VEER SURENDRA SAI UNIVERSITY OF TECHNOLOGY BURLA, ODISHA, INDIA

DEPARTMENT OF PRODUCTION ENGINEERING

Lecture Notes on
THEORY OF METAL FORMING
COURSE CODE: BPE:309
6th Semester B. Tech in Production Engineering

VEER SURENDRA SAI UNIVERSITY OF TECHNOLOGY
LESSON PLAN

Semester >>6th (B.Tech)	**Year >> 2015**	**Contact Hours per week >>4**
Sub: Theory Of Metal forming	**Branch >>** **Production Engineering**	**Total Credit >>4**
TEACHER	**Abinash Pujari**	
Period	**Jan 2015-May 2015**	
Recommended books >>	Text books 1. Mechanical Metallurgy: By- Dieter, Mc Graw Hill Book Co. 2. Plasticity- Chakraborty- McGraw Hill. References 1. Engineering Plasticity: BY- Johson & Mellor, Van Nostrand. 2. Metal working –Avitzur, Mc Graw Hill 3. Industrial Metal working- G.W. Rowe	

Sl. No.	**Lecture No.**	**Topics to be covered**
1	Lecture-01	Review of 2D stress and strain
2	Lecture-02	**State of stress in 3D** 3d stress,strain
3	Lecture-03	**State of stress in 3D** Derivation for normal & shear stress
4	Lecture-04	Stress tensor

5	Lecture-05	Mohr’s cicle for 3D state of stress

6	Lecture-06	Mohr’s circle for 3D state of strain
7	Lecture-07	Hydrostatic and deviatory component of stress
8	Lecture-08	Problems on constructing Mohr’s circle in 3D
9	Lecture-09	Problems on stress, invariants, tensors
10	Lecture-10	Elastic stress-strain relations
11	Lecture-11	Class Test on Module-1
12	Lecture-12	Flow curve, true stress and strain
13	Lecture-13	Yield criteria for ductile materials
14	Lecture-14	Von mises and tresca criteria
15	Lecture-15	Combined stress tests
16	Lecture-16	The yield locus, anisotropy in yielding, yield surface
17	Lecture-17	Levy-mises equation
18	Lecture-18	prandtl –Reuss stress strain relation
19	Lecture-19	Classification of forming process
20	Lecture-20	Class test on module 2

21	Lecture-21	Analysis of deformation process
22	Lecture-22	Slip line field theory
23	Lecture-23	Upper bound theory
24	Lecture-24	Lower bound theory
25	Lecture-25	Slab method of analysis
26	Lecture-26	Flow stress determination
27	Lecture 27	Hot and cold working
28	Lecture 28	Strain rate effect
29	Lecture 29	Friction & lubrication
30	Lecture 30	Deformation zone geometry, Workability, residual stress
31	Lecture 31	Class test on Module 3
32	Lecture 32	Analysis of metal forming process
33	Lecture 33	Load calculation in plane forging
34	Lecture-34	**Rolling :** Forces and geometrical relationship in rolling
35	Lecture-35	**Rolling :** Rolling load and torque in cold rolling
36	Lecture-36	Von-karman work equation

37	Lecture-37	**Extrusion :** Analysis of extrusion process, extrusion pressure
38	Lecture-38	**Drawing :** Drawing load
39	Lecture-39	Class test on module-4
40	Lecture-40	Tips for final exam

BMS 309: THEORY OF METAL FORMING: SYLLABUS

Module I

Review of two dimensional stress and strain, state of stress in three dimensions, Stress tensor, Invariants, Mohr's circle for 3-dimensional state of stress, strain at a point-Mohr's circle for strain, Hydrostatic & Deviatory components of stress, Elastic stress-strain relations. [08]

Module II

Elements of theory of plasticity; Flow curve, True stress & true strain, Yield criteria for ductile metals, Von Misses & Teresa yield criteria, combined stress tests. The yield locus, Anisotropy in yielding, Yield surface, levy-Misses, Prandtl-Reuss Stress-Strain relation, Classification of forming processes variables in metal forming and their optimization [10]

Module III

Analysis of deformation processes- Method based on homogeneous compression slip line field theory, Upper bounds and lower bounds, Slab method of analysis. [06]

Flow stress determination, Hot working, Cold working, Strain rate effect, Friction and lubrication, Deformation zone geometry, Workability, Residual stress. [06]

Module IV

Analysis of metal forming processes(only limited portion), Forging: Load calculation in plane strain forging, Rolling: Forces & geometrical relationship in rolling, Rolling load and torque in cold rolling, Von-Karman work equation, Extrusion: Analysis of extrusion process, extrusion pressure, Drawing: Drawing load [10]

TEXT BOOK(S):

3. Mechanical Metallurgy: By- Dieter, Mc Graw Hill Book Co.
4. Plasticity- Chakraborty- McGraw Hill.

REFERENCE(S):

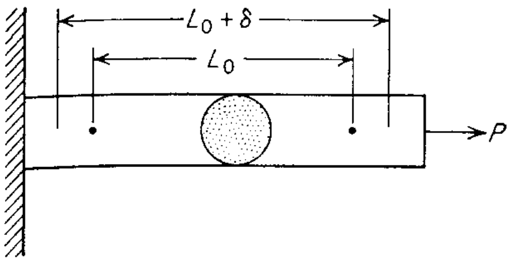
4. Engineering Plasticity: BY- Johson & Mellor, Van Nostrand.
5. Metal working –Avitzur, Mc Graw Hill
6. Industrial Metal working- G.W. Rowe

MODULE 1

Review of two dimensional stress and strain, state of stress in three dimensions, Stress tensor, Invariants, Mohr's circle for 3-dimensional state of stress, strain at a point- Mohr's circle for strain, Hydrostatic & Deviatory components of stress, Elastic stress-strain relations.

1.1 Review of two dimensional stress and strain:

As a starting point in the discussion of stress and strain, consider a uniform cylindrical bar which is subjected to an axial tensile load. Its unstrained state and that Lo is the gage length between these marks. A load P is applied to one end of the bar, and the gage length undergoes a slight increase in length and decrease in diameter. The distance between the gage marks has increased by an amount ΔL , called the deformation. The average linear strain e is the ratio of the change in length to the original length.



