MICROWAVE ENGINEERING

MICROWAVE ENGINEERING (3-1-0)

Module-I (14 Hours)

High Frequency Transmission line and Wave guides: The Lumped-Element Circuit model for a Transmission line. Wave propagation. The lossless line. Field Analysis of Co-ax Transmission Lines. R, L, C, G parameters of Co-axial & Two wire Transmission lines, Terminated lossless transmission line, Lowloss line, The Smith Chart. Solution of Transmission line problems using Smith chart. Single Stub and Double Stub matching.

Waveguides:

Rectangular waveguides, Field solution for TE and TM modes, Design of Rectangular waveguides to support Dominant TE only.

Module-II

(12 Hours)

TEM mode in Co-ax line. Cylindrical waveguides- Dominant mode. Design of cylindrical waveguides to support dominant TE mode. Microwave Resonator: Rectangular waveguides Cavities, Resonant frequencies and of cavity supporting. Dominant mode only.

Excitation of waveguides and resonators (in principle only). Waveguides Components: Power divider and Directional Couplers: Basic properties. The TJunction power divider, Waveguide-Directional Couplers.

Fixed and Precision variable Attenuator, Isolator, Circulator (Principle of Operation only).

Module-III (10 Hours)

Microwave Sources: Reflex Klystron: Velocity Modulation, Power output and frequency versus Reflector voltage Electronic Admittance. MultiCavity Magnetron: Principle of operation, Rotating field, Π-mode of operation, Frequency of oscillation. The ordinary type (O-type) traveling wave tube- Construction features, principle of operation as an amplifier, Gunn oscillator (principle).

Module-IV (6 Hours)

Microwave Propagation: Line of sight propagation. Attenuation of microwaves by Atmospheric gases, water vapors & precipitates.

Text Books:

1. Microwave Engineering by D.M.Pozor, 2nd Edition, John Willy & Sons. Selected portions from Chapters 2,3,4,6,7&9.
2. Principles of Microwave Engineering by Reich, Oudong and Others.
3. Microwave Devices and Circuits, 3rd Edition, Sammuel Y, Liao, Perason.

Microwave

The signal deals with very small wave wavelength is called microwave signal, this implies signal has:

Wavelength (ƛ) =speed/frequency

With due increase in frequency the wavelength decrease and vice versa; we can say that wavelength is inversely proportional to frequency.

In communication system,it generally consist of three main components: Transmitter, Receiver and Channel.

There are two type mediums: transmission line & waveguide.

Transmission line used for small range frequencies. Waveguide used for large range frequencies.

Transmission Line (TL)

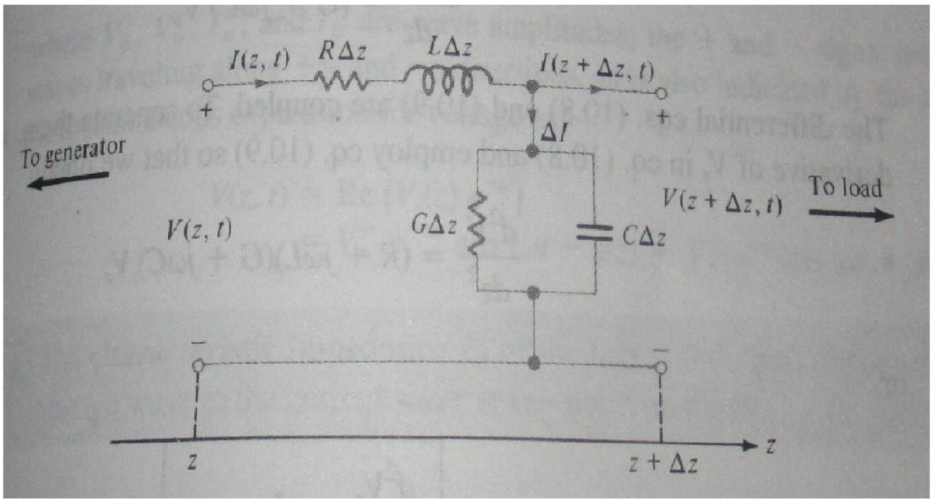
- The wave is bounded at low frequency in transmission line and hence called low pass filter.
- Transmission line mainly supports electromagnetic field.

Types of Transmission Line:

- Coaxial cable
- Parallel wire cable
- Microstrip line

Lumped Element Circuit Model of Transmission Line

- A current carrying conductor produce a magnetic field i.e., inductance which opposes the flow of current hence resistance 'R' is in series with inductance 'L'.
- Because of dielectric separation; there exist a capacitance and the loss in the dielectric medium give rise to conductance.
  R= Resistance of the conductor (Ω/m)
  L= Self Inductance of the conductor (μ/m)
  C= Capacitance across the conductor (F/m)
  G= Dielectric Loss between conductor (℧/m)



Applying KVL in the loop;

$v(z, t)= R\,\Delta z\, I(z, t) + L\,\Delta z\,(d\, I(z, t)/dt) + v(z + \Delta z, t)$

$v(z, t) - v(z + \Delta z, t)= R\,\Delta z\, I(z, t) + L\,\Delta z\,(d\, I(z, t)/dt)$

$(v(z, t) - v(z + \Delta z, t))/ \Delta z= R\, I(z, t) + L\,(d\, I(z, t)/dt)$

Taking limit;

$$\lim_{\Delta z \to 0} (v(z, t) - v(z + \Delta z, t))/ \Delta z= \lim_{\Delta z \to 0} R\, I(z, t) + L\,(d\, I(z, t)/dt)$$

$-dv(z, t)/dz= R\, I(z, t) + L\, dI(z, t)/dtC$

$-dV/dz= RI + LdI/dt$ -------------①

Applying KCL at first node

$I(z, t)= I(z + \Delta z, t) + \Delta I$

$I(z, t)= \Delta I_1 + \Delta I_2 + I(z + \Delta z, t)$

$I(z, t)= G\,\Delta z\, V(z + \Delta z, t) + C\,\Delta z\,(d\, V(z + \Delta z, t)/dt) + I(z + \Delta z, t)$

$I(z, t) - I(z + \Delta z, t)= G\,\Delta z\, V(z + \Delta z, t) + C\,\Delta z\,(d\, V(z + \Delta z, t)/dt)$

$(I(z, t) - I(z + \Delta z, t))/\Delta z= G\, V(z + \Delta z, t) + C\,(d\, V(z + \Delta z, t)/dt)$

$$\lim_{\Delta z \to 0} (I(z, t) - I(z + \Delta z, t))/\Delta z= \lim_{\Delta z \to 0} G\, V(z + \Delta z, t) + C\,(d\, V(z + \Delta z, t)/dt)$$

$-d\, I(z, t)/dz= G\, V(z,t) + C\,(d\, V(z, t)/dt)$

$-d\, I/dz= G\, V + C\, d\, V/dt$ -------------②

In equation ① & ②

$V= V(z, t)= Re\{V_s(z)e^{j\dot{\omega}t}\}$

$I= I(z, t)= Re\{I_s(z)e^{j\dot{\omega}t}\}$

Putting V & I in equation ①

$-dV/dz= R\,I + L\, dI/dt$

$-d\,[Re\{V_s(z)e^{j\dot{\omega}t}\}]/dz= R\,[Re\{I_s(z)e^{j\dot{\omega}t}\}] + L\, d\,[Re\{I_s(z)e^{j\dot{\omega}t}\}]/dt$

$- Re\{d\,V_s(z)e^{j\dot{\omega}t}/dz\}= R\,I_s(z)\,[Re\{ e^{j\dot{\omega}t}\}] + L\ \ [Re\{j\,\dot{\omega}\,I_s(z)e^{j\dot{\omega}t}\}]$

$-d\,V_s(z)/dz= (R + j\dot{\omega}L)I_s(z)$ -------------③

Similarly,

$-d\,I_s(z)/dz= (G + j\dot{\omega}C)V_s(z)$ -------------④

Equation ③ & ④ are called **Telegraphers Equation** or low frequency equation.

The transmission line discussed so far were of lossy type in which the conductors comprising the line are imperfect ($\omega_c = \infty$)and the dielectric in which the conductors are embedded is lossy ($\omega_c = 0$).

Having considered this general case , we may now consider two special cases:

1. Lossless line(R=0=G)
2. Distortionless line(R/l=G/c)

Case-1:Lossless line(R=0=G):- The transmission line is said to be lossless if the conductors of the line are perfect $\omega_c = \infty$ and the dielectric separating between them is lossless($\omega_c = 0$).

For such a line R=0=G .This is the necessary condition for a line to be lossless.

Hence for this line the attenuation constant α=0. But the propagation constant $\gamma=\sqrt{(R+j\omega L)(G+ j\omega C)}$

But we also know that $\gamma=\alpha+j\beta$.

so on solving we get:-

$\sqrt{\{(RG +R\,j\omega C +G\,j\omega L+j^2\omega^2LC}=j\beta$ $(\because\alpha=0)$

So we get $\sqrt{(j^2\omega^2 LC)}=j\beta$

this shows that the phase constant $\beta=\omega\sqrt{LC}$ and for the characteristic impedance $Z_0=\sqrt{\{(R+j\omega L)/(G+j\omega C)\}}$

so $Z_0=\sqrt{(L/C)}$

and the phase velocity $v_p=\omega/\beta=1/\sqrt{(LC)}=f\lambda$

Case-2:Distortionless Line(R/L=G/C):- A distortionless line is the one in which the attenuation constant α is frequency independent while the phase constant β is linearly dependent on frequency.

From the expression for α and β a distortionless line results if the parameters are such:- $R/L=G/C$

So $\gamma=\sqrt{(R+j\omega L)(G+j\omega C)}$

Hence $\gamma=\sqrt{RG(1+j\omega L/R)(1+j\omega C/G)}$

$=\sqrt{RG(1+j\omega C/G)^2}=\alpha+j\beta$

so $\alpha=\underline{\sqrt{RG}}$ and $\beta\underline{=\omega\sqrt{LC}}$

The above values shows that α is not dependent on frequency and β is a linearly dependent on frequency.

Also characteristic impedance=$Z_0=\sqrt{(R+j\omega L)/(G+j\omega C)}$

$Z_0=\sqrt{[R(1+j\omega L/R)]/[G(1+j\omega C/G)]}$

$Z_0\underline{=\sqrt{R/G}=\sqrt{L/C}}$

Also the phase velocity $v_p=\omega/\beta=1/\sqrt{LC}=f\lambda$

Or $\lambda=1/f\sqrt{LC}$

Microwave Line:- A microwave line is the one where the parameters are such :-$R<<\omega L$ and $G<<\omega C$

So the characteristic impedance $Z_0=\sqrt{(R+j\omega L)/(G+j\omega C)}$

So substituting the parameters we get:- $\underline{Z_0=\sqrt{L/C}}$

And propagation constant $\gamma=\sqrt{(R+j\omega L)(G+j\omega C)}$

So $\gamma=\sqrt{j\omega L(1+R/j\omega L)\, j\omega c(1+G/j\omega C)}$

$=j\omega\sqrt{LC}[(1+R/j\omega L)^{1/2}(1+G/j\omega C)^{1/2}$

$=j\omega\sqrt{LC}(1+R/2j\omega L)(1+G/2j\omega L)$

$=j\omega\sqrt{LC} \quad [1+R/2j\omega L+G/2j\omega C]$

$\gamma = j\omega\sqrt{LC} + (R/2)(\sqrt{C/L}) + G\sqrt{L/C}$

but we also know that $\gamma=\alpha+j\beta$

so we have $\alpha\underline{=(R/2)(\sqrt{C/L})}$ and $\underline{\beta=j\omega\sqrt{LC}}$

and the phase velocity $v_p=\omega/\beta=2\Pi/(j\omega\sqrt{LC})$

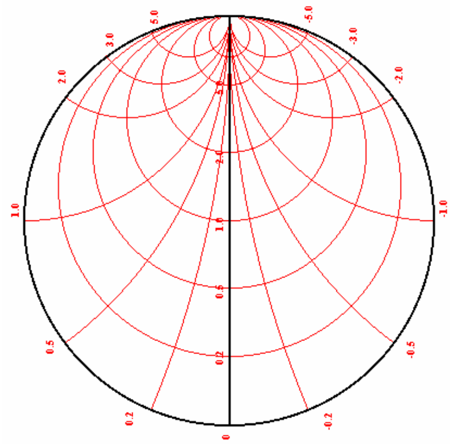
so $v_p= 2\Pi/(j2\Pi f\sqrt{LC}$

hence $v_p= \underline{1/(jf\sqrt{LC})}$

SMITH CHART

For evaluating the rectangular components, or the magnitude and phase of an input impedance or admittance, voltage, current, and related transmission functions at all points along a transmission line, including:

- Complex voltage and current reflections coefficients
- Complex voltage and current transmission coefficents
- Power reflection and transmission coefficients
- Reflection Loss
- Return Loss
- Standing Wave Loss Factor
- Maximum and minimum of voltage and current, and SWR
- Shape, position, and phase distribution along voltage and current standing waves
- Evaluating effects of shunt and series impedances on the impedance of a transmission line.
- For displaying and evaluating the input impedance characteristics of resonant and anti-resonant stubs including the bandwidth and Q.
- Designing impedance matching networks using single or multiple open or shorted stubs.
- Designing impedance matching networks using quarter wave line sections.
- Designing impedance matching networks using lumped L-C components.
- For displaying complex impedances verses frequency.
- For displaying s-parameters of a network verses frequency.



