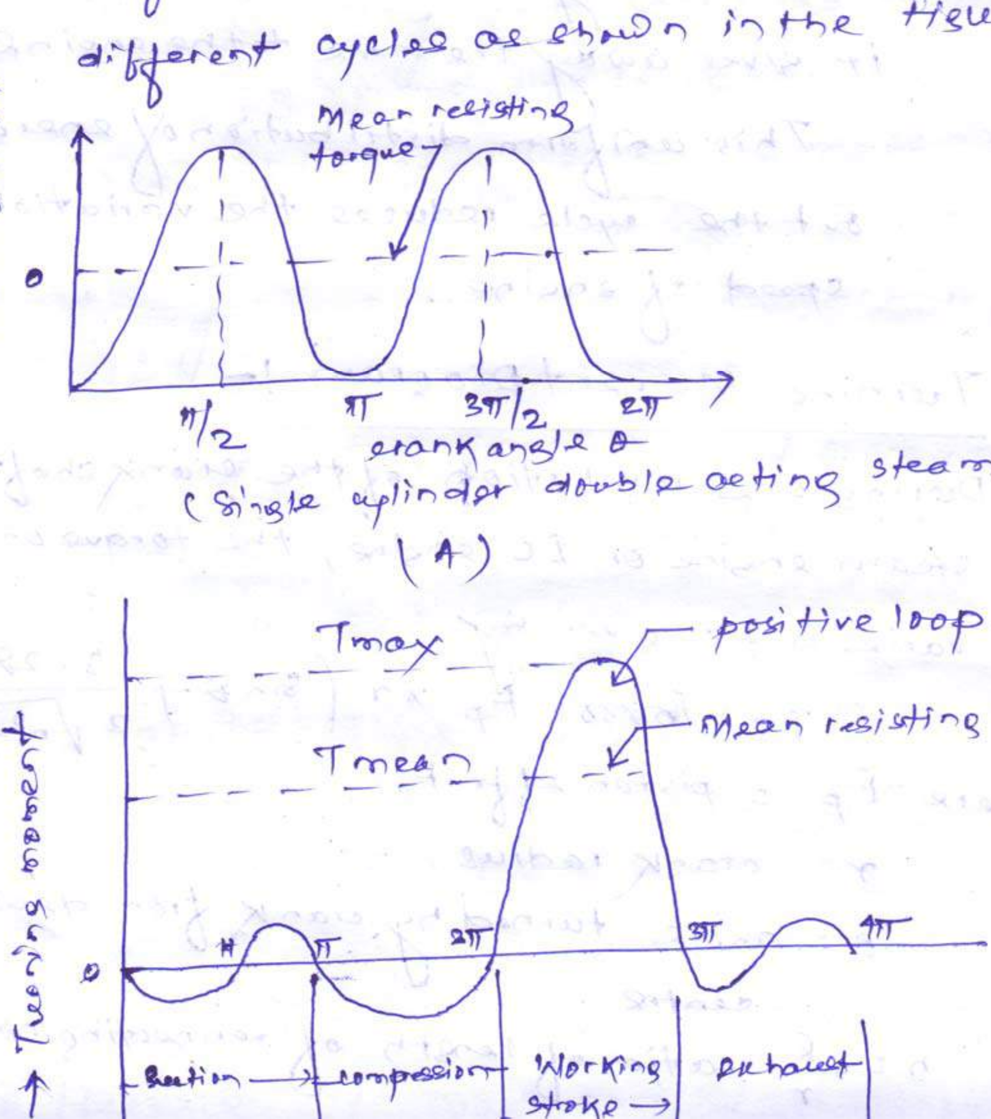
From the above expression it is clear that the turning moment T varies with the variation of crank rotation angle θ. If the value of T is plotted against crank angle θ in radians, the diagram obtained is called turning moment diagram. The turning moment diagram (or crank effort diagram) is the graphical representation of 'T' for various positions of crank (θ).

- The area of turning moment diagram, for a particular cycle represents the work done during that cycle of operation. There are different turning moment diagram for different cycles as shown in the figure.



(Single cylinder double acting steam engine)

(A)



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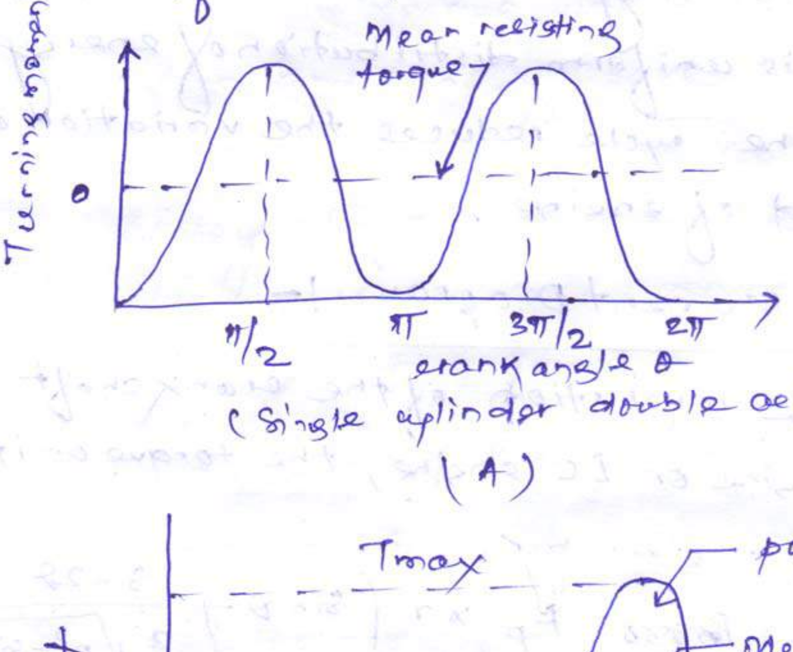
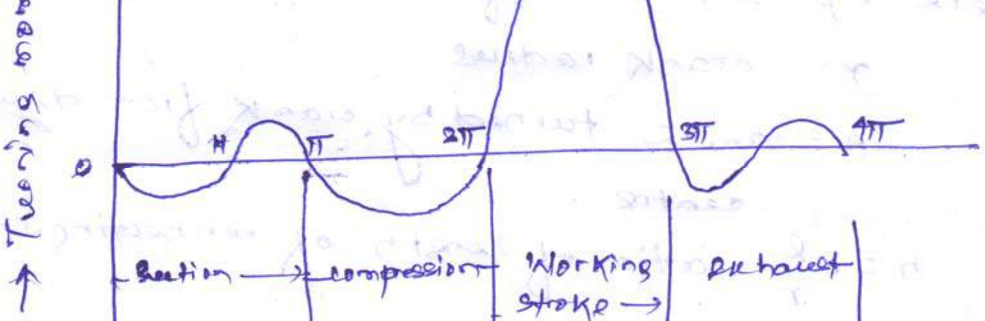
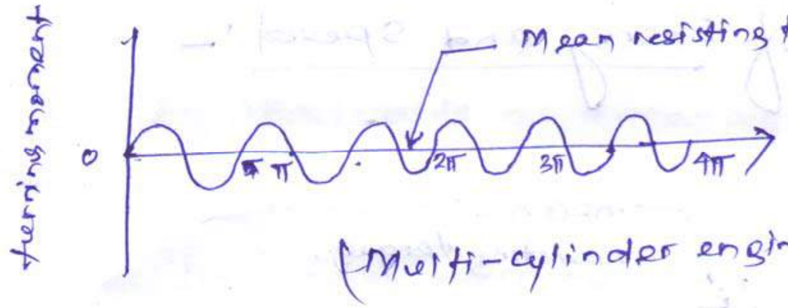