(Fig-1.1.1 Cylindrical bar subjected to axial load)

(Fig-1.1.2 Free body diagram)

$$e = \frac{\delta}{L_0} = \frac{\Delta L}{L_0} = \frac{L - L_0}{L_0} \qquad \text{---------(1.1.1)}$$

Strain is a dimensionless quantity since both 5 and Lo are expressed in units of length.
Figure 1.1.2 shows the free-body diagram for the cylindrical bar shown in Fig. 1.1.1. The external load P is balanced by the internal resisting force where a is the stress normal to the cutting plane and A is the cross-sectional area of the bar. The equilibrium equation is

$$P = \int \sigma \, dA \qquad \text{------------(1.1.2)}$$

If the stress is distributed uniformly over the area A, that is, if a is constant,
Eq. (1.1.2) becomes

$$P = \sigma \int dA = \sigma A$$

$$\sigma = \frac{P}{A} \qquad \text{--------------(1.1.3)}$$

In general, the stress will not be uniform over the area A, and therefore Eq. (1.1.3) represents an average stress. For the stress to be absolutely uniform, every longitudinal element in the bar would have to experience exactly the same strain, and the proportionality between stress and strain would have to be identical for each element. The inherent anisotropy between grains in a polycrystalline metal rules out the possibility of complete uniformity of stress over a body of macroscopic size. The presence of more than one phase also gives rise to non-uniformity of stress on a microscopic scale. If the bar is not straight or not centrally loaded, the strains will be different for certain longitudinal elements and the stress will not be uniform. An extreme disruption in the uniformity of the stress pattern occurs when there is an abrupt change in cross section. This result in a stress raiser or stress concentration. In engineering practice, the load is usually measured in pounds and the area in square inches, so that stress has units of pounds per square inch (psi). Since it is common for engineers to deal with loads in the thousands of pounds, an obvious simplification is to work with units of 1,000 lb, called kips. The stress may be expressed in units of kips per square inch (ksi). (1 ksi = 1,000 psi.) In scientific work stresses are often expressed in units of kilograms per square millimetre or dynes per square centimetre. (1 kg/mm^ = 9.81 X 10^ dynes/cml) Below the elastic limit Hooke's law can be considered vaHd, so that the average stress is proportional to the average strain,

$$\frac{\sigma}{e} = E = \text{constant}$$

The constant E is the modulus of elasticity, or Young's modulus.

1.1.1 Tensile Deformation of Ductile Metal:

The basic data on the mechanical properties of a ductile metal are obtained from a tension test, in which a suitably designed specimen is subjected to increasing axial load until it fractures. The load and elongation are measured at frequent intervals during the test and are expressed as average stress and strain according to the equations in the previous section. The data obtained from the tension test are generally plotted as a stress-strain diagram. Figure 1.1.3 shows a typical stress-strain curve for

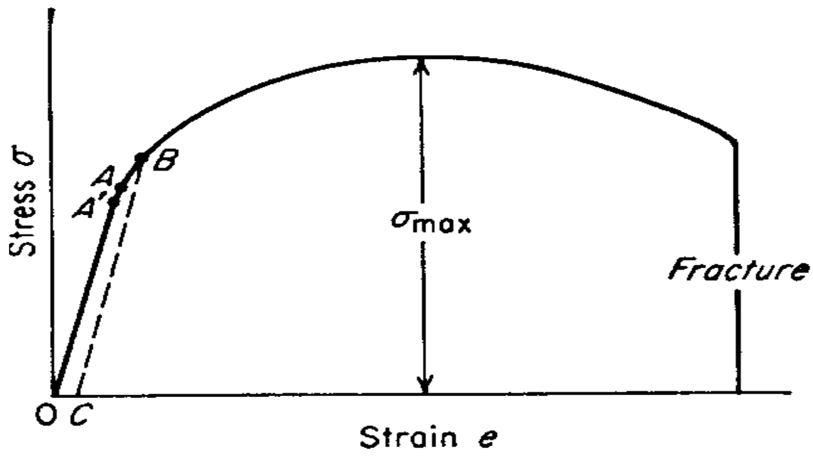


Fig1.1.3 Typical tension stress-strain curve

a metal such as aluminium or copper. The initial linear portion of the curve OA is the elastic region within which Hooke's law is obeyed. Point A is the elastic limit, defined as the greatest stress that the metal can withstand without experiencing a permanent strain when the load is removed. The determination of the elastic limit is quite tedious, not at all routine, and dependent on the sensitivity of the strain-measuring instrument.

For these reasons it is often replaced by the proportional limit, point A'. The proportional limit is the stress at which the stress strain curve deviates from linearity. The slope of the stress-strain curve in this region is the modulus of elasticity. For engineering purposes the limit of usable elastic behaviour is described by the yield strength, point B. The yield strength is defined as the stress which will produce a small amount of permanent deformation, generally a strain equal to 0.2 per cent or 0.002 inches per inch. In Fig. 1.1.3 this permanent strain, or offset, is OC. Plastic deformation begins when the elastic limit is exceeded. As the plastic deformation of the specimen increases, the metal becomes stronger (strain hardening) so that the load required to extend the specimen increases with further straining. Eventually the load reaches a maximum value. The maximum load divided by the original area of the specimen is the ultimate tensile strength. For a ductile metal the diameter of the specimen begins to decrease rapidly beyond maximum load, so that the load required to continue deformation drops off until the specimen fractures. Since the average stress is based on the original area of the specimen, it also decreases from maximum load to fracture.

1.1.2 Concept of Strain and the Types of Strain:

The average linear strain was defined as the ratio of the change in length to the original length of the same dimension.

$$e = \frac{\delta}{L_0} = \frac{\Delta L}{L_0} = \frac{L - L_0}{L_0}$$

where e = average linear strain
δ = deformation
By analogy with the definition of stress at a point, the strain at a point is the ratio of the deformation to the gage length as the gage length approaches zero. Rather than referring the change in length to the original gage length, it often is more useful to define the strain as the change in linear dimension divided by the instantaneous value of the dimension.

$$\varepsilon = \int_{L_0}^{L_f} \frac{dL}{L} = \ln \frac{L_f}{L_0}$$

The above equation defines the natural, or true, strain. True strain, which is useful in dealing with problems in plasticity and metal forming, will be discussed. For the present it should be noted that for the very small strains for which the equations of elasticity are valid the two definitions of strain give identical values.