- It is the most useful graphical tool for transmission line problems.
- From mathematical point of view, the Smith chart is simply a representation of all possible complex impedances with respect to coordinates defined by the reflection coefficient.
- It can be used to convert from reflection coefficients to normalized impedances (or admittances), and vice versa using the impedance (or admittance) circle printed on the chart. When dealing with impedances on a Smith chart, normalized quantities are generally used denoted by lowercase letters. The normalization constant is usually the characteristic impedance of the line($z=Z/Z_0$)
- The domain of definition of the reflection coefficient is a circle of radius 1 in the complex plane.

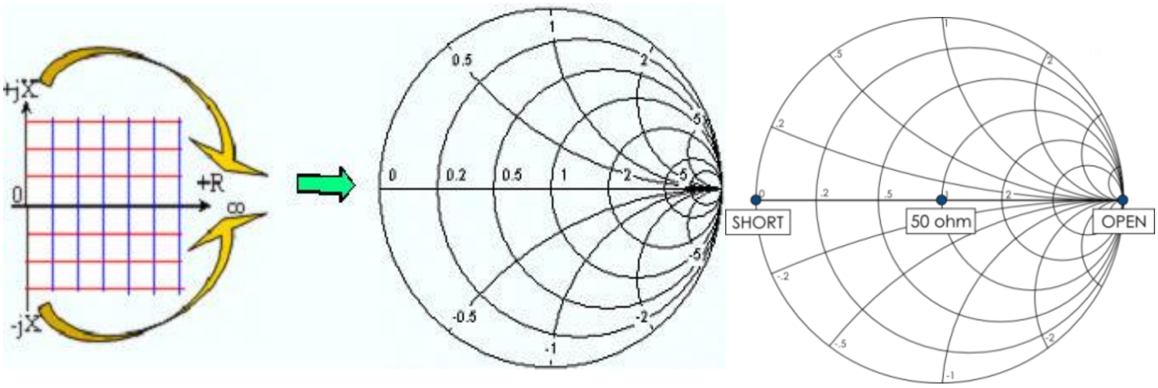


Fig-1: Smith Chart

- At point of Short, r = 0, x = 0 So $z_L = r+jx=0$
- At point of Open, r = ∞, x = ∞ So $z_L = r+jx=\infty$
- If a lossless line of characteristics impedance Z_0 is terminated with a load impedance Z_L, the reflection coefficient at the load can be written as

$$\Gamma = \frac{z_L - 1}{z_L + 1} = |\Gamma| e^{j\theta}$$

Where $z_L = Z_L / Z_0$ is the normalized impedance

$$z_L = \frac{1+|\Gamma| e^{j\theta}}{1-|\Gamma| e^{j\theta}}$$

Writing z_L and Γ in terms of their real and imaginary parts as $\Gamma = \Gamma_r + j\Gamma_i$ and $z_L = r_L + jx_L$ then

$$z_L = r_L + jx_L = \frac{1+\Gamma_r + j\Gamma_i}{1-\Gamma_r - j\Gamma_i}$$

Multiplying the numerator and denominator by the complex conjugate of the denominator and rearranging gives

$$\left(\Gamma_r - \frac{r_L}{1+r_L}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r_L}\right)^2$$

$$\left(\Gamma_i - \frac{1}{x_L}\right)^2 + (\Gamma_r - 1)^2 = \left(\frac{1}{x_L}\right)^2$$

The above two equation represents two families of a circle in the Γ_r, Γ_i plane.

Constant Resistance Circle : Center $\left(\dfrac{r}{r+1},0\right)$ and Radius $\left(\dfrac{1}{r+1}\right)$

- Constant Reactance Circle : Center $\left(1,\dfrac{1}{x}\right)$ and Radius $\left(\dfrac{1}{x}\right)$

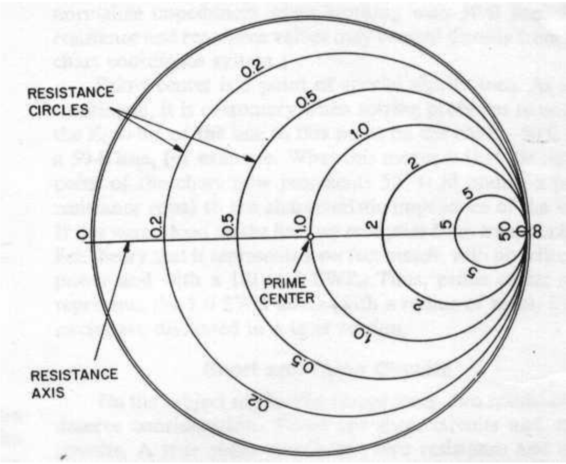


Fig-2: Constant Resistance Circle

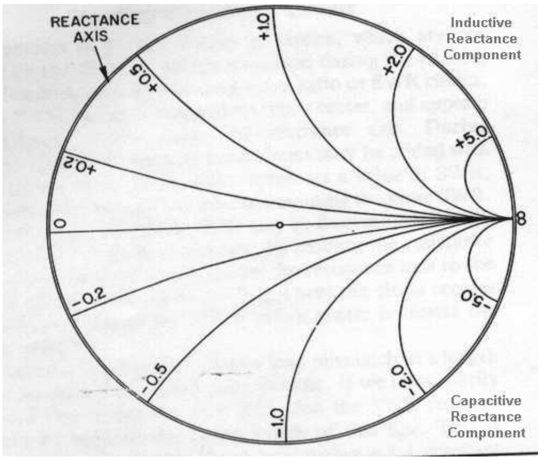


Fig-3: Constant Reactance Circle

- Complete revolution on the smith chart is $\dfrac{\lambda}{2}$ on Transmission line. Distance between Vmax and Vmin is $\dfrac{\lambda}{4}$ i.e 180.
- Movement from load towards generator is clockwise.

WAVEGUIDE

The transmission line can't propagate high range of frequencies in GHz due to skin effect. Waveguides are generally used to propagate microwave signal and they always operate beyond certain frequency that is called "*cut off frequency*". so they behaves as high pass filter.

SKIN EFFECT : $x_c = \frac{1}{(2*\pi*f*c)}$

According to this relation,as frequency increases, x_c tends to zero, that is short circuit. Hence signal becomes grounded and can't propagate further which is called skin effect.

Types of waveguides: - (1)rectangular waveguide (2)cylindrical waveguide (3)elliptical waveguide (4)parallel waveguide

RECTANGULAR WAVEGUIDE :

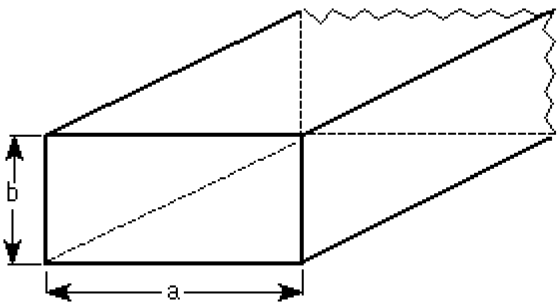
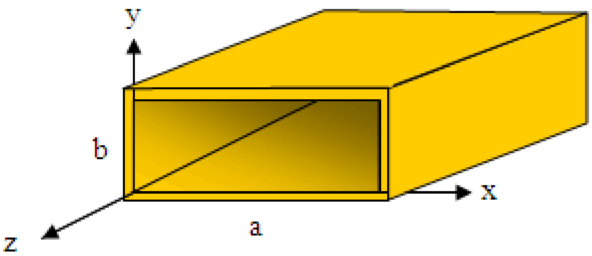


Figure 1. The Rectangular Waveguide



Let us assume that the wave is travelling along z-axis and field variation along z-direction is equal to $e^{-\Upsilon z}$,where z=direction of propagation and Υ= propagation constant.

Assume the waveguide is lossless (α=0) and walls are perfect conductor (σ=∞). According to maxwell's equation: $\nabla \times H = J + \partial D/\partial t$ and $\nabla \times E = -\partial B/\partial t$.

So $\nabla \times H = J\omega \in E ---(1.a)$, $\nabla \times E = -J\omega\mu H$. ---------(1.b)

Expanding equation (1),

$$\left|\begin{matrix} Ax & Ay & Az \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ Hx & Hy & Hz \end{matrix}\right| = J\omega \in [ExAx + EyAy + EzAz]$$

By equating coefficients of both sides we get,

$\frac{\partial}{\partial y} Hz - \frac{\partial}{\partial z} Hy = J\omega \in Ex$ -----------------2(a)

$-\frac{\partial}{\partial x} Hz + \frac{\partial}{\partial z} Hx = J\omega \in Ey$ -----------2(b)

$\frac{\partial}{\partial x}Hy - \frac{\partial}{\partial y}Hx = \text{J}\omega \in Ez$ -----------------2(c)

As the wave is travelling along z-direction and variation is along $-\Upsilon z$ direction.

=> $\frac{\partial}{\partial z}(e^{-\Upsilon z})$= $-\gamma e^{-\Upsilon z}$.

Comparing above equations, $\frac{\partial}{\partial z} = -\Upsilon$.

So by putting this value of $\frac{\partial}{\partial z}$ in equations 2(a,b,c),we will get

$\frac{\partial}{\partial y}Hz + \Upsilon\, Hy$ =j$\omega \in$Ex -----------------3(a)

$\frac{\partial}{\partial x}Hz + \Upsilon\, Hx = -\,\text{j}\omega \in Ey$ -----------3(b)

$\frac{\partial}{\partial x}Hy - \frac{\partial}{\partial y}Hx = \text{j}\omega \in Ez$ -----------------3(c)

Similarly from relation $\nabla \times \text{E} = -\text{j}\omega\mu H$ and $\frac{\partial}{\partial z} = -\Upsilon$,we will get

$\frac{\partial}{\partial y}Ez + \Upsilon\, Ey$ = - j$\omega\mu$Hx -----------------4(a)

$\frac{\partial}{\partial x}Ez + \Upsilon\, Ex = \text{j}\omega\mu Hy$ -----------4(b)

$\frac{\partial}{\partial x}Ey - \frac{\partial}{\partial y}Ex$ = - j$\omega\mu Hz$ -----------------4(c)

From equation sets of (3) , we will get : $\frac{\partial}{\partial y}Hz + \Upsilon\, Hy$ =j$\omega \in$Ex

Ex=$\frac{1}{\text{j}\,\omega\in}[\frac{\partial}{\partial y}Hz + \Upsilon\, Hy]$ ---------(5)

From equation sets of (4) ,we will get : $\frac{\partial}{\partial x}Ez + \Upsilon\, Ex = \text{j}\omega\mu Hy$

Ex=$\frac{1}{\Upsilon}[\text{j}\omega\mu Hy - \frac{\partial}{\partial x}Ez]$ ---------(6)

Equating equations (5) and (6), we will get

$=> \frac{1}{\text{j}\,\omega\in}[\frac{\partial}{\partial y}Hz + \Upsilon\, Hy] = \frac{1}{\Upsilon}[\text{j}\omega\mu Hy - \frac{\partial}{\partial x}Ez]$

$=> \frac{\Upsilon}{\text{j}\,\omega\in}\frac{\partial}{\partial y}Hz + \frac{\Upsilon^2}{\text{j}\,\omega\in}\text{Hy} = \text{j}\omega\mu Hy - \frac{\partial}{\partial x}Ez$

$=> (\frac{\Upsilon^2}{\text{j}\,\omega\in} - \text{j}\omega\mu)\text{Hy} = -\frac{\partial}{\partial x}Ez - \frac{\Upsilon}{\text{j}\,\omega\in}\frac{\partial}{\partial y}Hz$

$=> (\frac{Y^2+\omega^2\mu\in}{j\omega\in})$ Hy $=-\frac{\partial}{\partial x}Ez - \frac{Y}{j\,\omega\in}\frac{\partial}{\partial y}Hz$