(Single cylinder double acting steam engine)

(A)

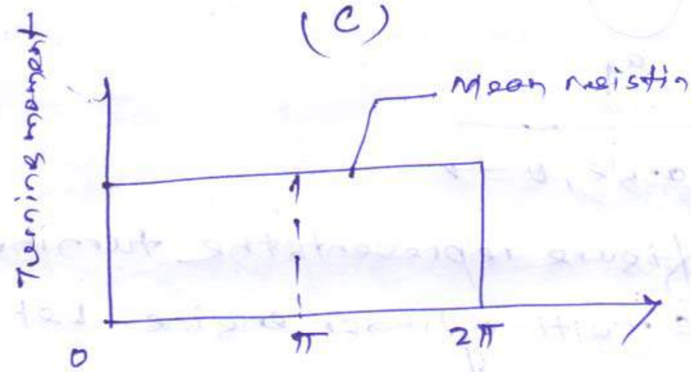


(B) (4-S Single cylinder Petrol engine)



(Multi-cylinder engine)

(C)

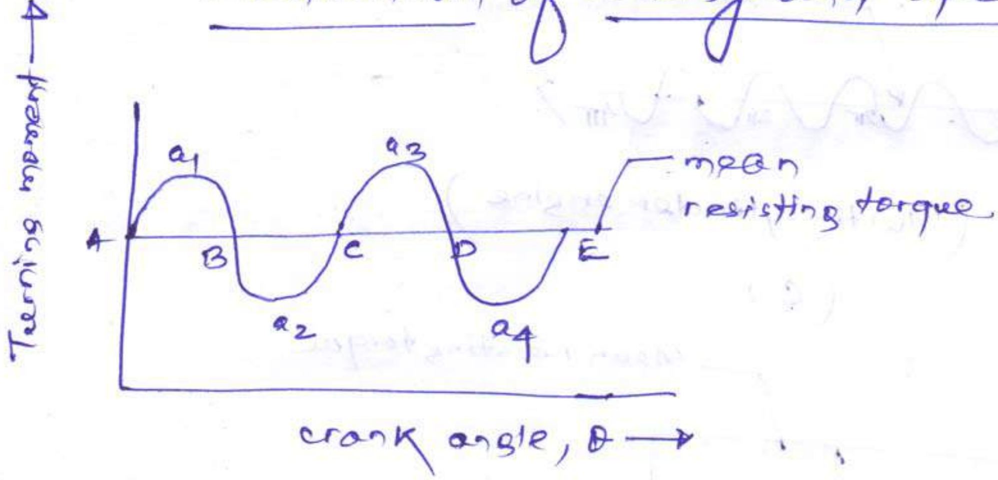


~~crank~~ (D)

for each cycle of operation the work done by the engine must be at least equal to the work required to overcome the load which is called mean resisting torque. If the work done by the engine is more than the mean resisting torque, the engine will accelerate. On the other hand the engine will retard or stop if the workdone is less than the resisting torque.

- As shown in fig (A) during suction stroke, pressure inside the cylinder is less than atmospheric pressure so in majority a negative loop is formed. During compression stroke, the work is done on the gas, so a higher negative loop is formed. In the working stroke work is done by gas, so a large possitive energy loop is obtained. During exhaust stroke the loop is negative as the work is done on the gas.

Fluctuation of Energy and Speed :-



The above figure represents the turning moment diagram of a multi-cylinder engine. Let AE is the mean resisting torque line. Let the area of energy loop between crank rotation from A to B, B to C, C to D and D to E, are a_1, a_2, a_3 and a_4 respectively.

Area of energy loop below the mean resisting torque line is taken with -ve sign and that of above the mean resisting torque line with +ve sign. The variation of energy above and below the mean resisting torque line is called ~~mean~~ fluctuation of energy. This fluctuation of energy may be obtained by turning moment diagram for one complete cycle of operation.

Let the energy at point A = U

at point B = $U + a_1$

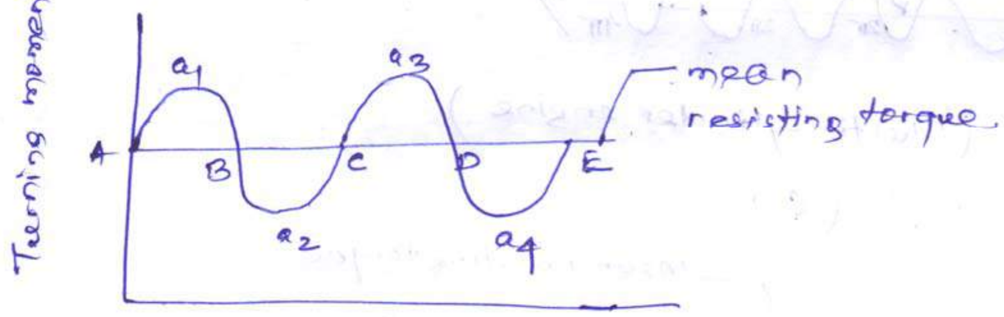
at point C = $U + a_1 - a_2$

at point D = $U + a_1 - a_2 + a_3$

at point E = $U + a_1 - a_2 + a_3 - a_4$

Difference betn max^m and min^m energy is called

Fluctuation of Energy and Speed :-



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at point E = $U + a_1 - a_2 + a_3 - a_4$

Difference betn maxm and minm energy is called maxm fluctuation of energy (E_f).