Not only will the elastic deformation of a body result in a change in length of a linear element in the body, but it may also result in a change in the initial angle between any two lines. The angular change in a right angle is known as shear strain. Figure 1-7 illustrates the strain produced by the pure shear of one face of a cube. The angle at A, which was originally 90°, is decreased by the application of a shear stress by a small amount 6. The shear strain y is equal to the displacement a divided by the distance between the planes, h. The ratio a/h is also the tangent of the angle through which the element has been rotated. For the small angles usually involved, the tangent of the angle and the angle (in radians) are equal. Therefore, shear strains are often expressed as angles of rotation.

$$\gamma = \frac{a}{h} = \tan\theta = \theta$$

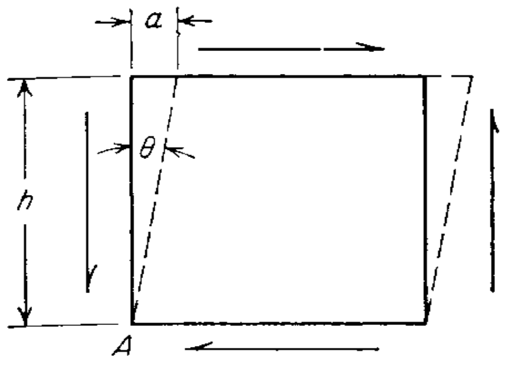


Fig 1.1.4 Shear strain

1.1.3 State of Stress in Two Dimensions (Plane Stress):

Many problems can be simplified by considering a two-dimensional state of stress. This condition is frequently approached in practice when one of the dimensions of the body is small relative to the others. For example, in a thin plate loaded in the plane of the plate, there will be no stress acting perpendicular to the plate. The stress system will consist of two normal stresses (Xx and ay and a shear stress Txy. A stress condition in which the stresses are zero in one of the primary directions is called plane stress. Figure 1.3 illustrates a thin plate with its thickness normal to the plane of the paper. In order to know the state of stress at point in the plate, we need to be able to describe the stress components at for any orientation of the axes through the point. To do this, consider an oblique plane normal to the plane of the paper at an angle 6 between the

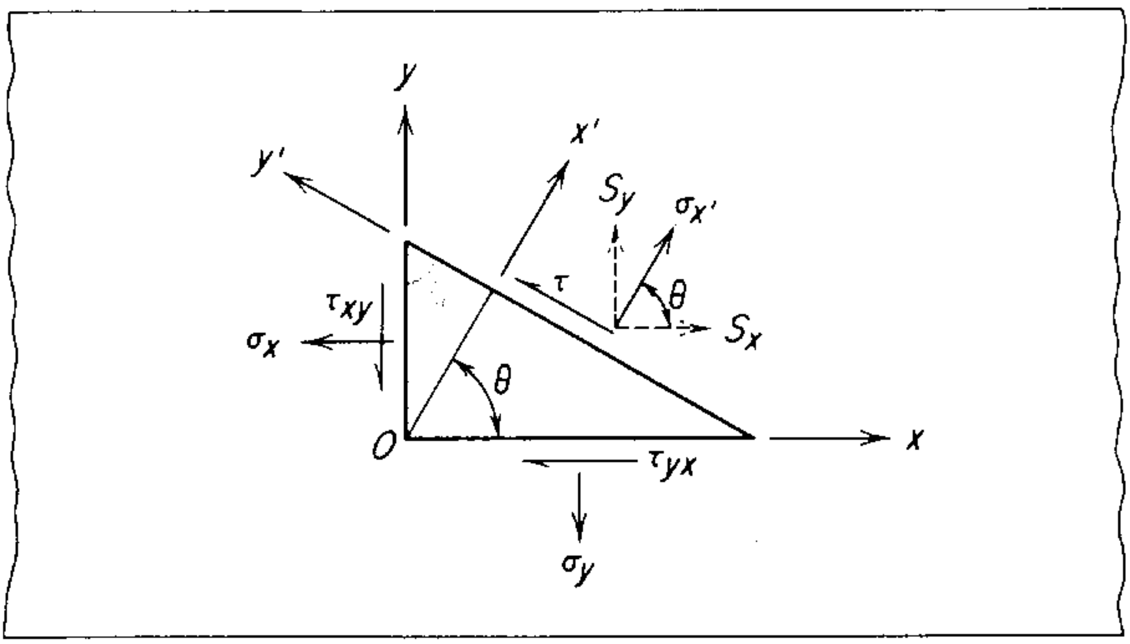


Fig 1.1.5 Stress on oblique plain

X axis and the normal A^ to the plane. It is assumed that the plane shown in Fig.1.1.5 is an infinitesimal distance from and that the element is so small that variations in stress over the sides of the element can be neglected. The stresses acting on the oblique plane are the normal stress a and the shear stress r. The direction cosines between N and the X and y axes are I and m, respectively. From the geometry of Fig. 1.1.5, it follows that I = cos 6 and m = sin 6. If A is the area of the oblique plane, the areas of the sides of the element perpendicular to the x and y axes are Al and Am. Let Sx and Sy denote the x and y components of the total stress acting on the inclined face. By taking the summation of the forces in the x direction and the y direction, we obtain

$$S_x A = \sigma_x Al + \tau_{xy} Am$$

$$S_y A = \sigma_y Am + \tau_{xy} Al$$

$$S_x = \sigma_x \cos\theta + \tau_{xy} \sin\theta$$

$$S_y = \sigma_y \sin\theta + \tau_{xy} \cos\theta$$

The components of Sx and Sy in the direction of the normal stress σ are:

$$S_{xN} = S_x \cos\theta \quad \text{and} \quad S_{yN} = S_y \sin\theta$$

so that the normal stress acting on the oblique plane is given by

$$\sigma_{x'} = S_x \cos\theta + S_y \sin\theta$$

$$\sigma_{x'} = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \sin\theta\cos\theta$$

The shearing stress on the oblique plane is given by

$$\tau_{x'y'} = S_y \cos\theta - S_x \sin\theta$$

$$\tau_{x'y'} = \tau_{xy}(\cos^2\theta - \sin^2\theta) + (\sigma_y - \sigma_x)\sin\theta\cos\theta$$

To aid in computation, it is often convenient to express both equations in terms of the double angle 2Ө. This can be done with the following
identities:

$$\cos^2\theta = \frac{\cos 2\theta + 1}{2}$$

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

$$2\sin\theta\cos\theta = \sin 2\theta$$

$$\cos^2\theta - \sin^2\theta = \cos 2\theta$$

Equations now become

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

1.1.4 Mohr's Circle of Stress—Two Dimensions

A graphical method for representing the state of stress at a point on any oblique plane through the point was suggested by O. Mohr.