Let $(Y^2 + \omega^2\mu \in) = h^2$

$=> (\frac{h^2}{j\omega\in})$Hy $=-\frac{\partial}{\partial x}Ez - \frac{Y}{j\omega\in}\ \frac{\partial}{\partial y}Hz$

$$\boxed{Hy = -\frac{j\omega\in}{h^2}\frac{\partial}{\partial x}Ez - \frac{Y}{h^2}\ \frac{\partial}{\partial y}Hz \text{ ------------ } (7)}$$

Similarly we will get by simplifying other equations

$$\boxed{Hx = -\frac{j\omega\in}{h^2}\frac{\partial}{\partial y}Ez - \frac{Y}{h^2}\ \frac{\partial}{\partial x}Hz \text{ ------------ } (8)}$$

$$\boxed{Ex = -\frac{j\omega\mu}{h^2}\ \frac{\partial}{\partial y}Hz - \frac{Y}{h^2}\ \frac{\partial}{\partial x}Ez \text{ ------------ } (9)}$$

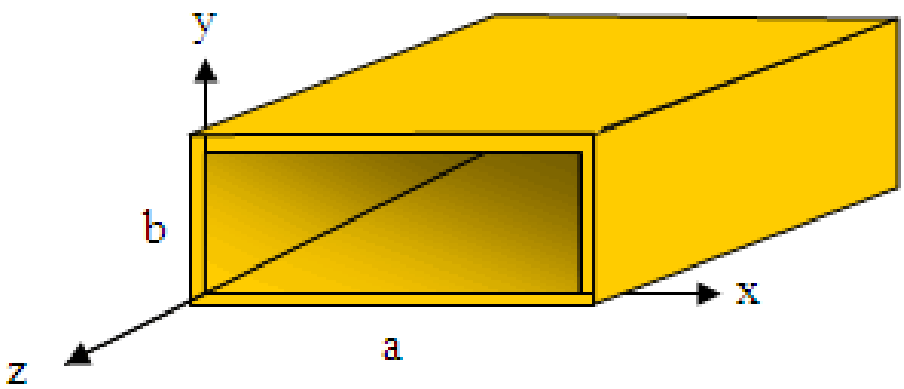
$$\boxed{Ey = \frac{j\omega\mu}{h^2}\frac{\partial}{\partial x}Hz - \frac{Y}{h^2}\ \frac{\partial}{\partial y}Ez \text{ ----------- } (10)}$$

IMPORTANT QUESTION :

(Q) Transverse electromagnetic mode(TEM mode)is not possible in a waveguide.why?

(A)Let the wave propagate along z-direction,then the waveguide field equation 7,8,9,10.as wave propagate along z-direction Ez and Hz=0.From equations 7,8,9,10 :Ex,Hx,Ey,Hy=0.hence it is not possible practically.so Ez,Hz both can't be zero.if Ez=0,transverse electric mode exists and Hz=0,transverse magnetic mode exists.

Field solutions of rectangular waveguide :



$*$To find the solution we have to assume waveguide is lossless that is $(\alpha = 0)$ and walls are perfect conductor$(\sigma = \infty)$. According the Poisson's equation $\nabla^2 E = \gamma^2 E\ and\ \nabla^2 H = \gamma^2 H$

*As $\gamma = \alpha + j\beta$ and as $\alpha = 0\ so$ the equations becomes $\gamma = j\beta$ and $\gamma^2 = -\beta^2 = -k^2$(let) *now the Poisson's equation are $\nabla^2 E + k^2 E = 0 - - - - - (a)$

$$and\ \nabla^2 H + k^2 H = 0 - - - -(b)$$

Expanding equation (a) $\frac{\partial^2 Ex}{\partial x^2} + \frac{\partial^2 Ey}{\partial y^2} + \frac{\partial^2 Ez}{\partial z^2} + k^2 E = 0 - - - -(c)$

$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} + k^2 E = 0$ -----it is a second order partial differential equation whose solution let it be E=XYZ where X=X(x),Y=Y(y),Z=Z(z)

Putting E in equation (b) we will get

$$\frac{\partial^2 (XYZ)}{\partial x^2} + \frac{\partial^2 (XYZ)}{\partial y^2} + \frac{\partial^2 (XYZ)}{\partial z^2} + k^2 (XYZ) = 0$$

=>$YZ\frac{\partial^2 X}{\partial x^2} + XZ\frac{\partial^2 Y}{\partial y^2} + XY\frac{\partial^2 Z}{\partial z^2} + k^2 (XYZ) = 0$

Dividing XYZ in both sides,We will get

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + \frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} + k^2 = 0$$

$Kx^2 + ky^2 + kz^2 = k^2$

As $\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = -\mathrm{Kx}^2$, $\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = -\mathrm{Ky}^2$, $\frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} = -\mathrm{Kz}^2$ ------------(d)

As the wave is travelling along z-direction Z(z)=$e^{-\gamma z}$

$\frac{\partial^2 Z(z)}{\partial z^2} = \gamma^2 e^{-\gamma z}$

⇨ $\frac{\partial^2 Z(z)}{\partial z^2} = Z(z)\,\gamma^2$

by comparing it we will get $\partial^2/\partial z^2 = \gamma^2$

⇨ $\frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} = -k_z^2 = \gamma^2$

Similarly $\frac{1}{x}\frac{\partial^2 X}{\partial x^2} + k_x^2 = 0$

⇨ $\frac{\partial^2 X}{\partial x^2} + x\, k_x^2 = 0$

It is a second order homogenous differential equation whose solution is

X(x)=$C_1 \cos kxx + C_2 \sin k_x x$ ------(e)

X(x)=$C_3 \cos kyy + C_4 \sin k_y y$ ------(f)

As E=XYZ ;

$E_{XYZ} = (C_1\cos kxx + C_2\sin k_x x)(C_3\cos kyy + C_4\sin k_y y)e^{-\gamma z}$ ------(g)

Similarly by solving for magnetic field we will get

$H_{XYZ} = (B_1\cos kxx + B_2\sin k_x x)(B_3\cos kyy + B_4\sin k_y y)\, e^{-\gamma z}$ ------(h)

Equations (g) and (h) are the field solutions foe rectangular waveguide.

CASE-1

FIELD SOLUTIONS FOR TRANSVERSE MAGNETIC FIELD IN RECTANGULAR WAVEGUIDE :

Hz=0 and $E_z \neq 0$

$E_{XYZ} = (C_1 \cos kxx + C_2\sin k_x x)(C_3\cos kyy + C_4\sin k_y y)\, e^{-\gamma z}$

The values of C1,C2,C3,C4,K_x,K_y are found out from boundary equations.as we know that the tangential component of E are constants across the boundary,then

$$E= \begin{cases} 0, x = 0\ and\ x = a \\ 0, y = 0\ and\ y = b \end{cases}$$

AT x=0 AND y=0 ;

$E=C_1C_3e^{-\gamma z} = 0$ but we know that $e^{-\gamma z} \neq 0$ wave is travelling along z-direction.

So either C_1=0 or C_3=0 oterwise $C_1C_{3=}0$

AT x=0 AND y=b ;

$E= C_1(C_3\cos kyb + C_4\sin k_y b)\, e^{-\gamma z}=0$

So $C_1C_{3=}0$

So equation (g)becomes

$E_{XYZ} = (C_2\sin k_x x \times C_4\sin k_y y)e^{-\gamma z}$ ------(i)

Hence for x=0,E=0

So $(C_2\sin k_x a \times C_4\sin k_y y)e^{-\gamma z} = 0$

$=>\sin kxa = 0 => k_x=\frac{m*\pi}{a}$

In equation (i) for y=b=>E=0;

So (C_2sin k_xx $\times$ C_4sin k_yb)$e^{-\gamma z} = 0$

=>sin $kyb = 0 => k_y$=$\frac{n*\pi}{b}$

So finally solutions for TRANSVERSE MAGNETIC MODE is given by

$\boxed{E_z = C\,(\sin(\frac{m*\pi}{a})x \times \sin(\frac{n*\pi}{b})y \times e^{-\gamma z}}$

Where $C_2 \times C_4$=C

CUT-OFF FREQUENCY :

It is the minimum frequency after which propagation occurs inside the waveguide.

As we know that => K_x^2 +k_y^2+k_z^2=k^2

=> K_x^2 +k_y^2= k^2-k_z^2

=> K_x^2 +k_y^2=k^2+γ^2

As we know that $\beta = -j\omega\sqrt{\mu\varepsilon}$ *and* $k^2 = \beta^2$

So we will get that : => K_x^2 +k_y^2=k^2+γ^2=$\omega^2\mu\varepsilon + \gamma^2$

So $\gamma = \sqrt{\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon\right]}$

At f=f_c or w=w_c ,at cut off frequency propagation is about to start. So $\gamma = 0$

=>0=$\sqrt{\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega c^2\mu\varepsilon\right]}$

=>$\omega c^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$

W_c=1/$\sqrt{\mu\varepsilon}$ $\left(\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right)^{1/2}$

$\boxed{\text{So } f_c = 1/2\pi\sqrt{\mu\varepsilon}\left(\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right)^{1/2}}$ ---------***cut off frequency equation***

where m=n=0,1,2,3……

At free space f_c=1/2$\pi\sqrt{\mu o\varepsilon o}$ $\left(\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right)^{1/2}$

$\boxed{f_c = c/2\left(\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right)^{1/2}}$ =>---------***cut off frequency equation in free space***

CUT – OFF WAVELENGTH:

This is given by

$$\lambda c = \frac{c}{f} = 2 \times \left(\frac{1}{\left(\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right)^{1/2}}\right)$$

DOMINANT MODE :

The mode having lowest cut-off frequency or highest cut-off wavelength is called DOMINANT MODE.

*the mode can be TM_{01},TM_{10},TM_{11},But for TM_{10} and TM_{01},wave can't exist.

*hence TM_{11}has lowest cut-off frequency and is the DOMINANT MODE in case of all TM modes only.

PHASE CONSTANT :

As we know that $\gamma = \left(\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon\right)^{1/2}$

So j$\beta = \sqrt{\omega^2\mu\varepsilon - \omega c^2\mu\varepsilon}$

This condition satisfies that only $\omega c^2\mu\varepsilon > \omega^2\mu\varepsilon$

So that $\beta = \sqrt{\omega^2\mu\varepsilon - \omega c^2\mu\varepsilon}$

PHASE VELOCITY :

It is given by Vp=ω/β

$V_p=\frac{\omega}{(\omega^2\mu\varepsilon - \omega c^2\mu\varepsilon)}$

$V_p=1/ [\sqrt{\omega\varepsilon}(1 - fc^2/f^2)]$

GUIDE WAVELENGTH :

It is given by $\lambda g = \frac{2\pi}{\beta} = \frac{2\pi}{\sqrt{\omega^2\mu\varepsilon - \omega c^2\mu\varepsilon}}$

$\lambda g = 1/\sqrt{(f^2\mu\varepsilon - fc^2\mu\varepsilon)}$

$$\lambda g = \frac{c}{f}/(1 - fc^2/f^2)$$

⇨ $\lambda g = \frac{\lambda o}{[1-\left(\frac{\lambda o}{\lambda c}\right)^2]^{\frac{1}{2}}}$

⇨ $\frac{1}{\lambda g^2}$=1/ $\lambda o^2 - \frac{1}{\lambda c^2}$

CASE -2

SOLUTIONS OF TRANSVERSE ELECTRIC MODE :

Here E_z=0 and $H_z \neq 0$

H_z=(B_1cos kxx +B_2sin k_xx)(B_3cos kyy +B_4sin k_yy) $e^{-\gamma z}$

B_1,B_2,B_3,B_4,K_X,K_Y are found from boundary conditions.

$$(Ex = 0|for\ y = 0\ and\ y = b)$$

$$(Ey = 0|for\ x = 0\ and\ x = a)$$

At x=0 and y=0 ;

$\text{Ey} = \frac{\text{J}\omega\mu}{\text{h}^2}\frac{\partial}{\partial x}Hz - \frac{\Upsilon}{h^2}\ \frac{\partial}{\partial y}Ez$,as $\frac{\Upsilon}{h^2}\ \frac{\partial}{\partial y}Ez = 0$

So $\text{Ey} = \frac{\text{J}\omega\mu}{\text{h}^2}\frac{\partial}{\partial x}Hz$

Here

$\frac{\partial}{\partial x}Hz = [\text{B1} * \text{kx} * (-\sin kxx) + \text{B2} * \text{kx} * \cos k\ \text{xx})(\ \text{B3} \cos kyy + \text{B4} \sin k\ \text{yy}\ )\ e^{-\gamma z}]$

So $\text{Ey} = \frac{\text{J}\omega\mu}{\text{h}^2}[\text{B1} * \text{kx} * (-\sin kxx) + \text{B2} * \text{kx} * \cos k\ \text{xx})(\ \text{B3} \cos kyy + \text{B4} \sin k\ \text{yy}\ )\ e^{-\gamma z}]$