So $\boxed{E_f = \text{max}^m \text{ energy} - \text{min}^m \text{ energy}}$

Coefficient of fluctuation of energy :-

Coefficient of fluctuation of energy

$$\boxed{C_E = \frac{\text{Max}^m \text{ fluctuation of energy}}{\text{Workdone / cycle}}}$$

Coefficient of fluctuation of speed :-

Coefficient of fluctuation of speed is indicated by C_f

$$C_f = \frac{N_1 - N_2}{N} = \frac{N_1 - N_2}{\left(\frac{N_1 + N_2}{2}\right)} = \frac{2\pi(\omega_1 - \omega_2) \times 2}{2\pi(\omega_1 + \omega_2)}$$

$$\Rightarrow \boxed{C_f = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2}}$$

The reciprocal of coefficient of fluctuation of speed is known as coefficient of steadiness and is denoted by m

$$\boxed{m = \frac{1}{C_f}}$$

$N_1 \rightarrow$ max^m speed of flywheel

$N_2 \rightarrow$ min^m speed of flywheel.

Energy stored in a Flywheel :-

Let m = mass of flywheel in kg.

K = radius of gyration, m.

moment of inertia of flywheel $I = mK^2$

N_1 and N_2 = max and min speeds of flywheel

fluctuation of speed = $N_1 - N_2$

mean speed $N = \frac{N_1 + N_2}{2}$

coefficient of fluctuation speed

$$C_f = \frac{N_1 - N_2}{N}$$

Mean energy stored in the flywheel

$$E = \frac{1}{2} I \omega^2$$

Max energy stored by flywheel

$$E_1 = \frac{1}{2} I \omega_1^2 \quad \left(\text{where } \omega_1 = \frac{2\pi N_1}{60} \right)$$

Min energy stored by flywheel

$$E_2 = \frac{1}{2} I \omega_2^2 \quad \left(\text{where } \omega_2 = \frac{2\pi N_2}{60} \right)$$

Max fluctuation of energy, E_f

$$E_f = E_1 - E_2 = \frac{1}{2} I (\omega_1^2 - \omega_2^2)$$

$$= \frac{1}{2} I (\omega_1 + \omega_2)(\omega_1 - \omega_2)$$

$$= \frac{1}{2} I (\omega_1 + \omega_2) \cdot \frac{(\omega_1 - \omega_2)}{\left(\frac{\omega_1 + \omega_2}{2}\right)} \cdot \left(\frac{\omega_1 + \omega_2}{2}\right)$$

$$= \frac{1}{2} I \left(\frac{\omega_1 + \omega_2}{2}\right) \cdot C_f \cdot \left(\frac{\omega_1 + \omega_2}{2}\right)$$

$$= I \cdot C_f \left(\frac{\omega_1 + \omega_2}{2}\right)^2$$

$$= I \cdot C_f \cdot \omega^2$$

$$= \frac{1}{2} I \omega^2 \cdot (C_f \cdot 2)$$

$$= \left(\frac{1}{2} I \omega^2\right) 2 C_f$$

[illegible]

moment of inertia of flywheel

N_1 and N_2 = max^m and min^m speeds of flywheel

fluctuation of speed = $N_1 - N_2$

mean speed $N = \frac{N_1 + N_2}{2}$

coefficient of fluctuation speed

$$C_f = \frac{N_1 - N_2}{N}$$

Mean energy stored in the flywheel

$$E = \frac{1}{2} I \omega^2$$

Max^m energy stored by flywheel

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Min^m energy stored by flywheel

$$E_2 = \frac{1}{2} I \omega_2^2 \quad \left(\text{where } \omega_2 = \frac{2\pi N_2}{60} \right)$$

Max^m fluctuation of energy, E_f

$$E_f = E_1 - E_2 = \frac{1}{2} I (\omega_1^2 - \omega_2^2)$$

$$= \frac{1}{2} I (\omega_1 + \omega_2)(\omega_1 - \omega_2)$$

$$= \frac{1}{2} I (\omega_1 + \omega_2) \cdot \frac{(\omega_1 - \omega_2)}{\left(\frac{\omega_1 + \omega_2}{2}\right)} \cdot \left(\frac{\omega_1 + \omega_2}{2}\right)$$

$$= \frac{1}{2} I \left(\frac{\omega_1 + \omega_2}{2}\right) \cdot C_f \cdot \left(\frac{\omega_1 + \omega_2}{2}\right)$$

$$= I \cdot C_f \left(\frac{\omega_1 + \omega_2}{2}\right)^2$$

$$= I \cdot C_f \cdot \omega^2$$

$$= \frac{1}{2} I \omega^2 \cdot (C_f \cdot 2)$$

$$= \left(\frac{1}{2} I \omega^2\right) 2 C_f$$

$$\Rightarrow \boxed{E_f = 2 E C_f}$$

Now $\boxed{\text{workdone / cycle} = \dfrac{P \times 60}{n}}$

where P = Power transmitted in watt.

n = no of working strokes / minute.

= $N/2$ in case of 4-S IC engine

= N in case of steam engine and 2-S IC engines.

moment of inertia of solid flywheel disc

$$\boxed{I = m \cdot \frac{D^2}{8}}$$

mass of flywheel rim

$$\boxed{m = A \cdot \pi D \cdot \rho}$$

and $\sigma = \rho \cdot v^2$

$$\Rightarrow \boxed{\sigma = \rho \cdot \left(\frac{\pi D N}{60}\right)^2}$$

area of flywheel $\boxed{A = b \times t}$

where A = cross-sectional area of flywheel rim

b = width of rim

t = thickness of rim.