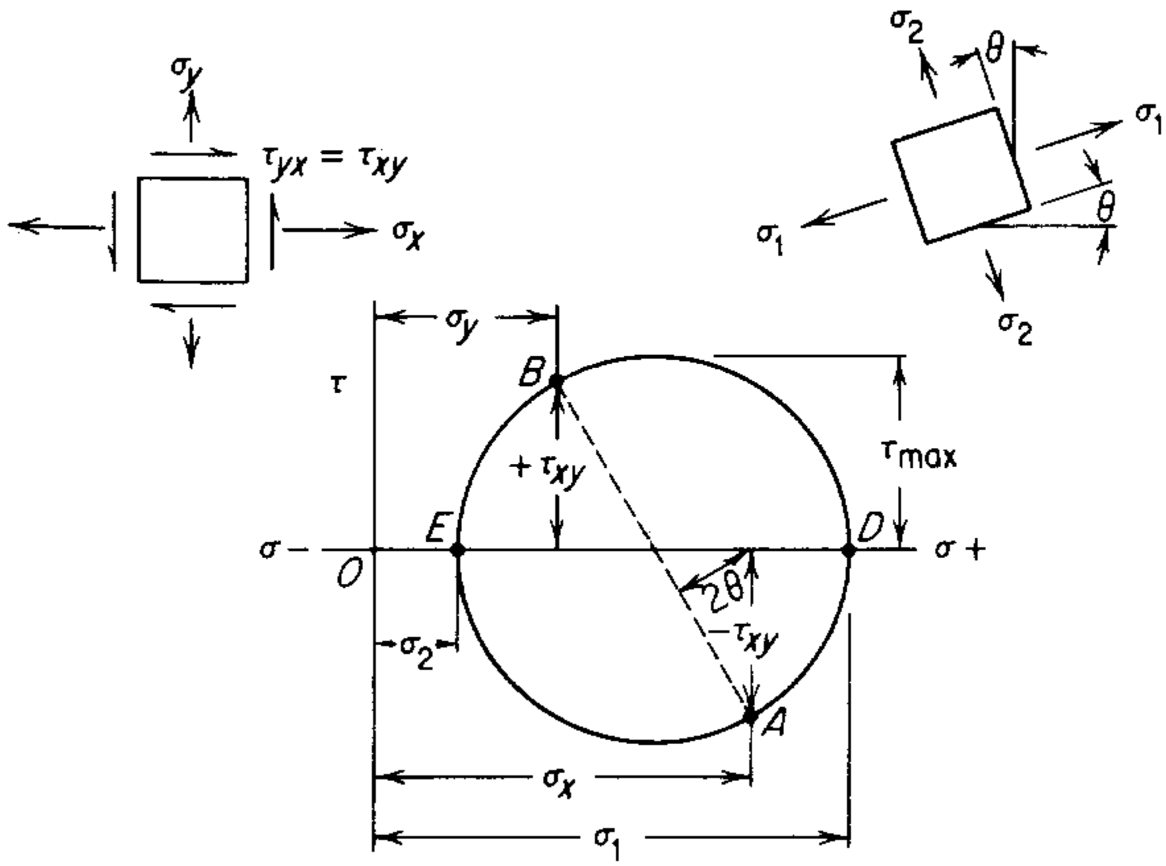


Fig 1.1.6 Mohr's circle for two dimension state of stress

Normal stresses are plotted along the x axis, shear stresses along the y axis. The stresses on the planes normal to the x and y axes are plotted as points A and B. The intersection of the line AB with the x axis determines the center of the circle. At points D and E the shear stress is zero, so that these points represent the values of the principal stresses. The angle between the x axis and ax is determined by angle ACD in Fig. This angle on Mohr's circle is equal to twice the angle between ai and the x axis on the actual stressed body.

$$\sigma_{x'} - \frac{\sigma_x + \sigma_y}{2} = \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{y'x'} = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

The radius of Mohr's circle is

$$\left(\sigma_{x'} - \frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{x'y'}^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$

This value is given by the maximum ordinate of the circle. Note that it acts on a plane for which 6 = x/4 (26 = 7r/2 on Mohr's circle); i.e., the plane on which r^ax acts bisects the angle between the principal stresses. Mohr's circle can also be used to determine the stresses acting on any oblique plane mm. Using the convention that 6 is positive when it is measured clockwise from the positive x axis, we find that to determine the stresses on the oblique plane whose normal is at an angle 6 we must advance an angle 2ϴ from point A in the Mohr's circle. The normal and shearing stresses on the oblique plane are given by the coordinates of point F. The stresses on a plane perpendicular to mm would be obtained by advancing an additional 180° on the Mohr's circle to point G. This shows that the shearing stresses on two perpendicular planes are numerically equal. It can also be seen from Fig. 1.1.6 that OF' + OG' = 20C. Therefore, the sum of the normal stresses on two perpendicular planes is a constant, independent of the orientation of the planes.

1.2 State of stress in three dimensions:

1.2.1 Description of Stress at a Point:

As described it is often convenient to resolve the stresses at a point into normal and shear components. In the general case the shear components are at arbitrary angles to the coordinate axes, so that it is convenient to resolve each shear stress further into two components. The general case is shown in Fig. 1.2.1. Stresses acting normal to the faces of the elemental cube are identified by the subscript which also identifies the direction in which the stress acts; that is the normal stress acting in the x direction. Since it is a normal stress, it must act on the plane perpendicular to the x direction. By convention, values of normal stresses greater than zero denote tension; values less than zero indicate compression. All the normal stresses shown in Fig. 1.2.1 are tensile. Two subscripts are needed for describing shearing stresses. The first subscript indicates the plane in which the stress acts and the second the direction in which the stress acts. Since a plane is most easily defined by its normal, the first subscript refers to this normal. For example, Tyz is the shear stress on the plane perpendicular to the y axis in the direction of the z axis. t„x is the shear stress on a plane normal to the y axis in the direction of the x axis. Shearing stresses oriented in the positive directions of the coordinate axes are positive if a tensile stress on the same cube face is in the positive direction of the corresponding axis. All the shear stresses shown in Fig. 1.2.1 are positive. The notation given above is the one used by Timoshenko and most American workers in the field of elasticity. The reader is reminded, however, that several other systems of notation are in use. Before attempting to read papers in this field it is important to become familiar with the notation which is used. It can be seen from Fig. 1.2.1 that nine quantities must be defined in order to establish the state of stress at a point. They are Txy, Txz, Tyx, Tyz, Tx, and Tzy. However, some simplification is possible. If we assume that the areas of the faces of the unit cube are small enough so that the change in stress over the face is negligible, by taking the sum.