At x=0, $\frac{\partial}{\partial x}Hz = 0$

0=[B_2*K_x][(B_3cosK_yy+B_4sinK_yy)$e^{-\gamma z}$]

From this B_2=0

So , $\text{Ex} = -\frac{\text{j}\omega\mu}{\text{h}^2}\ \frac{\partial}{\partial y}Hz - \frac{\Upsilon}{h^2}\ \frac{\partial}{\partial x}Ez$ [as $\frac{\Upsilon}{h^2}\ \frac{\partial}{\partial x}Ez = 0$];

$\text{Ex} = -\frac{\text{j}\omega\mu}{\text{h}^2}\ \frac{\partial}{\partial y}Hz$

Here

$\frac{\partial}{\partial y}Hz = [\text{B1}\ (\cos kxx) + \text{B2} \sin k\ \text{xx})(-\ \text{B3} * \text{ky} * \sin kyy + \text{B4} * \text{ky} * \cos k\ \text{yy}\ )\ e^{-\gamma z}]$

$\text{Ex} = -\frac{\text{J}\omega\mu}{\text{h}^2}[\text{B1}\ (\cos kxx) + \text{B2} \sin k\ \text{xx})(-\ \text{B3} * \text{ky} * \sin kyy + \text{B4} * \text{ky} * \cos k\ \text{yy}\ )\ e^{-\gamma z}]$

$$\text{B1}\ (\cos kxx) + \text{B2} \sin k\ \text{xx})(-\ \text{B3} * \text{ky} * \sin kyy + \text{B4} * \text{ky} * \cos k\ \text{yy}\ )\ e^{-\gamma z} = 0$$

At y=0 , $\frac{\partial}{\partial y} Hz = 0$

$$[\mathrm{B1}\,(\cos kxx) + \mathrm{B2} \sin k\,\mathrm{xx})(\mathrm{B4} * \mathrm{ky}\,)e^{-\gamma z} = 0$$

From this B_4=0

So H_z= $\mathrm{B1}\,(\cos kxx) * \mathrm{B3} \cos kyy * e^{-\gamma z}$

$$\frac{\partial}{\partial x} Hz = [\mathrm{B1} * \mathrm{kx} * (-\sin kxx)\,(\,\mathrm{B3} \cos kyy\,)\,e^{-\gamma z}]$$

Here we know that at x=a,E_y=0

So E_y= $\frac{\mathrm{J}\omega\mu}{\mathrm{h}^2}\,[-\mathrm{B1} * \mathrm{kx} * (-\sin kxa) * \mathrm{B3} \cos kyy * e^{-\gamma z}]$=0

$$\sin kxa = 0 => kx = \frac{m\pi}{a}$$

At y=b,E_x=0

So E_x=$-\frac{\mathrm{J}\omega\mu}{\mathrm{h}^2}[\mathrm{B1}\,(\cos kxx)\,(\mathrm{B3} * \mathrm{ky} * \sin kyb)\,e^{-\gamma z}]$=0

So here

$$\sin kyb = 0 => ky = \frac{n\pi}{b}$$

So the general TRANSVERSE ELECTRIC MODE solution is given by

H_z=B $(\cos \frac{m\pi}{a} x)\,(\cos \frac{n\pi}{b} y)\,e^{-\gamma z}$

Where B=B_1B_3

CUT-OFF FREQUENCY :

The cut-off frequency is given as

⇨ f_c=c/2 $\left(\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right)^{1/2}$ ---------***cut off frequency equation in free space***

DOMINANT MODE :

The mode having lowest cut-off frequency or highest cut-off wavelength is called DOMINANT MODE.here TE_{00} where wave can't exist.

So $f_{c(TE01)}$=c/2b

$f_{c(TE10)}$=c/2a

for rectangular waveguide we know that a>b so TE_{10} is the dominant mode in all rectangular waveguide.

DEGENERATE MODE :

The modes having same cut-off frequency but different field equations are called degenerate modes.

WAVE IMPEDANCE :

Impedance offered by waveguide either in TE mode or TM mode when wave travels through ,it is called wave impedance.

For TE mode

$$\eta TE = \frac{\eta i}{\sqrt{(1 - \frac{fc^2}{f^2})}}$$

And

$$\eta TM = \eta i * \sqrt{(1 - \frac{fc^2}{f^2})}$$

Where ηi = intrinsic impedance = 377ohm = 120π

CYLINDRICAL WAVEGUIDES

A circular waveguide is a tubular, circular conductor. A plane wave propagating through a circular waveguide results in transverse electric (TE) or transverse magnetic field(TM) mode.

Assume the medium is lossless(α=0) and the walls of the waveguide is perfect conductor($\sigma=\infty$).

The field equations from MAXWELL'S EQUATIONS are:-

∇xE=-jωμH ------(1.a)

∇xH=jωϵE -----(1.b)

Taking the first equation,

∇xE=-jωμH

Expanding both sides of the above equation in terms of cylindrical coordinates, we get

$$\frac{1}{\rho}*\begin{vmatrix} A\rho & \rho A\varphi & Az \\ \partial/\partial\rho & \partial/\partial\varphi & \partial/\partial z \\ E\rho & \rho E\varphi & Ez \end{vmatrix} = -j\omega\mu[H\rho A\rho + H\varphi A\varphi + HzAz]$$

Equating :-

$$\frac{1}{\rho}\left\{\frac{\partial(Ez)}{\partial\varphi} - \frac{\partial(\rho E\varphi)}{\partial z}\right\} = -j\omega\mu H\rho \quad \text{------ (2.a)}$$

$$\left\{\frac{\partial(E\rho)}{\partial z} - \frac{\partial(Ez)}{\partial\rho}\right\} = -j\omega\mu H\varphi \quad \text{------ (2.b)}$$

$$\frac{1}{\rho}\left\{\frac{\partial(\rho E\varphi)}{\partial\rho} - \frac{\partial(E\rho)}{\partial\varphi}\right\} = -j\omega\mu Hz \quad \text{------- (2.c)}$$

Similarly expanding ∇xH=jωϵE,

$$\frac{1}{\rho}*\begin{vmatrix} A\rho & \rho A\varphi & Az \\ \partial/\partial\rho & \partial/\partial\varphi & \partial/\partial z \\ H\rho & \rho H\varphi & Hz \end{vmatrix} = j\omega\epsilon[E\rho A\rho + E\varphi A\varphi + EzAz]$$

Equating:

$$\frac{1}{\rho}\left\{\frac{\partial(Hz)}{\partial\varphi} - \frac{\partial(\rho H\varphi)}{\partial z}\right\} = j\omega\epsilon E\rho \quad \text{------ (3.a)}$$

$$\left\{\frac{\partial(H\rho)}{\partial z} - \frac{\partial(Hz)}{\partial\rho}\right\} = -j\omega\epsilon E\varphi \quad \text{------- (3.b)}$$

$$\frac{1}{\rho}\left\{\frac{\partial(\rho H\varphi)}{\partial\rho} - \frac{\partial(H\rho)}{\partial\varphi}\right\} = j\omega\epsilon Ez \quad \text{------- (3.c)}$$

Let us assume that the wave is propagating along z direction. So,

$$Hz = e^{-\gamma z} \;;$$

$$=> \frac{\partial Hz}{\partial z} = -\gamma e^{-\gamma z}$$

$$\boxed{\frac{\partial}{\partial z} = -\gamma}$$

Putting in equation 2 and 3:

$$\left\{\frac{\partial (Ez)}{\partial \varphi} + \gamma\rho E\varphi\right\} = -j\omega\mu\rho H\rho \quad \text{------ (4.a)}$$

$$\left\{\gamma E\rho + \frac{\partial (Ez)}{\partial \rho}\right\} = j\omega\mu\rho H\varphi \quad \text{------- (4.b)}$$

$$\left\{\frac{\partial (\rho E\varphi)}{\partial \rho} - \frac{\partial (E\rho)}{\partial \varphi}\right\} = -j\omega\mu\rho Hz \quad \text{------- (4.c)}$$

And

$$\frac{1}{\rho}\left\{\frac{\partial (Hz)}{\partial \varphi} + \gamma\rho H\varphi\right\} = j\omega\epsilon E\rho \quad \text{------ (5.a)}$$

$$\left\{\gamma H\rho + \frac{\partial (Hz)}{\partial \rho}\right\} = j\omega\epsilon E\varphi \quad \text{------- (5.b)}$$

$$\frac{1}{\rho}\left\{\frac{\partial (\rho H\varphi)}{\partial \rho} - \frac{\partial (H\rho)}{\partial \varphi}\right\} = j\omega\epsilon Ez \quad \text{------- (5.c)}$$

Now from eq(4.a) and eq(5.b),we get

$$H\rho = \frac{1}{-j\omega\mu\rho}\left\{\frac{\partial (Ez)}{\partial \varphi} + \gamma\rho E\varphi\right\}; \quad H\rho = \frac{1}{\gamma}\left[-j\omega\epsilon E\varphi - \frac{\partial (Hz)}{\partial \rho}\right]$$

$$\therefore \frac{1}{-j\omega\mu\rho}\left\{\frac{\partial (Ez)}{\partial \varphi} + \gamma\rho E\varphi\right\} = \frac{1}{\gamma}\left[-j\omega\epsilon E\varphi - \frac{\partial (Hz)}{\partial \rho}\right]$$

$$=> \frac{1}{\rho}\frac{\partial (Ez)}{\partial \varphi} + \gamma E\varphi = -\frac{\omega^2\mu\varepsilon}{\gamma}E\varphi + \frac{j\omega\mu}{\gamma}\frac{\partial Hz}{\partial \rho}$$

$$=> \left(\frac{\gamma^2+\omega^2\mu\varepsilon}{\gamma}\right)E\varphi = \frac{j\omega\mu}{\gamma}\frac{\partial Hz}{\partial \rho} - \frac{1}{\rho}\frac{\partial Hz}{\partial \rho}$$

Let $(\gamma^2 + \omega^2\mu\varepsilon) = h^2 = Kc^2$;

For lossless medium $\alpha=0; \gamma=j\beta$;

Now the final equation for $E\varphi$ is

$$E\varphi = \frac{-j}{Kc^2}\left(\frac{\beta}{\rho}\frac{\partial Ez}{\partial \varphi} - \omega\mu\frac{\partial Hz}{\partial \rho}\right) \quad \text{----- (6.a)}$$

$$H\varphi = \frac{-j}{Kc^2}\left(\omega\varepsilon\frac{\partial Ez}{\partial \rho} + \frac{\beta}{\rho}\frac{\partial Hz}{\partial \varphi}\right) \quad \text{----- (6.b)}$$

$$E\rho = \frac{-j}{Kc^2}\left(\frac{\omega\mu}{\rho}\frac{\partial Hz}{\partial \varphi} + \beta\frac{\partial Ez}{\partial \rho}\right) \quad \text{----- (6.c)}$$

$$H\rho = \frac{j}{Kc^2}\left(\frac{\omega\varepsilon}{\rho}\frac{\partial Ez}{\partial \varphi} - \beta\frac{\partial Hz}{\partial \rho}\right) \quad \text{------ (6.d)}$$

Equations (6.a),(6.b),(6.c),(6.d) are the field equations for cylindrical waveguides.

TE MODE IN CYLINDRICAL WAVEGUIDE :-

For TE mode, $E_z=0$, $H_z \neq 0$.

As the wave travels along z-direction, $e^{-\gamma z}$ is the solution along z-direction.

As, $\gamma^2 + \omega^2\mu\varepsilon = h^2$;

- ⇨ $-\beta^2+\omega^2\mu\epsilon = Kc^2$;
- ⇨ $-\beta^2+K^2 = Kc^2$ (as $K^2 = \omega^2\mu\varepsilon$)

According to maxwell's equation, the laplacian of H_z :

$$\nabla^2 H_z = -\omega^2\mu\varepsilon H_z;$$

- ⇨ $\nabla^2 H_z + \omega^2\mu\varepsilon H_z = 0$;
- ⇨ $\nabla^2 H_z + K^2 H_z = 0$

Expanding the above equation, we get:

$$\frac{\partial^2 Hz}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial Hz}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 HZ}{\partial \varphi^2} + \frac{\partial^2 Hz}{\partial z^2} + K^2 H_z = 0;$$

Now, $H_Z = e^{-\gamma z}$; $\frac{\partial^2 Hz}{\partial z^2} = (-\gamma)^2\ e^{-\gamma z}$; $\frac{\partial^2 Hz}{\partial z^2} = -\beta^2\ e^{-\gamma z}$;

$$\frac{\partial^2 Hz}{\partial z^2} = -\beta^2 Hz \quad => \frac{\partial^2}{\partial z^2} = -\beta^2;$$

Putting this value in the above equations, we get:-

$$\frac{\partial^2 Hz}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial Hz}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 HZ}{\partial \varphi^2} - \beta^2 Hz + K^2 H_z = 0;$$

$$\frac{\partial^2 Hz}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial Hz}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 HZ}{\partial \varphi^2} + Kc^2 H_z = 0; \quad [\text{as } -\beta^2+K^2 = Kc^2\ ]$$

$$\frac{\partial^2 Hz}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial Hz}{\partial\rho}+Kc^2H_z = -\frac{1}{\rho^2}\frac{\partial^2 HZ}{\partial\varphi^2};$$

The partial differential with respect to ρ and φ in the above equation are equal only when the individuals are constant (Let it be K_o^2).