ρ = density of rim material kg/m³

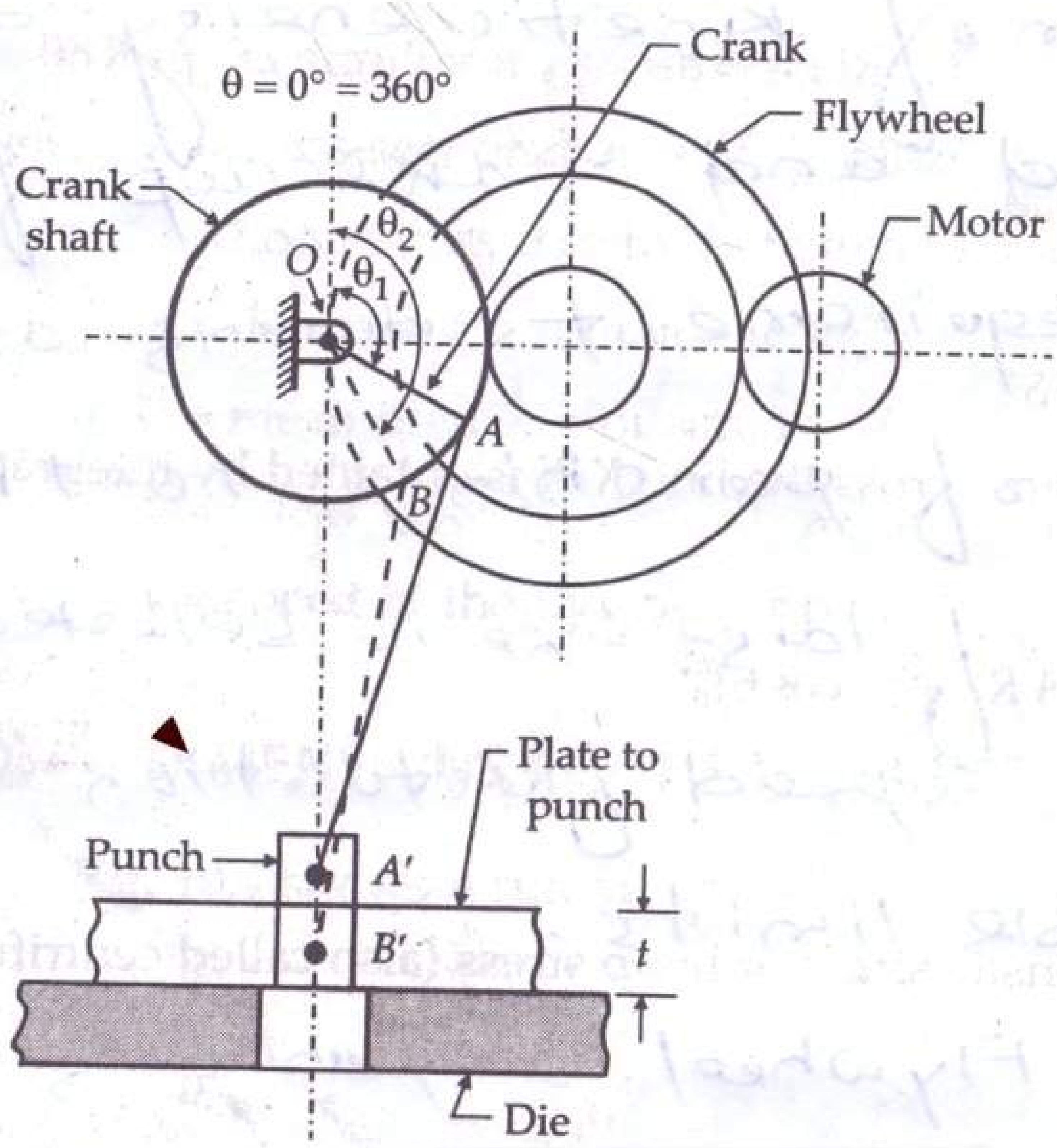
D = mean diameter of rim in m

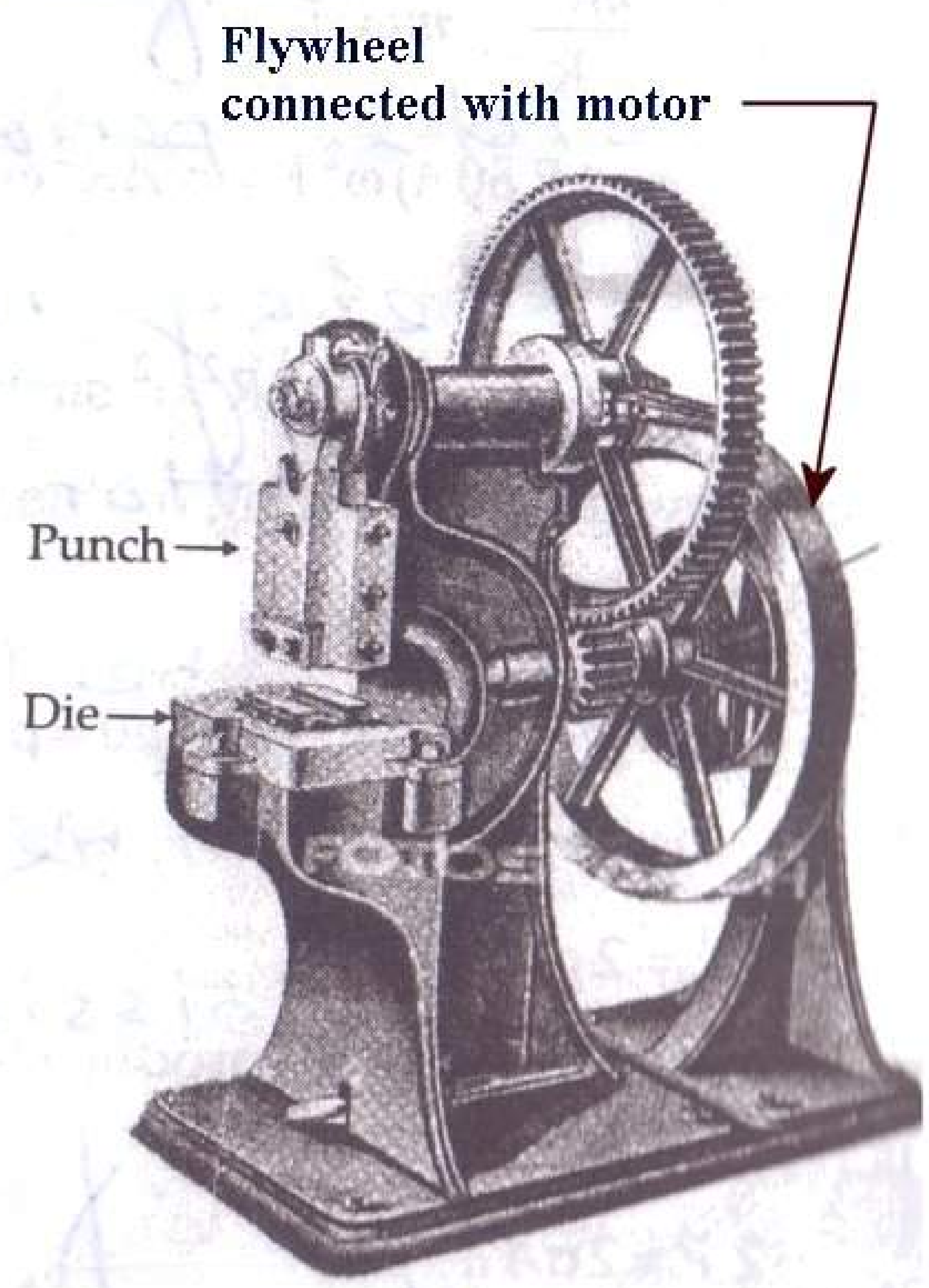
v = linear velocity of flywheel in m/s.