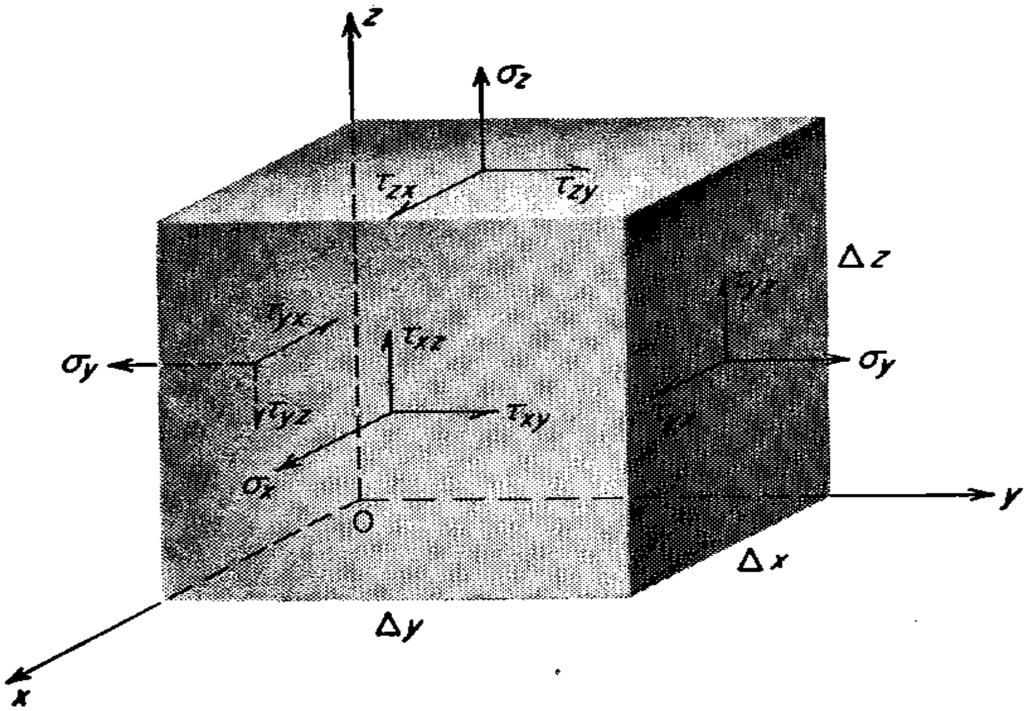


Fig 1.2.1 Stresses acting on a elemental cube

1.2.2 State of Stress in Three Dimensions:

The general three-dimensional state of stress consists of three unequal principal stresses acting at a point. This is called a tri axial state of stress. If two of the three principal stresses are equal, the state of stress is known as cylindrical, while if all three principal stresses are equal, the state of stress is said to be hydrostatic, or spherical. The determination of the principal stresses for a three-dimensional state of stress in terms of the stresses acting on an arbitrary Cartesian coordinate system is an extension of the method described for the two-dimensional case. Figure 1.2.2 represents an elemental free body similar to that shown in Fig.1.2.1 with a diagonal plane JKL of area A. The plane JKL is assumed to be a principal plane cutting. Through the unit cube, a is the principal stress acting normal to the plane JKL. Let l, m, n be the direction cosines of a, that is, the cosine

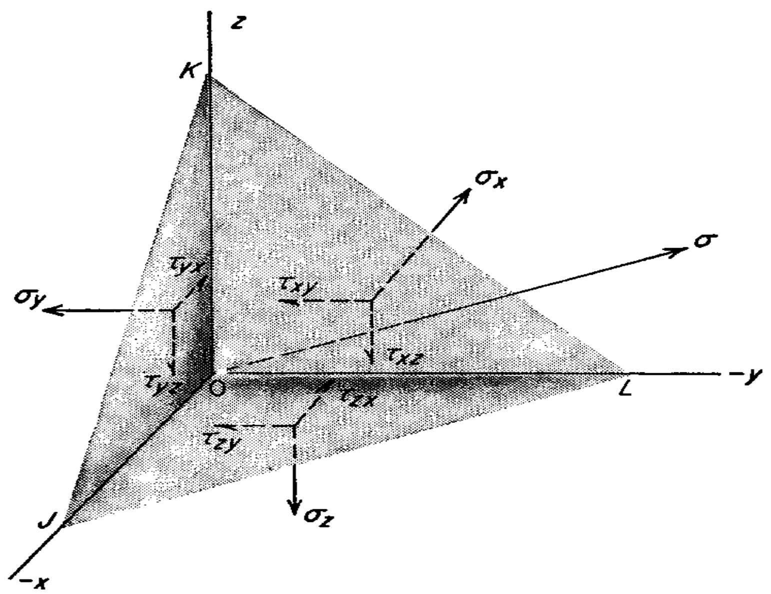


Fig 1.2.2 Stresses acting on a elemental cube

1.3 Mohr's circle of stress- Three Dimension:

While the only physical significance of Mohr's circle is that it gives a geometrical representation of the equations that express the transformation of stress components to different sets of axes, it is a very convenient way of visualizing the state of stress. Figure 1.2.2 shows Mohr's circle for a number of common states of stress. Note that the application of a tensile stress ai at right angles to an existing tensile stress results in a decrease in the principal shear stress on two of the three sets of planes on which a principal shear stress acts. However, the maximum shear stress is not decreased from what it would be for uniaxial tension, although if only the two-dimensional Mohr's circle had been used, this would not have been apparent. If a tensile stress is applied in the third principal direction the maximum shear stress is reduced appreciably. For the limiting case of equal triaxial tension (hydrostatic tension) Mohr's circle reduces to a point, and there are no shear stresses acting on any plane in the body. The effectiveness of biaxial- and tri-axial- tension stresses in reducing the shear stresses results in a considerable decrease in the ductility of the material, because plastic deformation is produced by shear stresses. Thus, brittle fracture is invariably associated with triaxial stresses developed at a notch or stress raiser.