⇨ $$-\frac{1}{\rho^2}\frac{\partial^2 HZ}{\partial\varphi^2} = K_o^2$$

⇨ $$\frac{\partial^2 HZ}{\partial\varphi^2}+\rho^2 K_o^2=0$$

Solutions to the above differential equation is:-

Hz = $B_1\sin(K_o\varphi)+B_2\cos(K_o\varphi)$-----{solution along φ direction}.

Now,

$$\frac{\partial^2 Hz}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial Hz}{\partial\rho}+(Kc^2H_z - K_o^2)= 0$$

This equation is similar to BESSEL'S EQUATION,so the solution of this equation is

Hz=CnJn(K_c ρ) --------{solution along ρ-direction}

Hence,

$Hz=Hz(\rho)Hz(\varphi)e^{-\gamma z}$;

So, the final solution is,

$$\boxed{Hz= CnJn(K_c\,\rho)[\,B_1\sin(K_o\varphi)+B_2\cos(K_o\varphi)]\,e^{-\gamma z}}$$

Applying boundary conditions:-

At ρ=a, Eφ=0 $\Rightarrow \frac{\partial Hz}{\partial\rho}=0$,

=> $J_n(K_c\,\rho)=0$

=> $J_n(K_c\,a)=0$

If the roots of above equation are defined as P_{mn}',then

$$K_c=\frac{Pmn'}{a};$$

∴ $$\boxed{Hz= CnJn\left(\frac{Pmn'}{a}\rho\right)[\,B_1\sin(K_o\varphi)+B_2\cos(K_o\varphi)]\,e^{-\gamma z}}$$

<u>CUT-OFF FREQUENCY</u> :- It is the minimum frequency after which the propagation occurs inside the cavity.

∴ γ=0;

But we know that $\gamma^2 + \omega^2\mu\varepsilon = Kc^2$;

- ⇨ $\omega^2\mu\varepsilon = Kc^2$
- ⇨ $\omega^2 = \frac{Kc^2}{\mu\varepsilon}$
- ⇨ $2\pi f_c = \sqrt{\frac{Kc^2}{\mu\varepsilon}}$
- ⇨ $f_c = \frac{1}{2\pi}\sqrt{\frac{Kc^2}{\mu\varepsilon}}$

$$\therefore \quad f_c = \frac{Pmn'}{2\pi a\sqrt{\mu\varepsilon}}$$

CUTOFF WAVELENGTH :- $\frac{c}{fc} = \frac{\frac{1}{\sqrt{\mu\varepsilon}}}{\frac{Pmn'}{2\pi a\sqrt{\mu\varepsilon}}} = \frac{2\pi a}{Pmn'}$

The experimental values of Pmn' are:-

n \ m	1	2	3
0	3.832	7.016	10.173
1	1.841	5.331	8.536
2	3.054	6.706	9.969
3	3.054	6.706	9.970

As seen from this table, TE_{11} mode has the lowest cut off frequency, hence **TE_{11}** is the dominating mode.

WAVE IMPEDANCE(Z_{TE}) $= \frac{\eta k}{\beta}$

TM MODE IN CYLINDRICAL WAVEGUIDE :-

For TM mode Hz=0,Ez≠0.

From the calculation in TE mode, we got the equation as,

$\frac{\partial^2 Ez}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial Ez}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 Ez}{\partial \varphi^2} + Kc^2 E_z = 0$; -------(2)

⇨ $\frac{\partial^2 Ez}{\partial \varphi^2} + \rho^2 K_1^2 = 0$ (let k_1 be a constant)

So, the general solution of the above equation is

$E_z = C_1 \sin(k_1\varphi) + C_2 \cos(k_1\varphi)$

Applying the boundary value conditions,i.e at $\rho=a, Ez=0$.

We get $J_n(K_c\, a)=0$;

So,if P_{mn} is the root of the above equation, then

$$K_c = \frac{Pmn}{a}$$

The experimental values of Pmn' are:-

n \ m	1	2	3
0	2.405	5.520	8.654
1	3.832	7.016	10.174
2	5.135	8.417	11.620

So, we can conclude that the TE_{01} having a value of 2.405 is the lowest one. But since this value is greater than the TE_{11} mode, so mode **TE_{11}** is the dominant mode for the cylindrical TM modes. Here $m \geq 1$,so there is no TM_{10} mode.

CUTOFF FREQUENCY:

The cut off frequency is given by

$$f_c = \frac{Pmn}{2\pi a\sqrt{\mu\varepsilon}}$$

CUTOFF WAVELENGTH:-

The cut off wavelength is given by

$$\frac{c}{fc} = \frac{\frac{1}{\sqrt{\mu\varepsilon}}}{\frac{Pmn}{2\pi a\sqrt{\mu\varepsilon}}} = \frac{2\pi a}{Pmn}$$

WAVE IMPEDANCE :-

The wave impedance is given by $\frac{\eta k}{\beta}$.

MICROWAVE COMPONENTS

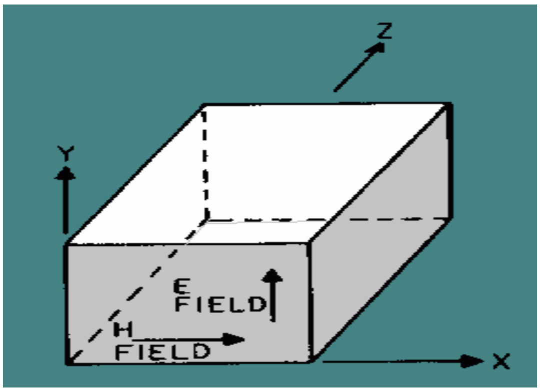
MICROWAVE RESONATOR:

- They are used in many applications such as oscillators, filters, frequency meters, tuned amplifiers and the like.
- A microwave resonator is a metallic enclosure that confines electromagnetic energy and stores it inside a cavity that determines its equivalent capacitance and inductance and from the energy dissipated due to finite conductive walls we can determine the equivalent resistance.
- The resonator has finite number of resonating modes and each mode corresponds to a particular resonant frequency.
- When the frequency of input signal equals to the resonant frequency, maximum amplitude of standing wave occurs and the peak energy stored in the electric and magnetic field are calculated.

RECTANGULAR WAVEGUIDE CAVITY RESONATOR:

- Resonator can be constructed from closed section of waveguide by shorting both ends thus forming a closed box or cavity which store the electromagnetic energy and the power can be dissipated in the metallic walls as well as the dielectric medium

DIAGRAM:



- The geometry of rectangular cavity resonator spreads as

$$0 \leq x \leq a;$$

$$0 \leq y \leq b;$$

$$0 \leq z \leq d$$

- Hence the expression for cut-off frequency will be

$$\omega_0^2 \mu \epsilon = (m\pi/a)^2 + (n\pi/b)^2 + (l\pi/d)^2$$

or $$\omega_0 = \frac{1}{\sqrt{\mu\epsilon}} [(m\pi/a)^2 + (n\pi/b)^2 + (l\pi/d)^2]^{1/2}$$

or $$f_0 = \frac{1}{2\pi\sqrt{\mu\epsilon}} [(m\pi/a)^2 + (n\pi/b)^2 + (l\pi/d)^2]^{1/2}$$

or $$f_0 = \frac{c'}{2} [(m\pi/a)^2 + (n\pi/b)^2 + (l\pi/d)^2]^{1/2}$$

- This is the expression for resonant frequency of cavity resonator.
- The mode having lowest resonant frequency is called DOMINANT MODE and for TE AND TM the dominant modes are TE-101 and TM-110 respectively.

QUALITY FACTOR OF CAVITY RESONATOR:

- $Q=2\pi \times \frac{\text{maximum energy stored per cycle}}{\text{energy dissipated per cycle}}$

FACTORS AFFECTING THE QUALITY FACTOR:

Quality factor depends upon 2 factors:

- Lossy conducting walls
- Lossy dielectric medium of a waveguide

1) LOSSY CONDUCTING WALL:

- The Q-factor of a cavity with lossy conducting walls but lossless dielectric medium i.e. $\sigma_c \neq \infty$ and $\sigma = 0$

Then $Q_c = (2\omega_0 W_e/P_c)$

Where w_0-resonant frequency

W_e-stored electrical energy

P_c-power loss in conducting walls

- From dimensional point of view:

$$Q_c = (kad^3)(b\eta) \times \{ 1/(2l^2a^3d + l^3a^3d + ad^3 + 2bd^3) \} /(2\pi^2R_s)$$

Where $k = (\omega^2\mu\epsilon)^{1/2}$

$$\eta = \eta_i/\sqrt{\epsilon_r} = 377/\sqrt{\epsilon_r}$$

2) LOSSY DIELECTRIC MEDIUM:

The Q-factor of a cavity with lossy dielectric medium but lossless conducting walls

i.e. $\sigma_c=\infty$ and $\sigma\neq0$

$Q_d = 2\omega_0 W_e / P_d = (1/\tan\delta)$

Where $\tan\delta = \frac{\sigma}{\omega\epsilon}$

P_d=power loss in dielectric medium

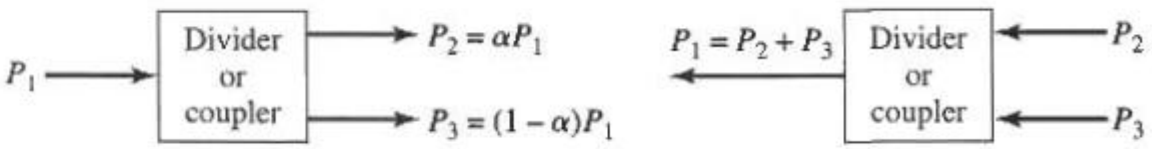
- When both the conducting walls and the dielectric medium are lossy in nature then

Total power loss = $P_c + P_d$

$$1/Q_{total} = 1/Q_c + 1/Q_d$$

$$\text{or } Q_{total} = 1/(1/Q_c + 1/Q_d)$$

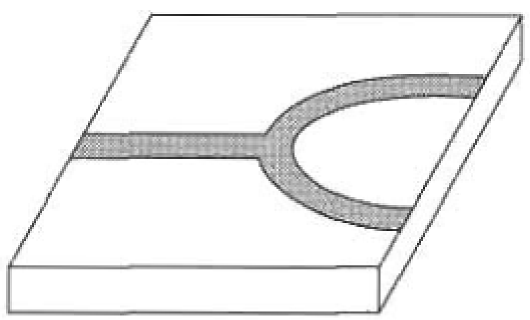
POWER DIVIDERS:



- Power dividers and combiners are the passive microwave components that are used for power division and combination in microwave frequency range
- In this case, the input power is divided into two or more signals of lesser power.
- The power divider has certain basic parameters like isolation, coupling factor and directivity.

T-JUNCTION POWER DIVIDER USING WAVEGUIDE:

The T-junction power divider is a 3-port network that can be constructed either from a transmission line or from the waveguide depending upon the frequency of operation.