N = speed of flywheel in rpm.

FLYWHEEL AND PUNCHING PRESS :-

The crank of punching press is driven by a motor which supplies a constant torque. The mechanism is just like slider crank mechanism where the punch is at the position of slider. A punching press is shown in the figure.





Punching operation is performed when the crank rotates from θ_1 to θ_2 and only during this rotation of crank load is maxim. It represents the period of actual operation. There is no load on the crank when it rotates from θ_2 to θ and θ to θ_1. It represents period of idle operations.

The load is max^m when punching operation takes place and it's zero during the rest of the cycle. Thus there are high variations in load and hence there is high fluctuations in speed.

The speed of crank increases during idle operations and decreases during actual operation. The flywheel absorbs energy in the form of kinetic energy during idle period and makes up for the energy requirements during actual operation of punching, for this purpose a flywheel of large size is selected which keeps the speed fluctuation within permissible limits.

Analysis of Flywheel in punching operation

Let E_1 = energy required in punching operation

σ_s = max^m shear stress of plate material

d = dia. of hole to be punched

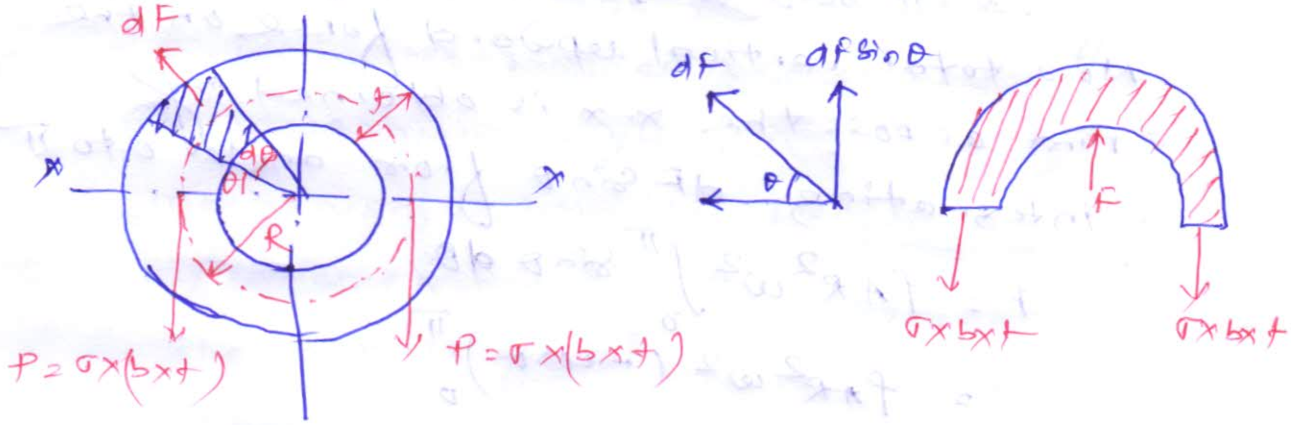
t = thickness of plate.

Max^m shear force

$$F_s = \sigma_s \times \pi d t \quad \text{---(1)}$$

Assuming that during punching operation shear force decreases from max^m value to zero value.

Dimensions of flywheel Rim



Let D = mean diameter of rim, m

R = mean radius of rim, m

A = cross-sectional area of rim, m^2

N = speed of flywheel, rpm

$w = \frac{2\pi N}{60}$ rad/s

v = linear velocity at the mean radius, m/s

$= wR = \frac{\pi D N}{60}$

m = mass of element, Kg;

σ = tensile or hoop stress due to centrifugal force, N/m^2

ρ = density of rim material, Kg/m^3

consider a small element of the rim as shown in the shaded portion in the fig. Let the small element substend an angle $\delta\theta$ at the centre of the flywheel. The centrifugal force dF acts radially outwards. The force dF acting on the centre of the element is given by

$$dF = \frac{mv^2}{R} = m\omega^2 R$$

$$= \rho(R\,\delta\theta A)\omega^2 R = \rho A R^2 \omega^2 \delta\theta$$

vertical component of dF

$= dF \sin\theta = \rho A R^2 \omega^2 \sin\theta \, \delta\theta$

Now total vertical upward force on the rim across the x-x is obtained by integrating $dF \sin\theta$ from angle 0 to π

$$F = \rho A R^2 \omega^2 \int_0^{\pi} \sin\theta \, d\theta$$

$$= \rho A R^2 \omega^2 \left(-\cos\theta\right)_0^{\pi}$$

$$\Rightarrow \boxed{F = 2\rho A R^2 \omega^2} \quad \text{--- (1)}$$

This vertical upward force will produce tensile stress or hoop stress (also called centrifugal stress) and it is resisted by $2P$, such that

$$2P = 2\sigma A \quad \text{--- (2)}$$

Equating (1) and (2)

$$2\sigma A = 2\rho A R^2 \omega^2$$

$$\Rightarrow \sigma = \rho R^2 \omega^2 = \rho v^2$$

$$\Rightarrow \boxed{v = \sqrt{\frac{\sigma}{\rho}}}$$

We know that mass of the rim

$$\boxed{m = \text{volume} \times \text{density} = \pi D A \rho}$$

So area $\boxed{A = \frac{m}{\pi D \rho}}$

For rectangular cross-sectional area of rim $\boxed{A = b \times t}$

The average energy of punching one hole

$$\boxed{E_1 = \frac{1}{2} F_s \cdot t}$$

Let E_2 = energy supplied by motor during punching a hole / revolution

$$= \boxed{E_1 \left(\frac{\theta_2 - \theta_1}{2\pi}\right)}$$

Max.m fluctuation of energy

$$\boxed{\begin{aligned} E_f &= E_1 - E_2 \\ &= E_1 - E_1\left(\frac{\theta_2 - \theta_1}{2\pi}\right) \end{aligned}}$$

If s is the stroke length of punch, then punch moves 2s in one revolution

so $$\boxed{\frac{\theta_2 - \theta_1}{2\pi} = \frac{t}{2s} = \frac{t}{4r}}$$

where r = crank radius.