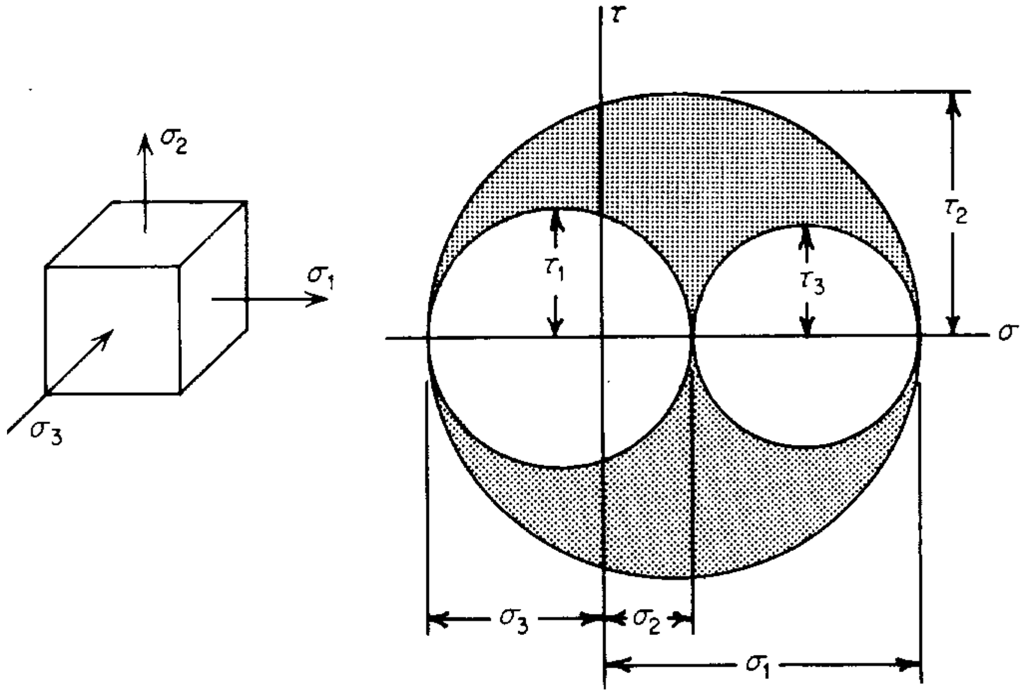
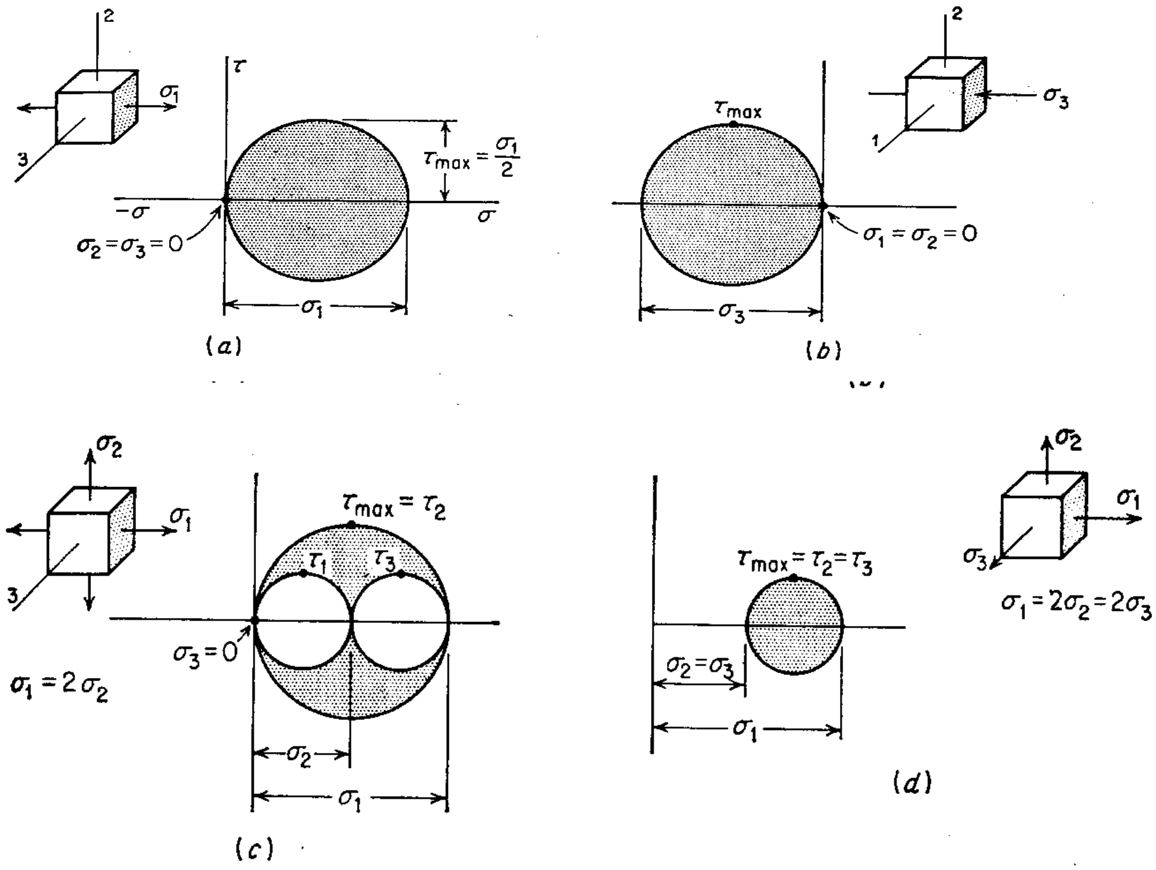


Fig 1.2.3 Mohr's circle representation of a Three dimensional state of stress



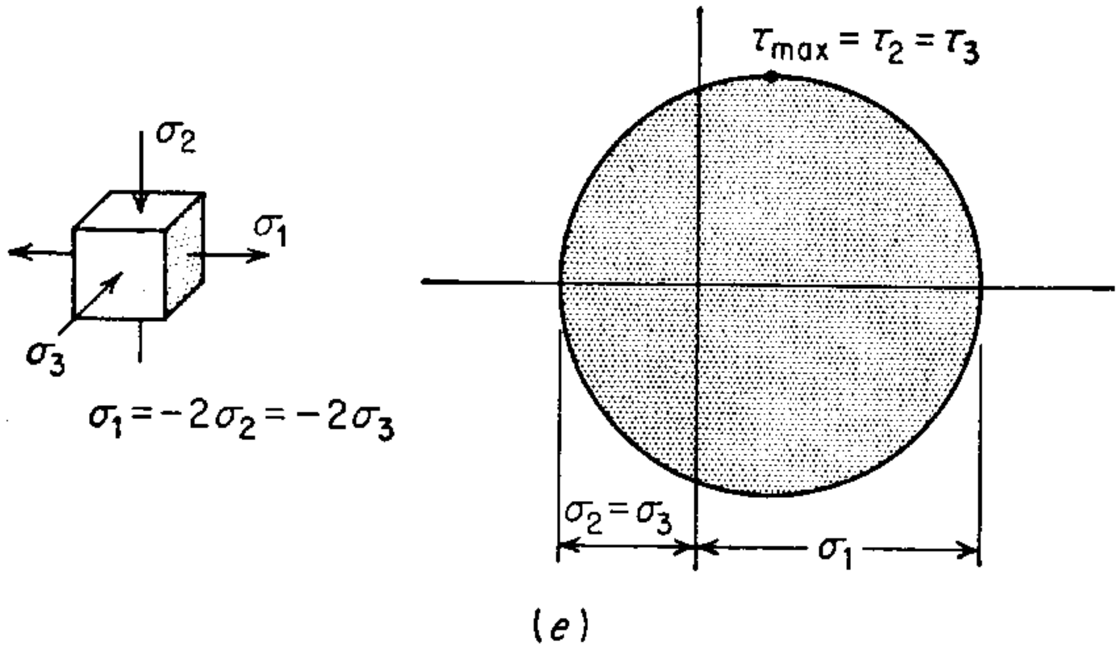


Fig 1.2.3

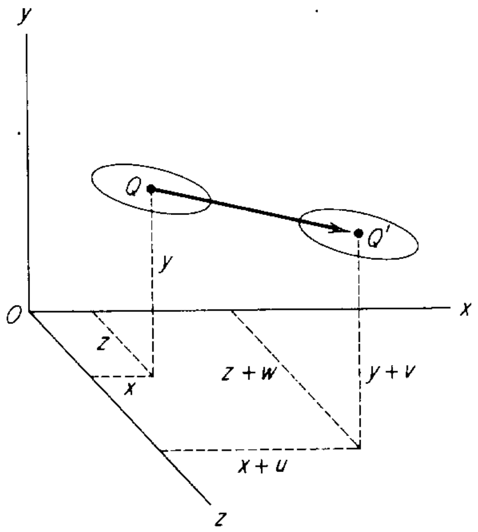
Mohr's circles (three-dimensional) for various states of stress, (a) Uniaxial tension; (6) uniaxial compression; (c) biaxial tension; (d) tri axial tension (unequal); (e) uniaxial tension plus biaxial compression.