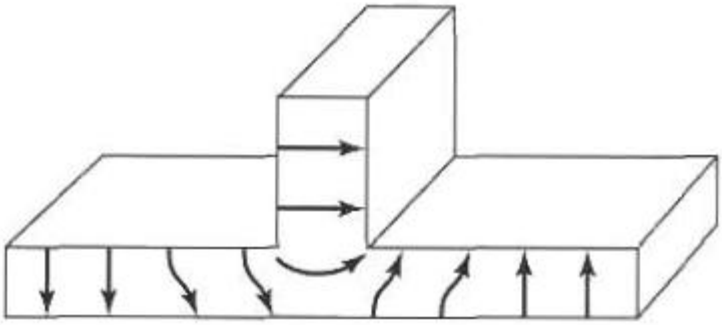


For very high frequency, power divider using waveguide is of 4 types

- ⇨ E-Plane Tee
- ⇨ H-Plane Tee
- ⇨ E-H Plane Tee/Magic Tee
- ⇨ Rat Race Tee

E-PLANE TEE:

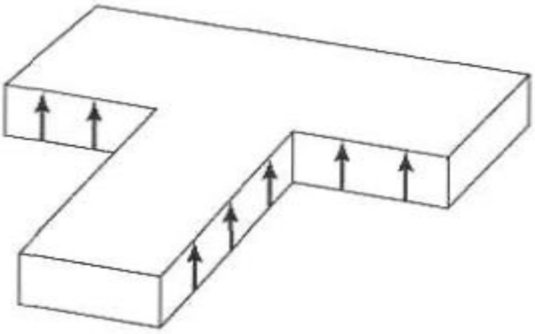
Diagram



- It can be constructed by making a rectangular slot along the wide dimension of the main waveguide and inserting another auxiliary waveguide along the direction so that it becomes a 3-port network.
- Port-1 and Port-2 are called collinear ports and Port-3 is called the E-arm.
- E-arm is parallel to the electric field of the main waveguide.
- If the wave is entering into the junction from E-arm it splits or gets divided into Port-1 and Port-2 with equal magnitude but opposite in phase
- If the wave is entering through Port-1 and Port-2 then the resulting field through Port-3 is proportional to the difference between the instantaneous field from Port-1 and Port-2

H-PLANE TEE:

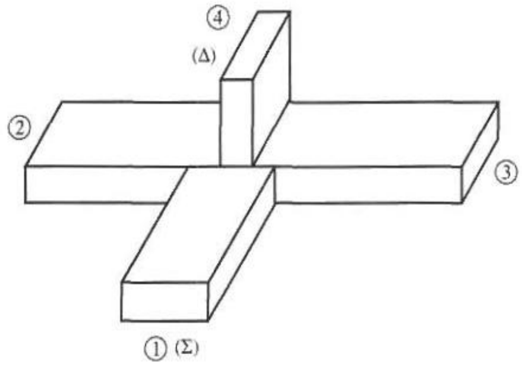
Diagram:



- An H-plane tee is formed by making a rectangular slot along the width of the main waveguide and inserting an auxiliary waveguide along this direction.
- In this case, the axis of the H-arm is parallel to the plane of the main waveguide.
- The wave entering through H-arm splits up through Port-1 and Port-2 with equal magnitude and same phase
- If the wave enters through Port-1 and Port-2 then the power through Port-3 is the phasor sum of those at Port-1 and Port-2.
- E-Plane tee is called PHASE DELAY and H-Plane tee is called PHASE ADVANCE.

E-H PLANE TEE/MAGIC TEE:

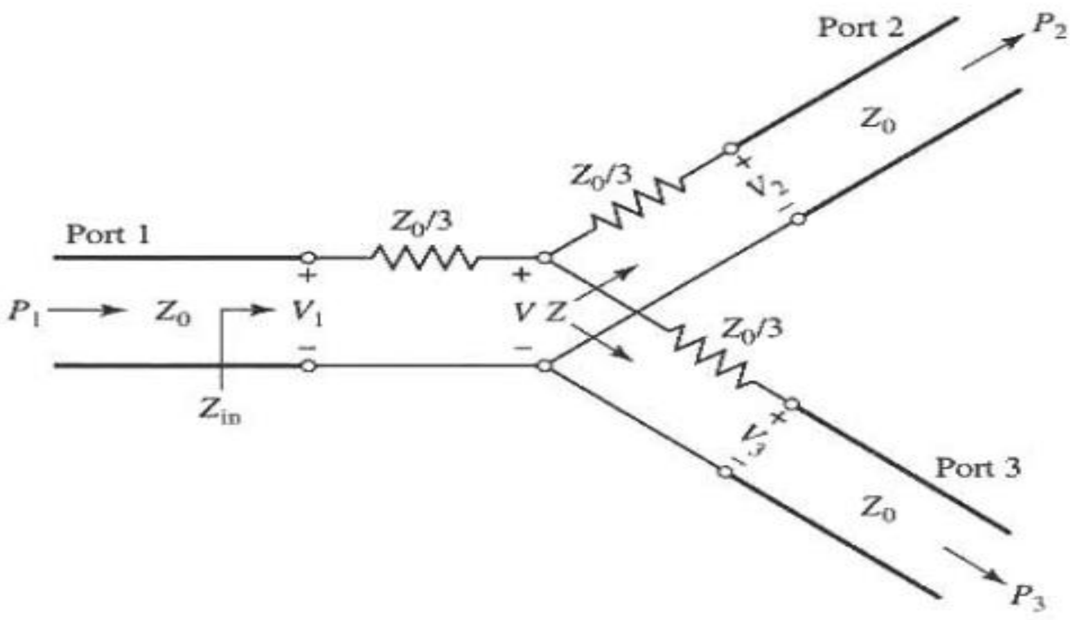
Diagram:



- It is a combination of E-Plane tee and H-Plane tee.
- If two waves of equal magnitude and the same phase are fed into Port-1 and Port-2, the output will be zero at Port-3 and additive at Port-4.
- If a wave is fed into Port-4 (H-arm) then it will be divided equally between Port-1 and Port-2 of collinear arms (same in phase) and will not appear at Port-3 or E-arm.
- If a wave is fed in Port-3 then it will produce an output of equal magnitude and opposite phase at Port-1 and Port-2 and the output at Port-4 will be zero.
- If a wave is fed in any one of the collinear arms at Port-1 or Port-2, it will not appear in the other collinear arm because the E-arm causes a phase delay and the H-arm causes phase advance.

T-JUNCTION POWER DIVIDER USING TRANSMISSION LINE:

Diagram:



- It is a junction of 3 transmission lines

- In this case, if P1 is the input port power then P2 and P3 are the power of output Port-2 and Port-3 respectively.
- To transfer maximum power from port-1 to port-2 and port-3 the impedance must match at the junction.

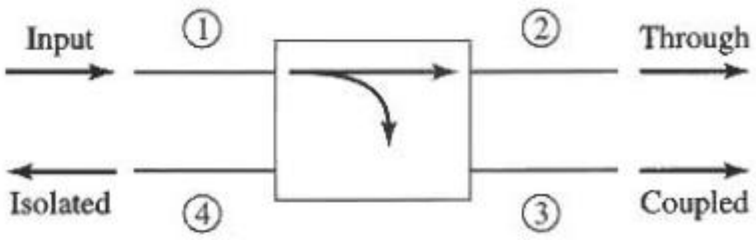
⇨ For maximum power transfer

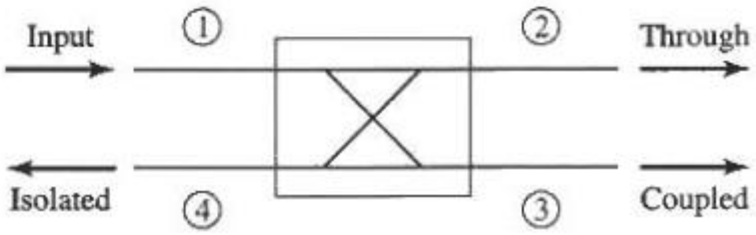
$Z_{01} = Z_{02} \parallel Z_{03}$

⇨ $1/Z_{01} = 1/Z_{02} + 1/Z_{03}$ (Condition for lossless power division)

DIRECTIONAL COUPLER:

Diagram:





- It is a 4- port waveguide junction consisting of a primary waveguide 1-2 and a secondary waveguide 3-4.
- When all the ports are terminated in their characteristic impedance there is free transmission of power without reflection between port-1 and port-2 and no power transmission takes place between port-1 and port-3 or port-2 and port-4 a sno coupling exists.
- The characteristic of a directional coupler is expressed in terms of its coupling factor and directivity.
- The coupling factor is the measure of ratio of power levels in primary and secondary lines.
- Directivity is the measure of how well the forward travelling wave in the primary waveguide couples only to a specific port of the secondary waveguide.
- In ideal case, directivity is infinite i.e. power at port-3 =0 because port-2 and port-4 are perfectly matched.
- Let wave propagates from port-1 to port-2 in primary line then:

Coupling factor (dB) =10 $\log_{10}$ (P1/P4)

Directivity (dB) = 10 $\log_{10}$ (P4/P3)

Where P1=power input to port-1

P3=power output from port-3 and P4=power output from port-4

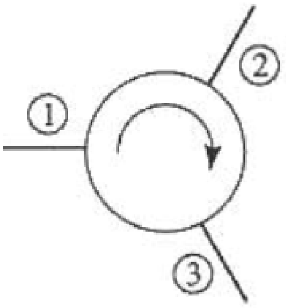
CIRCULATORS AND ISOLATORS:

- Both microwave circulators and microwave isolators are non-reciprocal transmission devices that use Faraday rotation in the ferrite material.

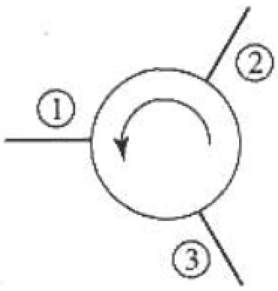
CIRCULATOR:

- A microwave circulator is a multiport waveguide junction in which the wave can flow only in one direction i.e. from the nth port to the (n+1)th port.
- It has no restriction on the number of ports
- 4-port microwave circulator is most common.
- One of its types is a combination of two 3-dB side hole directional couplers and a rectangular waveguide with two non reciprocal phase shifters.

Diagram:



$$[S] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$



$$[S] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

- Each of the two 3db couplers introduce phase shift of 90 degrees
- Each of the two phase shifters produce a fixed phase change in a certain direction.
- Wave incident to port-1 splits into 2 components by coupler-1.
- The wave in primary guide arrives at port-2 with 180 degrees phase shift.
- The second wave propagates through two couplers and secondary guide and arrives at port-2 with a relative phase shift of 180 degrees.
- But at port-4 the wave travelling through primary guide phase shifter and coupler-2 arrives with 270 degrees phase change.
- Wave from coupler-1 and secondary guide arrives at port-4 with phase shift of 90 degrees.

- Power transmission from port-1 to port-4 =0 as the two waves reaching at port-4 are out of phase by 180 degrees.

$$\mathbf{w_1\text{-}w_3 = (2m+1)\ \pi\ rad/s}$$

$$\mathbf{w_2\text{-}w_4 = 2n\pi\ rad/s}$$

Power flow sequence: 1-> 2 -> 3 -> 4-> 1

MICROWAVE ISOLATOR:

- A non reciprocal transmission device used to isolate one component from reflections of other components in the transmission line.
- Ideally complete absorption of power takes place in one direction and lossless transmission is provided in the opposite direction
- Also called UNILINE, it is used to improve the frequency stability of microwave generators like klystrons and magnetrons in which reflections from the load affects the generated frequency.
- It can be made by terminating ports 3 and 4 of a 4-port circulator with matched loads.
- Additionally it can be made by inserting a ferrite rod along the axis of a rectangular waveguide.

REFLEX KLYSTRON

The reflex klystron is a single cavity variable frequency time-base generator of low power and load effiency

APPLICATION:

- It is widely used as in radar receiver
- Local oscillators in microwave receiver
- Portable microwave rings
- Pump oscillator in parametric amplifier

CONSTRUCTION:

- Reflex cavity klystron consists of an electron gun , filament surrounded by cathode and a floating electron at cathode potential
- Electron gun emits electron with constant velocity

$$\frac{1}{2}\text{m}v^2\text{=qVa}$$

$$\text{V=}\sqrt{\frac{2qVa}{m}}\ \text{m/s}$$

OPERATION:

- The electron that are emitted from cathode with constant velocity enter the cavity where the velocity of electrons is changed or modified depending upon the cavity voltage.
- The oscillations is started by the device due to high quality factor and to make it sustained we have to apply the feedback.