Above relation is obtained by calibration of crank motion with punch motion,

Q.1 A flywheel of a steam engine weighs 2000N and has a radius of gyration of 76cm. The starting torque of steam engine is 130 kg. and m is assumed to be constant. Determine the angular acceleration of flywheel along with speed and kinetic energy after 10s.

Given data:- $K = 0.76\,m$

starting torque $T = 130\,kgm$

$= 130 \times 9.81 = 1274\ N\cdot m$

mass of flywheel $m = \frac{W}{g} = \frac{2000}{9.81} = 204.05\,kg$.

moment of inertia of flywheel

$I = mK^2 = 204.08 \times (0.76)^2 = 117.88\,kgm^2$

angular acceleration of flywheel (α)

we have $T = I \cdot \alpha$

$$\Rightarrow \alpha_1 = \frac{T}{I} = \frac{1274}{117.88} = 10.8 \text{ rad/s}^2$$

The flywheel started from rest, so $\omega_1 = 0$

and $\omega_2 = \omega_1 + \alpha t$

$$= 0 + 10.8 \times 10 = 108 \text{ rad/s}.$$

Kinetic energy $K.E = \frac{1}{2} I \omega_2^2$

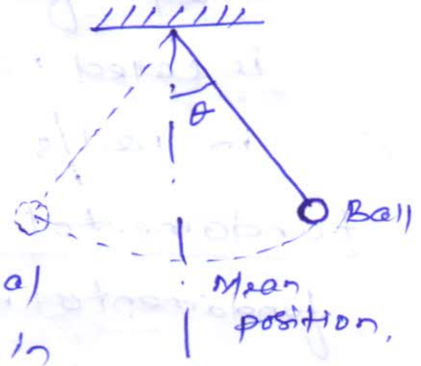
$$= \frac{1}{2} \times 117.88 \times 108^2$$

$$= \boxed{687476.16 \text{ N-m}} \quad \text{(Ans)}$$

— 'VIBRATION' —

Basic concept :-

The mass is inherent of body and elastricity causes relative motion among it's parts. When the body particles are displaced by the application of external force, the internal forces in



the form of elacitic energy are present in the body. These forces try to bring the body to it's original position. At equilibrium position, the whole of the elastic energy is converted into kinetic energy and body continues to move in opposite direction because of it. The whole of the kinetic energy is again converted into elastic energy due to which the body again returns to the equilibrium position. This way vibratory motion is repeated with exchange of energy. This phenomenon is called vibration.

— Swinging of simple pendulum shown in the fig. is an example of vibration.

Definitions :-

Periodic motion → a motion repeating itself after equal interval of time.

Time period → time taken to complete one cycle.

frequency → No of cycles per unit time.

Amplitude → max^m displacement of vibrating body from its equilibrium position

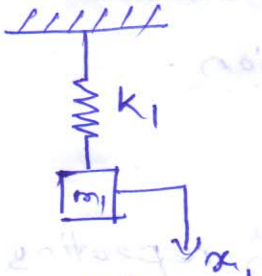
Natural frequency:- When no external force acts on the system after giving it an initial displacement the body vibrates. These vibrations are called free vibrations and their frequency is called natural frequency. It is expressed in rad/s or Hertz.

Fundamental mode of vibration:- The fundamental mode of vibration of a system is the mode having the lowest natural frequency.

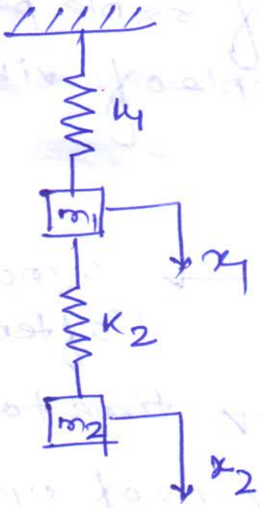
Degree of freedom:- The min^m no of independent coordinates required to specify the motion of a system at any instant is known as degrees of freedom of the system.

- In general it is equal to the no of independent displacements that are possible. This number varies from zero to infinity.

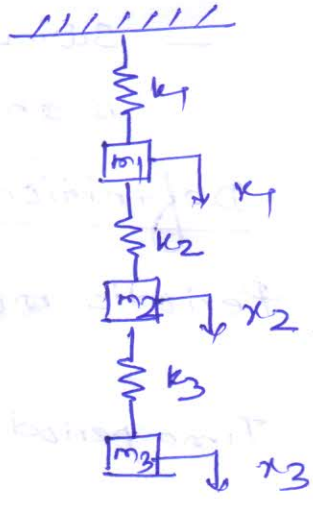
Example of one, two and three degree of freedom system are shown in the figures.



(One DOF)

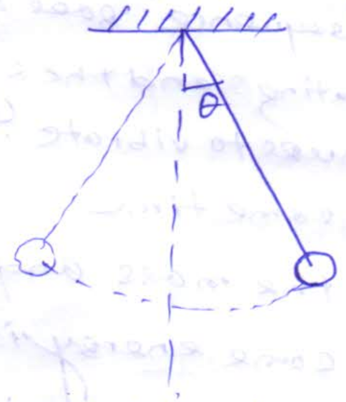


(Two DOF)



(Three DOF)

Simple Harmonic Motion:- The motion of a body to and fro about a fixed point is called Simple Harmonic motion. The motion is periodic and it's acceleration is always directed towards the mean position and is proportional to it's distance from mean position. The motion of a simple pendulum is an example of SHM.



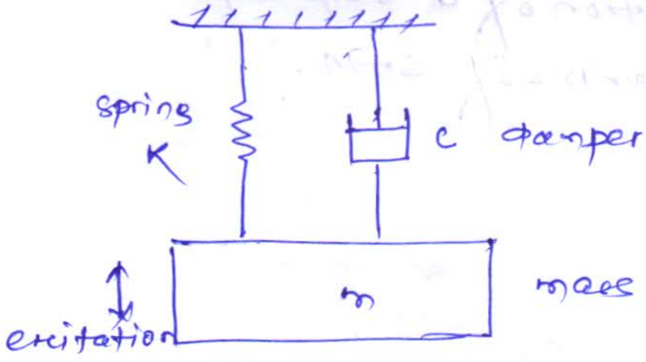
Damping:- It is the resistance to the motion of a vibrating body. The vibrations associated with this resistance are known as damped vibrations.

Resonance:- When the frequency of external excitation is equal to the natural frequency of a vibrating body, the amplitude of vibration becomes excessively large. This phenomenon is called resonance.