Either uniaxial tension or compression because of the high value of shear stress relative to the applied tensile stress the material has an excellent opportunity to deform plastically without fracturing under this state of stress. Important use is made of this fact in the plastic working of metals. For example, greater ductility is obtained in drawing wire through a die than in simple uniaxial tension because the reaction of the metal with the die will produce lateral compressive stresses.

An important state of stress is pure shear. Figure Fig 1.2.3a illustrates Mohr's circle for a two-dimensional state of pure shear. This state of stress is readily obtained by twisting a cylindrical bar in torsion. The Mohr's circle for this state of stress shows that the maximum and minimum normal stresses are equal to the shear stress and occur at 45° to the shear stresses. The maximum shear stress is equal to the applied shear stress Txii, but it occurs only on the set of planes parallel to the z axis. On the other two sets of planes the principal shear stress is rxi//2. Note that for three-dimensional pure shear two out of the three sets of shear planes have a value of Tn,ax = ci. An identical state of stress to pure shear can be obtained when equal tensile and compressive stresses are applied to a unit cube. Once again Xmax = cri, but to obtain complete identity with a state of two-dimensional pure shear, the axes must be rotated 45° in space or 90° on the Mohr's circle.

1.4 Strain at a Point:

The brief description of linear strain and shear strain can be expanded into a more generalized description of the strain at a point in a rigid body.



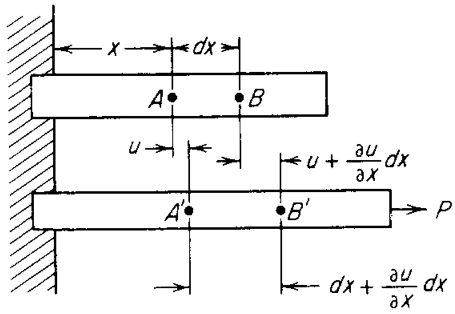


Fig 1.4.1 Displacement of Point Q **Fig 1.4.2 One dimensional strain**

It is, however, a simple matter to generalize the relationship obtained from this figure to the three-dimensional case. Let the coordinates of a point in an unstrained rigid body be defined by X, y, and z. After strain is applied, the point will undergo displacements u, V, and w in the directions x, y, and z. In order that the displacement of the entire body be geometrically compatible, it is necessary that no two particles occupy the same point in space or that no voids be created within the body. In order to satisfy these requirements, the displacement components u, v, and w must vary continuously from point to point. This can be accomplished if their gradients with respect to x, y, and z have no discontinuities, and therefore the partial derivatives of u, v, and w with respect to x, y, and z enter into the analysis.

1.5 Strain at a point- Mohr's circle for strain:

Except in a few cases involving contact stresses, it is not possible to measure stress directly. Therefore, experimental measurements of stress are actually based on measured strains and are converted to stresses by means of Hooke's law and the more general relationships. The most universal strain-measuring device is the bonded wire resistance gage, frequently called the SR-4 strain gage. These gages are made up of several loops of fine wire or foil of special composition, which are bonded to the surface of the body to be studied. When the body is deformed, the wires in the gage are strained and their electrical resistance is altered. The change in resistance, which is proportional to strain, can be accurately determined with a simple Wheatstone-bridge circuit. The high sensitivity, stability, comparative ruggedness, and ease of application make resistance strain gages a very powerful tool for strain determination.