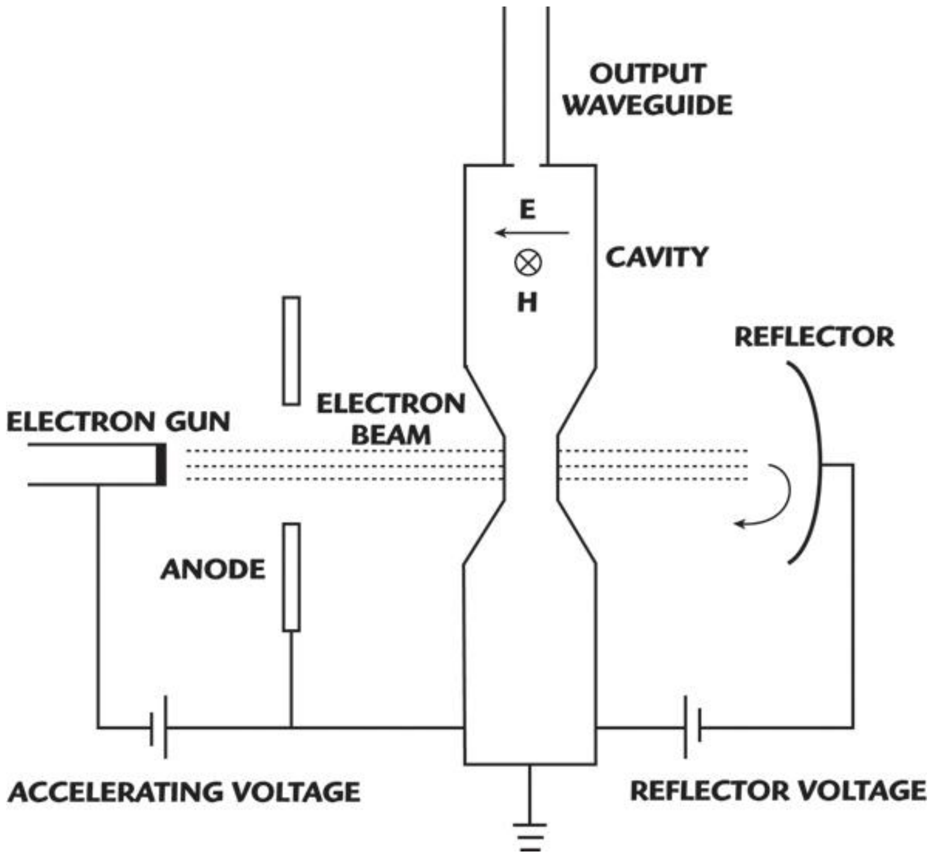
  Hence there are the electrons which will bunch together to deliver the energy act a time to the RF signal.

  - Inside the cavity velocity modulation takes place. Velocity modulation is the process in which the velocity of the emitted electrons are modified or change with respect to cavity voltage. The exit velocity or velocity of the electrons after the cavity is given as

$$\text{V'=}\ \sqrt{\frac{2q\left(Va+Vi\sin\left(\omega t\right)\right)}{m}}$$

  - In the cavity gap the electrons beams get velocity modulated and get bunched to the drift space existing between cavity and repellar.
  - Bunching is a process by which the electrons take the energy from the cavity at a different time and deliver to the cavity at the same time.

Diagram:



Bunching continuously takes place for every negative going half cycle and the most appropriate time for the electrons to return back to the cavity ,when the cavity has positive peak .So that it can give maximum retardation force to electron.

- It is found that when the electrons return to the cavity in the second positive peak that is 1 whole ¾ cycle.(n=1π).It is obtained max power and hence it is called dominant mode.

The electrons are emitted from cathode with constant anode voltage Va, hence the initial entrance velocity of electrons is

$$V=\sqrt{\frac{2qVa}{m}}\ \text{m/s}$$

Inside the cavity the velocity is modulated by the cavity voltage Visin(ωt) as,

$$V=\sqrt{\frac{2q(Va+Vi\sin(\omega t))}{m}}$$

$$V=\sqrt{\frac{2qVa}{m}+\frac{2qVaKaVi\ \sin\ \omega t}{Vam}}$$

$$V=\sqrt{Vo^2+\frac{Vo^2KiVi\ \sin\ \omega t}{Va}}$$

$$V= Vo\sqrt{1+\frac{KiVi\ \sin\omega t}{Va}}$$

$$V= \sqrt{1+\frac{Vo^2 iVi \sin\omega t}{Va}}$$

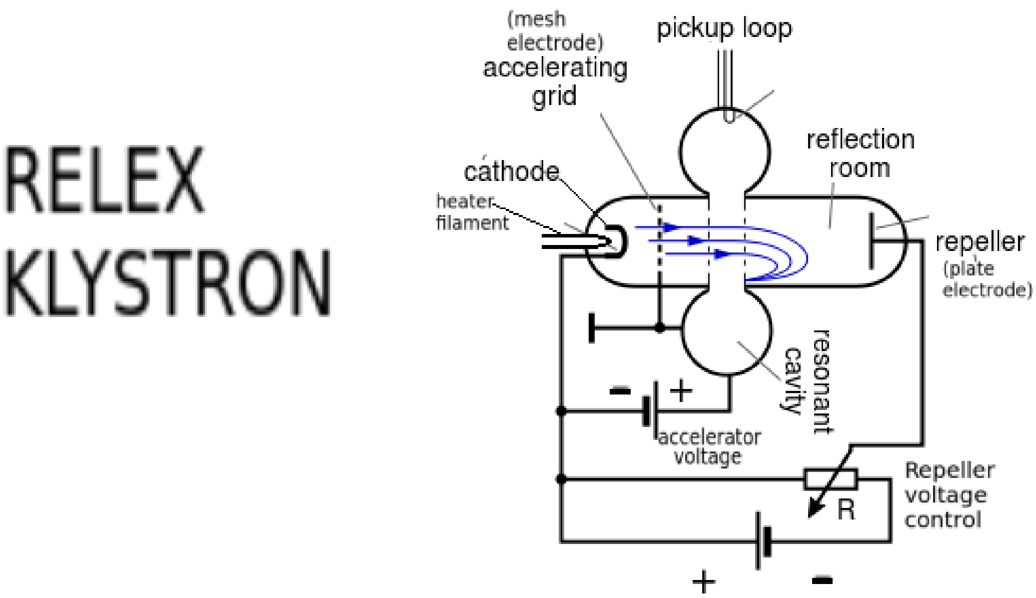
When, $\frac{kivi}{Va}$ =Depth of velocity

$$V=Vo\ (1+\frac{Vo^2 iVi \sin\omega t}{2Va})$$

<u>TRANSIENT TIME:</u>

Transit time is defined as the time spent by the electrons in the cavity space or, time taken by the electrons to leave the cavity and again return to the cavity.

(FIGURE)



If t_1 is the time at which electrons leave the cavity and t_2 is the time at which electrons bunch in the cavity then, transit time

$$t_{r=}t_1 - t_2$$

During this time the net displacement by electrons is zero. That the potential of two point A and B is VA and VB(plate) as known in figure,then,

$$V_{AB} = V_A - V_B$$

$$= V_A + V_i \sin wt + VR$$

$$=V_A + VR + V_i \sin wt$$

Neglecting the AC compoment,

$$V_{AB} = V_a + V_R$$

$$E = \frac{\partial V_{AB}}{\partial x} = \frac{-V_{AB}}{S} = \frac{-(V_a+V_R)}{S} \quad \text{-------(a)}$$

The force experienced on an electron

$F=qe=\frac{-q\,(V_a+V_R)}{S}$

$F=\frac{m\,\partial^2 x}{\partial t^2}$ -------(b)

From equations a and b we get

$\frac{m\,\partial^2 x}{\partial t^2} = \frac{-q\,(V_a+V_R)}{S}$

Integrating both sides we get,

$\int \frac{m\,\partial^2 x}{\partial t^2} = \frac{-q\,(V_a+V_R)}{S}$

$\int m\,\partial x^2 = \frac{-q\,(V_a+V_R)}{S} \begin{cases} t \\ t1 \end{cases} \partial t$

$m\frac{\partial x}{\partial t} = \frac{-q\,(V_a+V_R)}{S}\ {}_{t1}^{t}t$

$\frac{\partial x}{\partial t} = \frac{-q\,(V_a+V_R)}{mS}\,(t-t_1) + k_1$

$\therefore k_1$ is called velocity constant and assumed to velocity at (t-t_1)

$\frac{\partial x}{\partial t} = \frac{-q\,(V_a+V_R)}{mS}\,(t-t_1) + V(t1)$

$$\int \partial x = \frac{-q(V_a+V_R)}{mS} \int (t - t_1)\,\partial t + V(t1)$$

$X=\frac{-q\,(V_a+V_R)}{mS}\ {}_{t1}^{t}[\frac{t^2}{2}] - t_1\ {}_{t1}^{t}[t] + \int V(t1)\ \partial t$

$=\frac{-q\,(V_a+V_R)}{mS}\,[\left[\frac{t^2-t1^2}{2}\right] - (t_2 - t_2)t_1] + \int V(t1)\ \partial t$

$= \frac{-q\,(V_a+V_R)}{mS}\left[\frac{t^2-t1^2-2t_2t_1+2t_2{}^2}{2}\right] + \int V(t1)\ \partial t$

$=\frac{-q(V_a+V_R)}{mS}\left[\frac{t^2+t1^2-2t_2t_1}{2}\right]+\int V(t1)\ \partial t$

$=\frac{-q(V_a+V_R)}{mS}\left[\frac{(t-t1)^2}{2}\right]+\int V(t1)\ \partial t$

$=\frac{-q(V_a+V_R)}{mS}\left[\frac{(t-t1)^2}{2}\right]+\int V(t1)\ (t-t1)+k_2$

Where,k_2 is displacement constant at $t=t_2$,x=0 .In practice k_2=the cavity width which is negligible with respect to cavity space s.Here we can neglect k_2 in the expression of x

At $t=t_2$,x=0

$0=\frac{-q(V_a+V_R)}{mS}\left[\frac{(t-t1)^2}{2}\right]+V(t1)\ (t-t1)$

- $\frac{-q(V_a+V_R)}{mS}\left[\frac{(t_2-t1)^2}{2}\right]=\ V(t1)\ (t_2-t1)$
- $(t_2-t1)=\frac{2V(t1)ms}{q(V_a+V_R)}$

TRANSIT ANGLE:

$$\omega t_r\ =\omega(t_2-t1)=\omega\frac{2V(t1)ms}{q(V_a+V_R)}$$

$$\omega t_r\ =\ \omega\frac{2V(t1)ms}{q(V_a+V_R)}$$

We know,

n=+3/4, for n=0,1,2,3,4…..

n-1/4. for n=0,1,2,3,4…..

$2\times\pi(t_2-t1)=\left(n-\frac{1}{4}\right)2\pi\times T$

$2\times\pi\times f(t_2-t1)=\left(2n\pi-\frac{\pi}{2}\right)$

$$\omega t_r\ =\left(2n\pi-\frac{\pi}{2}\right)=\ \omega\frac{2V(t1)ms}{q(V_a+V_R)}$$

OUTPUT POWER:

The beam current of Reflex klystron is given as

$Ib=I_O+\sum_{n=1}^{\infty}(2I_0I_n(x')\cos(n\omega t-\varphi)$

$I_0$is dc current due to cavity voltage is given by

$P_{dc=}v_aI_o$ -------(1)

The ac component of the current is given by

$I_{ac}= \sum_{n=1}^{\infty}(2I_0 I_n(x') \cos(n\omega t - \varphi)$

For (n=1),we have fundamental current component ie,

If $(2I_0 I_1(x') \cos(n\omega t - \varphi)$

For n=2; $\cos(n\omega t - \varphi) = 1$

$I_2=2I_o K_i J_i(X_i)$

Power= $\frac{v_i I_2}{2}= \frac{v_i}{2}(2I_o k_i Ji(Xi))$

$$\omega(t_2 - t1) = \omega \frac{2V(t1)ms}{q(V_{a+}V_R)}$$

$\omega t_2= \omega t_1+\frac{2V_0 ms\omega}{q(V_{a+}V_R)}[1 + \frac{k_i v_i}{2v_a}\sin \omega t_1]$

$\therefore [\frac{2V_0 ms\omega}{q(V_{a+}V_R)}] = \alpha$

→ $\omega t_2= \omega t_1+\alpha[1 + \frac{k_i v_i}{2v_a}\sin \omega t_1]$

→ $\omega t_2= \omega t_1+\alpha + \frac{k_i v_i \alpha}{2v_a}\sin \omega t_1]$

Where , $\frac{k_i v_i \alpha}{2v_a}$=x, bunching parameter

$V_i=\frac{2v_a x}{\alpha k_i} = \frac{2v_a x}{(2n\pi-\frac{\pi}{2})k_i}$

$P_{o/p}=\frac{2v_a x I_0 \times J_i(x)}{(2n\pi-\frac{\pi}{2})}$

Efficiency: $\eta\% = \frac{P_{output}}{P_{input}} \times 100\%$

$=\frac{2v_a x I_0 \times J_i(x)}{(2n\pi-\frac{\pi}{2})v_a x I_0}$

$=\frac{2\times J_i(x)}{(2n\pi-\frac{\pi}{2})} \times 100\%$

Electronics admittance of reflex klystron

It is defined as the ratio of current induced in the cavity by the modulation of electron beam to the voltage across the cavity gap.