× Parts of a Vibrating system:-

A simple vibratory system consists of three elements namely mass, the spring and damper. In a vibratory body there is exchange of energy from one form to another. Energy is stored by mass in the form of KE ($\frac{1}{2}m\dot{x}^2$), in the spring in the form of PE ($\frac{1}{2}Kx^2$) and dissipated in the damper in the form of heat energy which opposes the motion of the system.

Energy enters the system with the application of external force, known as excitation. The excitation disturbs the mass from its mean position and it goes up and down from its mean position.



The kinetic energy is converted to potential energy and vice versa. This sequence goes on repeating and the system continues to vibrate. At the same time damping force $c\dot{x}$ acts on the mass and opposes its motion. Thus some energy is dissipated in each cycle of vibration due to damping. After some time free vibration die out and the system remains at its static equilibrium position. A basic vibratory system is shown in figure.

The equation of motion for such a vibratory system is

$$m\ddot{x} + c\dot{x} + Kx = 0$$

where $c\dot{x}$ = damping force

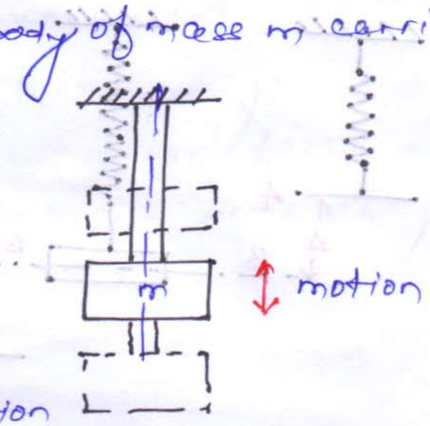
Kx = spring force

$m\ddot{x}$ = inertia force.

Types of vibrations:-

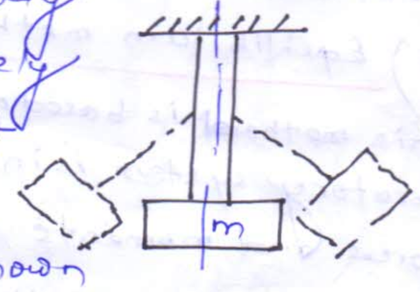
(i) Longitudinal vibrations:-

considering a case where a body of mass m carried on one end of a weightless spindle, the other end being fixed. If the mass 'm' moves up and down parallel to the spindle axis, it is said to execute longitudinal vibration,



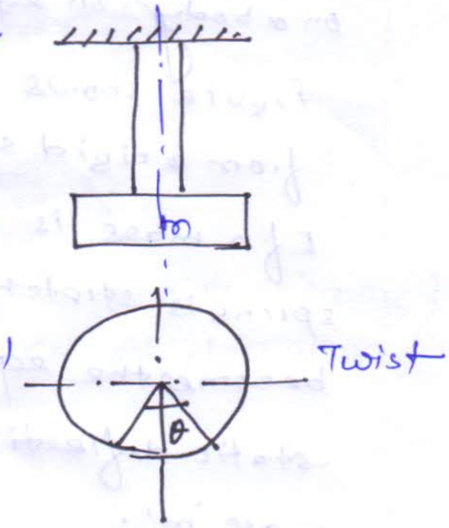
(ii) Transverse vibrations:-

When the particles of the body or shaft move approximately ⊥r to the axis of the shaft as shown in the figure, the vibration caused is known as transverse vibration,



(iii) Torsional Vibrations:-

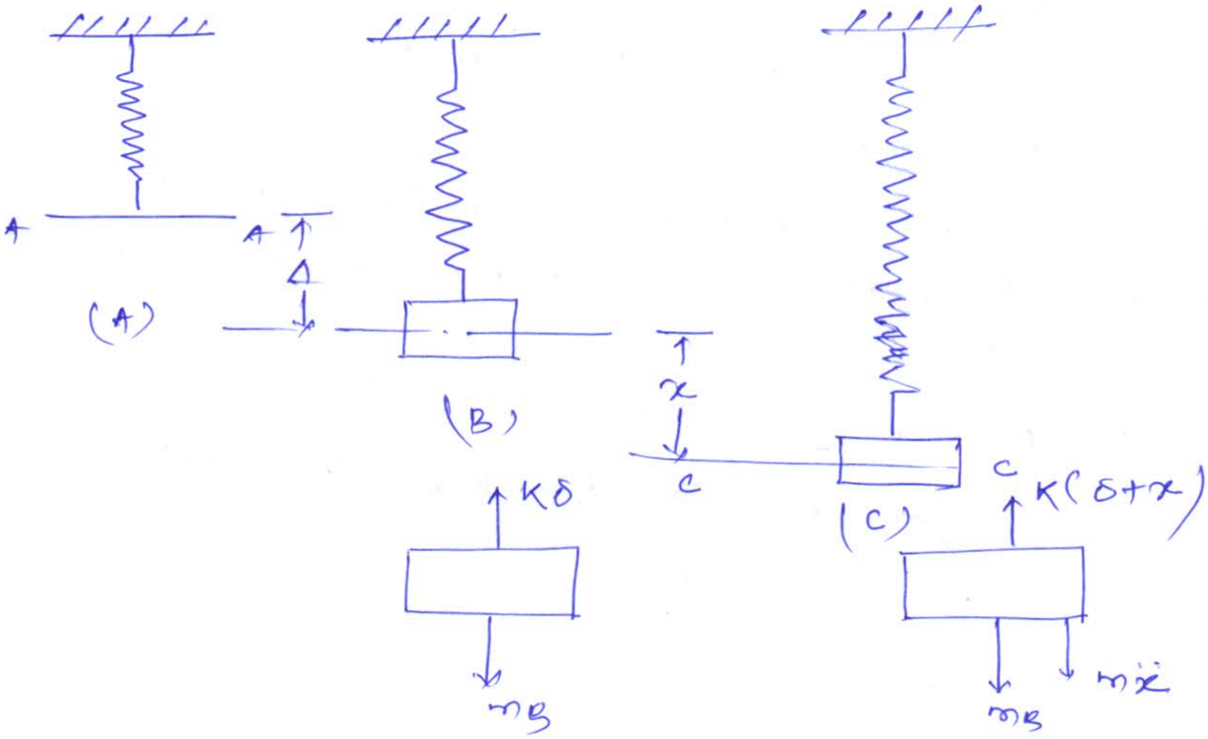
If the spindle get alternately twisted and untwisted on account of vibratory motion of the suspended disc, then it is undergoing torsional vibration



Transient vibrations:-

In ideal systems the free vibration continues indefinitely as there is no damping. The amplitude of vibration decays continuously because of damping (in practical system) and vanishes ultimately. Such vibration is real system is called transient vibration.