$Y_e = \frac{I_2}{v_2}$,

- $I_2 = 2I_o k_i J_i(x') \cos(\omega t - \varphi)$
- $= 2I_o k_i J_i e^{-j\varphi}$
- $V_{2=} V_1 e^{-\pi/2} = \frac{2\, v_{ax'}}{\alpha k_i} e^{-j\pi/2}$
- $Y_e = \frac{\alpha I o k i^2}{v_a} \frac{J_{i(x)'} \sin\varphi}{x'} + j \frac{\alpha I o k i^2}{v_a} \frac{J_{i(x)'} \cos\varphi}{x'}$

  $Y_e = G_e + j\beta_e$

MAGNETRON

DIFFERENCE BETWEEN REFLEX KLYSTRON AND MAGNETRON:

REFLEX KLYSTRON	MAGNETRON
❖ It is a linear tube in which the magnetic field is applied to focus the electron and electric field is applied to drift the electron. ❖ In klystron the bunching takes places only inside the cavity which is very small ,hence generate low power and low frequency.	❖ In magnetron the magnetic field and electric field are perpendicular to each other hence it is called as cross field device. ❖ In magnetron the interacting or bunching space is extended so the efficiency can be increase.

APPLICATION:

- Used as oscillator.
- Used in radar communication.
- Used in missiles.
- Used in microwave oven (in the range of frequency of 2.5Ghz).

Types of magnetron:

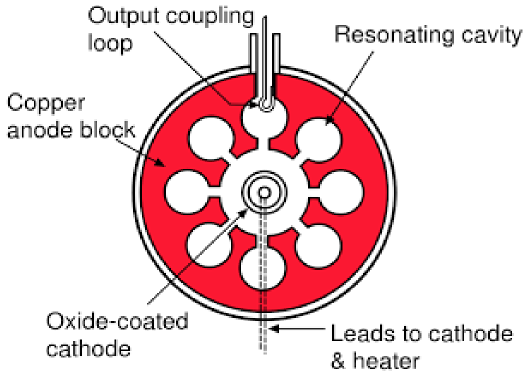
Magnetron is of 3 types:

- ✓ Negative resistance type.
- ✓ Cyclotron frequency type.
- ✓ Cavity type.

Construction of cavity magnetron:

Cavity type magnetron depends upon the interaction of electron with a rotating magnetic field with constant angular velocity.

FIGURE:



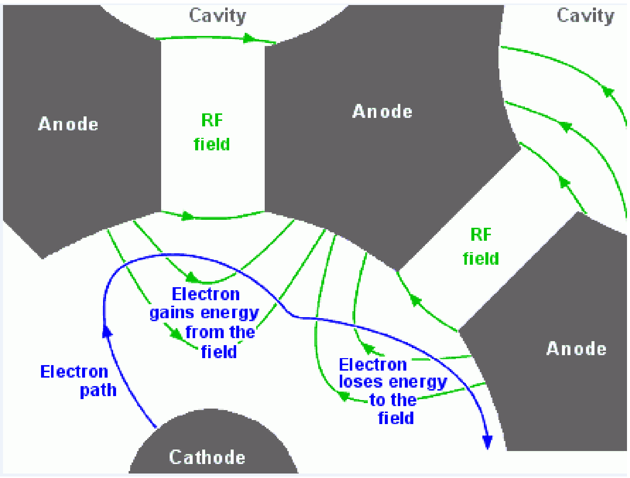
Resonant cavity magnetron high-power high-frequency oscillator

A magnetron consists of a cathode which is used to emit electrons and a number of anode cavities a permanent magnet is placed on the backside of cathode. The space between anode cavity and cathode is called interacting space. The electron which are emitted from cathode moves in different path in the interacting space depending upon the strength of electron and magnetic field applied to the magnetron.

OPERATION:

EFFECT OF ELECTRIC FIELD ONLY:

- In the absence of magnetic field(B=0) the electron travel straight from the cathode to the anode due to the radial electric field force acting on it(indicated by path A).
- If the magnetic field strength increases slightly it will exert a lateral force which bends the path of the as indicated in path B.



- The radius of the path is given as

$$r=\frac{mv}{eB}$$

where v=velocity of electron

B=magnetic field strength

- If from reaching the anode current become zero(indicated by path D).the strength of magnetic field is made sufficiently high enough, so to prevent the electron
- The magnetic field required to return the electron back to the cathode just touching the surface of anode is called critical magnetic field or cut off magnetic field(Bc).
- If B>Bc the electron experiences a grater rotational force and may return back to the cathode quite faster this results is heating of cathode.

Effect of magnetic field:

The force experience by the electron because of magnetic field only.

$$F= q(v\times B)$$

= qvBsinθ

- For maximum force θ=90°

  Fmax = qvB
- And hence the electron which are emitted, moved in a right angle with respect to force.
- If the magnetic field strength is sufficiently large enough, then the electrons emitted will return back to the cathode with high velocity which may destroy the cathode this effect is called Back heating of cathode.

π MODE OF OSCILLATION:

- The shape consisting of oscillation can maintain if the phase difference between anode cavity is $\frac{n\pi}{4}$ where n is the mode of operation and the best result can be obtained for n=4 => $\frac{n\pi}{4}=\pi$ (for n=4 hence it is called π mode operation).
- It is assume that each anode cavity is of $\frac{\lambda}{4}$ length, hence a voltage antinodes will exist at the opening of anode cavity and the lines of forces present due to the oscillation started by high quality factor device.
- In the above figure the electron followed by path 'b' is so emitted that is not influenced by the electric lines of forces hence it will spend very less time inside the cavity and doesn't contribute to the oscillation so it is called unfavourable electron.
- The electron followed by path a is so emitted that is influence by the electric lines of forces at position 1,2&3 respectively where the velocity increases or decreases, hence more time spend inside the cavity therefore it is called favourable electron.
- Any favourable electron which are emitted earlier or later with respect to reference electron (let a) may be bunch together due to change in velocity by the effect of electric lines of forces. This type of bunching is called phase focusing effect.
- Electrons emitted from the cathode may rotate around itself in a confined area in a shape of spoke (spiral) at a angular velocity and before delivering the energy to the anode cavity. They will rotate until they reach the anode and completely absorbed by them. Hence the magnetron are also called travelling wave magnetron.

CUT OFF MAGNETIC FIELD (B_C):

- Assume a cylindrical magnetron whose inner radius is 'a' and outer radius is 'b' and the magnetic field is as shown in the figure. Under the effect of magnetic field the electrons will rotate in a circular path at any point the force electron will be balance by the centrifugal force.

$$\frac{mv^2}{r} = qvB_z$$

$$\Rightarrow v = \frac{q\mathrm{Bz}}{m}r$$

$$\Rightarrow \frac{v}{r} = \frac{q\mathrm{Bz}}{m}$$

$$\Rightarrow \omega = \frac{q\mathrm{Bz}}{m}$$

$$\Rightarrow f = \frac{1}{2\pi}\left(\frac{q\mathrm{Bz}}{m}\right)$$

In cylindrical coordinate system the equation of motion is given as

$$=> \frac{1}{\partial}\frac{d}{dt}(\gamma^2\frac{d\theta}{dt})=\omega\frac{dr}{dt}$$

$$=>\int \frac{d}{dt}(r^2\frac{d\theta}{dt})=w\int \frac{dr}{dt}.r$$

$$=> r^2 (\frac{d\theta}{dt}) =\frac{\omega r^2}{2}+k$$

The constant k can be calculated using boundary condition.

At r=a, $\frac{d\theta}{dt}=0$

- ⇨ $0=\frac{\omega a^2}{2} + k$
- ⇨ $K=-\frac{\omega a^2}{2}$...........(1)

The value of k in equation (1)

- ⇨ $r^2\frac{d\theta}{dt} = \frac{\omega}{2}(r^2 - a^2)$..............(2)

At r=b the equation (2) becomes

- ⇨ $b^2\frac{d\theta}{dt}=\frac{\omega}{2}(b^2 - a^2)$
- ⇨ $b\frac{d\theta}{dt}=\frac{\omega}{2b}(b^2 - a^2)$................(3)

Due to electric field only, the electrons move radially from cathode to anode.

- ⇨ $\frac{1}{2}mv^2 = q\,V_{dc}$
- ⇨ $v = \sqrt{\frac{2qV_{dc}}{m}}$

But inside the cavity the velocity of electron has two components

- ⇨ $v^2 = v_r^2 + v\theta^2$
- ⇨ $\frac{2qV_{dc}}{m}=(\frac{dr}{dt})^2+(r\frac{d\theta}{dt})$

At r=b, $\frac{dr}{dt}=0$

- ⇨ $\frac{2qV_{dc}}{m}=0+(b\frac{d\theta}{dt})^2$
- ⇨ $b\frac{d\theta}{dt}=\sqrt{\frac{2qV_{dc}}{m}}$..............(4)

Comparing equation (3) & (4)

$$\frac{\omega}{2b}(b^2 - a^2)=\sqrt{\frac{2qV_{dc}}{m}}$$

- ⇨ $\frac{1}{2b}\frac{qB_z}{m}(b^2-a^2)=\sqrt{\frac{2qV_{dc}}{m}}$(at r=b , $B_z = B_c$)

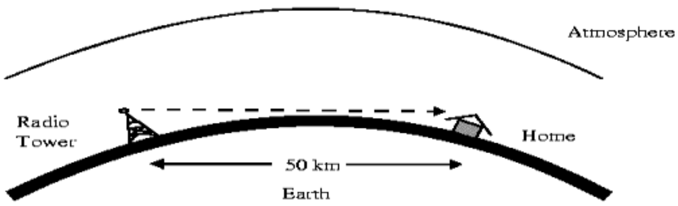
$$B_c = \frac{\frac{8mVdc}{b(1-a^2/b^2)}}{b(1-a^2/b^2)}$$

$$V_{dc} = \frac{q}{8m} b^2 \left(1 - a^2/b^2\right) B_z^2$$

MICROWAVE PROPAGATION

Line of Sight (LOS) Propagation:

Analog microwave communication system include LOS and over the horizon (OTH) analog communication systems. LOS communication is limited by horizon due to earth's curvature i.e. the transmission distance is determined by the height of the antenna above the earth in view of the horizon limitation. Microwaves are normally bent or refracted beyond the optical horizon (horizon visible to eyes). However due to atmospheric refractive changes, the radio horizon could even be less than the optical horizon at times.



The height of the microwave tower should be such that the radio beam is not obstructed by high rise buildings, trees or mountains. LOS microwave systems have shorter installation time, high flexible channel capacity and better adaptation to difficult terrains and natural barriers. LOS microwave system operates in the 14Hz to 10GHz frequency range. Above 10GHz however absorption due to rain, fog or snow may affect the system performance. Above 20GHz it is the absorption due to water vapour and atmospheric oxygen that limits the performance of an LOS system. Frequency Division Multiplexing (FDM) is used in all analog microwave LOS systems that enables several telephone signals to be transmitted over the same carrier. Normally the 4 KHz telephone channels are grouped as a 12 channel basic group in the frequency band 60KHz to 108KHz. Further , 5 basic groups form a 60-channel super group in the frequency band from 312KHz to 552KHz. 16 super groups are clubbed together to obtain a 960 channel master group.

ABSORPTION OF MICROWAVES BY ATMOSPHERIC GASES:

The transfer of electromagnetic radiation through an atmosphere is linked to its state (temperature, pressure, and composition) by the refractive index and by coefficient of absorption and scattering, if any. The absorption coefficient in a medium is a macroscopic parameter that represents the interaction of incident electromagnetic energy with the constituent molecules. The interaction is governed by three general principles.

1. Bohr's Frequency Condition:
   The frequency γ of a photon emitted or absorbed by the gas is equal to the difference of two energy levels (E_a-E_b) of the gas, divided by Plank's constant *h*.

2. Einstein's laws of emission and absorption:
   If E_a is higher than E_b, the probability, given initial state a, of stimulated emission of a photon by a transition from state a to state b is equal to the probability, given initial state b, of absorption of a photon by a transition from b to a. Hence the net absorption is proportional to the difference in thermodynamic probabilities (P_a-P_b) of two states.

3. Dirac's Perturbation Theory:
   For the electromagnetic field to include transitions between states a and b, the operator with which the field interacts must have a non-zero matrix element linking the two states. For wavelengths that are very long compared to molecular dimensions, this operator is the dipole moment.

ABSORPTION OF MICROWAVES BY WATER VAPOUR AND PRECIPITATES:

Water vapor and fog are the most influencing parameters when microwave propagates through the atmosphere. At high frequencies above about 10GHz, electromagnetic radiation starts interacting with the neutral atmosphere and also with the various metrological parameters, in particular, precipitation, producing absorption of energy and thus attenuation of signal levels. In addition to absorption by molecular oxygen, molecules of water vapor also interact with electromagnetic radiation in the microwave and millimeter wave regions. The water vapor molecule being a permanent electric dipole produces rotational transitions of the order of 10^4 times stronger than that of the magnetic transitions of the oxygen molecule. So, even though the abundance of water vapor in the atmosphere is considerably less than the oxygen, it can produce significant level of attenuation near the resonant frequencies. Water vapor attenuation is found to depend quadratically on water vapor density, particularly at high densities, above about $12g/m^3$.