Calculation of Natural frequency of undamped Single DOF system



The natural frequency of a free undamped single DOF system can be obtained from any of the following methods:

(i) Equilibrium method (D'Alembert's principle)

This method is based on the principle that ~~were~~ whenever a vibrating system is in equillibrium, the alzebric sum of force and moments acting on it is zero. This is in accordance with D'Alembert's principle that the sum of external forces and inertia forces acting on a body in equilibrium, must be zero.

- Figure (A) shows a helical spring suspended vertically from a rigid support with it's free end at A-A. If a mass is suspended from the free end the spring is stretched by a distance Δ and B-B becomes the equilibrium position. so Δ is the static deflection of the spring under the wt. of mass 'm'.

In equilibrium position, the gravitational pull W is balanced by a spring force, such that

$$mg = W = K\delta \quad -(1)$$

Where δ = static deflection of spring in, m,

K = spring stiffness.

- When the mass is displaced from its equilibrium position by a distance x and released, so after time t

$$\text{Resoring force} = W - K(\delta + x)$$

$$\Rightarrow m\ddot{x} = \cancel{W} - \cancel{K\delta} - Kx \quad (\because W = K\delta)$$

$$\Rightarrow m\ddot{x} = -Kx$$

$$\Rightarrow m\ddot{x} + Kx = 0 \quad -(2)$$

where $\ddot{x}$ acceleration of mass m. It is recognised as equation of a SHM.

The solution ~~to~~ eq. (2) is

$$x = A\cos\omega_n t + B\sin\omega_n t \quad -(3)$$

Where $A, B \rightarrow$ constants, which can be obtained by substituting initial conditions.

$\omega_n \rightarrow$ circular frequency of motion

~~substituting equation (3) in equation (2)~~ we have

~~we have~~ We have

$$\dot{x} = -\omega_n A\sin\omega_n t + \omega_n B\cos\omega_n t$$

$$\ddot{x} = -\omega_n^2 A\cos\omega_n t - \omega_n^2 B\sin\omega_n t$$

$$= -\omega_n^2(A\cos\omega_n t + B\sin\omega_n t) \quad -(4)$$

substituting the value of x and $\ddot{x}$ in eq. (2), we have

$$-\omega_n^2(A\cos\omega_n t + B\sin\omega_n t) + \frac{K}{m}(A\cos\omega_n t + B\sin\omega_n t) = 0$$

Since $A\cos\omega_n t + B\sin\omega_n t \neq 0$

$$-\omega_n^2 + \frac{K}{m} = 0$$

$$\Rightarrow \boxed{\omega_n = \sqrt{\frac{K}{m}}} \text{ rad/s} \quad \text{—(5)}$$

The frequency of vibration $f_n = \frac{\omega_n}{2\pi}$

$$\Rightarrow \boxed{f_n = \frac{1}{2\pi}\sqrt{\frac{K}{m}}} \text{ Hz.} \quad \text{—(6)}$$

Time period $\boxed{T = \frac{1}{f_n} = 2\pi\sqrt{\frac{m}{K}}}$ —(7)

(ii) Energy method:-

According to conservation of energy law

$$T + V = \text{constant} \quad \text{—(8)}$$

Where T = kinetic energy

V = potential energy

Differentiation of above equation w.r.t time will be zero.

$$\frac{d}{dT}(T+V) = 0$$

Now $T = \frac{1}{2} m\dot{x}^2$

$V = \frac{1}{2} Kx^2$

so $\frac{d}{dt}\left(\frac{1}{2} m\dot{x}^2 + \frac{1}{2} Kx^2\right) = 0$

$$\frac{1}{2} m\, 2\dot{x}\ddot{x} + \frac{1}{2} K\, 2x \cdot \dot{x} = 0$$

$$\Rightarrow m\ddot{x} + Kx = 0$$

so circular frequency $\boxed{\omega_n = \sqrt{\frac{K}{m}}}$ —(9)

Rayleigh's Method:-

In this method it is assumed that max KE at the mean position is equal to the max potential energy at the extreme position. The motion is assumed to be SHM, then

$$x = A\sin\omega_n t \quad \text{—(10)}$$

where x = displacement of the body from mean position after time t

A = max^m displacement from mean position to the extreme position.

Differenciating eq. (10)

$$\dot{x} = \omega_n A \cos \omega_n t$$

max^m velocity at mean position

$$\dot{x} = \omega_n A$$

so max^m KE at mean position $= \frac{1}{2} m \dot{x}^2$

$$= \frac{1}{2} m \omega_n^2 A^2 \quad \text{— (11)}$$

max^m potential energy at the extreme position

$$= \frac{1}{2} K A^2 \quad \text{— (12)}$$

Equating equation (11) and (12)

$$\frac{1}{2} m \omega_n^2 A^2 = \frac{1}{2} K A^2$$

$$\Rightarrow \omega_n^2 = \frac{K}{m}$$

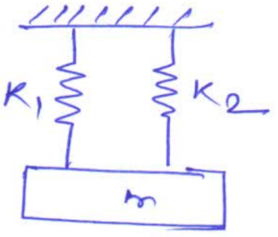
$$\Rightarrow \boxed{\omega_n = \sqrt{\frac{K}{m}}} \quad \text{— (13)}$$

Equivalent spring stiffness for different combinations

Certain springs have more than one spring, which are joined either in series or in parallel. They can be replaced by a single spring of same stiffness.

(a) springs in parallel :-

The deflection of individual spring is equal to the deflection of the system



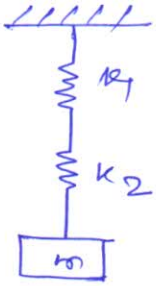
$$K_1 x + K_2 x = K_e x$$

$$\Rightarrow \boxed{K_e = K_1 + K_2}$$

where K_e is the equivalent spring stiffness.

(b) springs in series:-

Total deflection of the system is equal to the sum of deflection of individual springs.



$$x = x_1 + x_2$$

$$\Rightarrow \frac{\text{Force}}{k_e} = \frac{\text{force}}{k_1} + \frac{\text{Force}}{k_2}$$

$$\Rightarrow \frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$= \frac{k_1+k_2}{k_1 k_2}$$

$$\Rightarrow \frac{1}{k_e} = \frac{k_1+k_2}{k_1 k_2}$$

$$\Rightarrow \boxed{k_e = \frac{k_1 k_2}{k_1+k_2}}$$

Natural frequency of spring

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