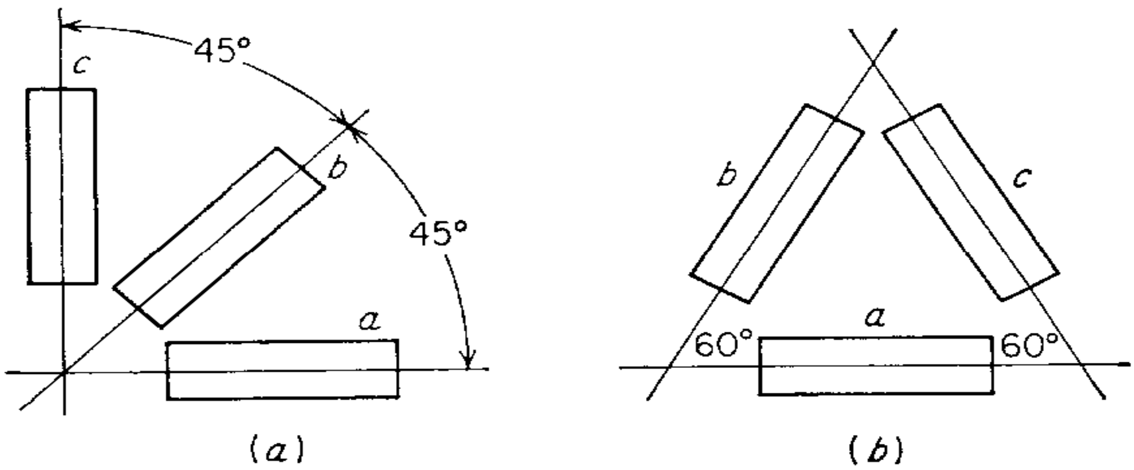


Fig 1.5.1. Typical strain-gage rosettes (a) Rectangular (b) Delta

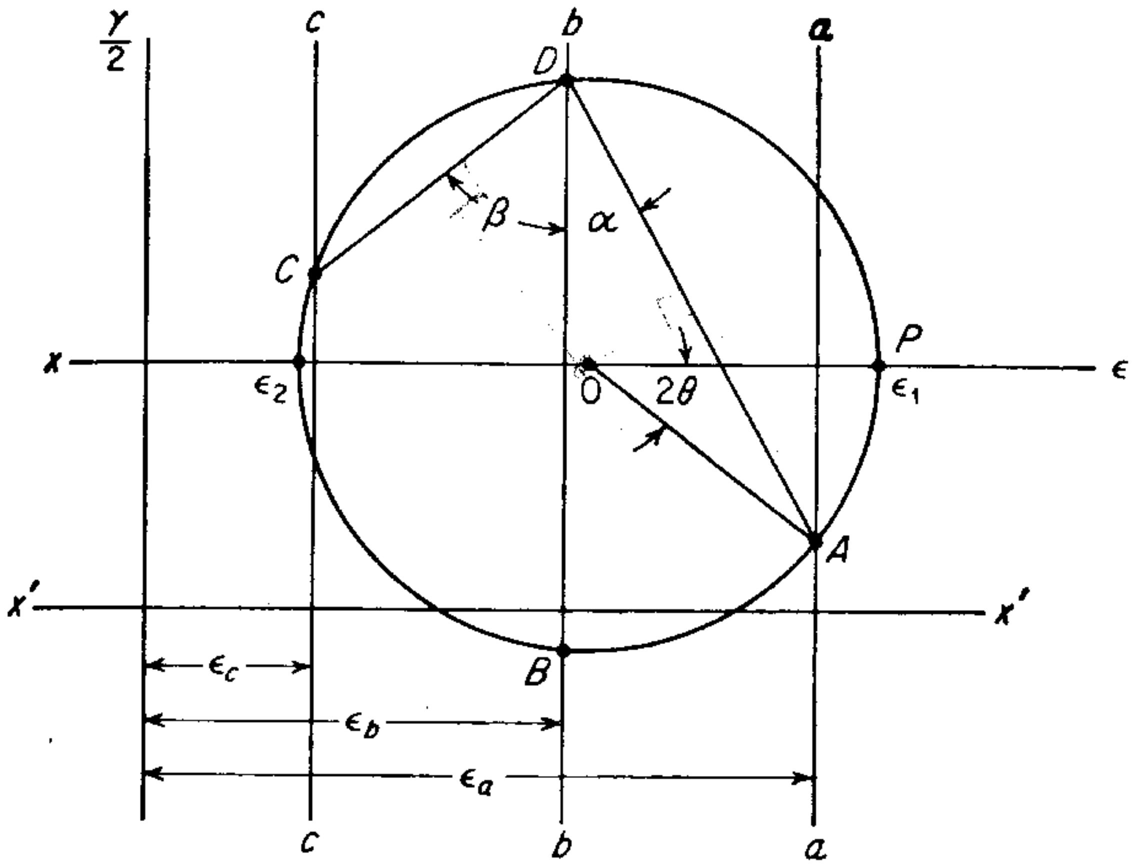
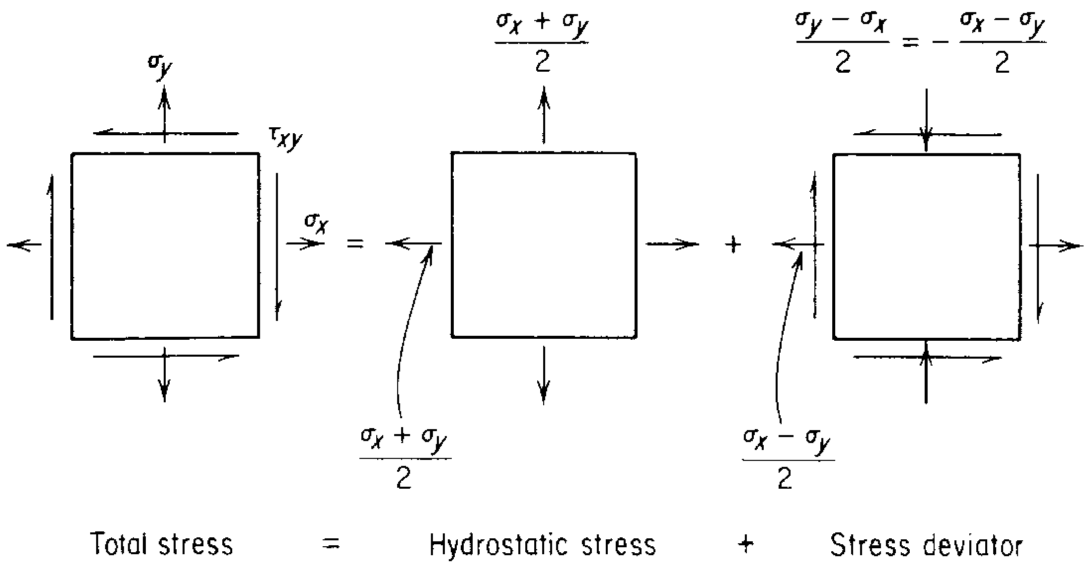


Fig 1.5.2 Mohr's circle for determination of Principal strain

1.6 Hydrostatic & Deviatory components of stress:

Experiment shows that materials can withstand very large hydrostatic pressures (spherical state of stress) without undergoing plastic deformation. In many problems, particularly in the theory of plasticity, it is desirable to designate the part of the total stress which can be effective in producing plastic deformation. This is known as the stress deviator. The other component is the spherical, or hydrostatic, component of stress.



1.7 Elastic stress-strain relations:

In the first chapter it was shown that uniaxial stress is related to uniaxial strain by means of the modulus of elasticity. This is Hooke's law in its simplest form,

$$\sigma_x = E\varepsilon_x$$

where E is the modulus of elasticity in tension or compression. While a tensile force in the x direction produces a linear strain along that axis, it also produces a contraction in the transverse y and z directions. The ratio of the strain in the transverse direction to the strain in the longitudinal direction is known as Poisson's ratio.

$$\varepsilon_y = \varepsilon_z = -\nu\varepsilon_x = -\frac{\nu\sigma_x}{E}$$

Only the absolute value of v is used in calculations. Poisson's ratio is 0.25 for a perfectly isotropic elastic material, but for most metals the values^ are closer to 0.33. The generalized description of Hooke's law says that for a body acted upon by a general stress system the strain along any principal axis is due to the stress acting along that axis plus the superimposed strain resulting from the Poisson effect of the principal stresses acting along the other two axes.

MODULE 2

Elements of theory of plasticity; Flow curve, True stress & true strain, Yield criteria for ductile metals, Von Misses & Teresa yield criteria, combined stress tests. The yield locus, Anisotropy in yielding, Yield surface, levy-Misses, Prandtl-Reuss Stress-Strain relation, Classification of forming processes variables in metal forming and their optimization.

4.1 Elements of Theory of plasticity:

The theory of plasticity deals with the behaviour of materials in the region of strain beyond which Hooke's law is no longer valid. The mathematical description of the plastic deformation of metals is not nearly so well developed as the description of elastic deformation by means of the theory of elasticity because plastic deformation is much more complicated than elastic deformation. For example, in the plastic region of strain, there is no simple relationship between stress and strain as there is for elastic deformation. Moreover, elastic deformation depends only on the initial and final states of stress and is independent of the loading path, but for plastic deformation the plastic strain depends not only on the final load but also on the path by which it was reached.

The theory of plasticity is concerned with a number of different types of problems. From the viewpoint of design, plasticity is concerned with predicting the maximum load which can be applied to a body without causing excessive yielding. The yield criterion^ must be expressed in terms of stress in such a way that it is valid for all states of stress. The designer is also concerned with plastic deformation in problems where the body is purposely stressed beyond the yield stress into the plastic region. For example, plasticity must be considered in designing for processes such as auto frottage, shrink fitting, and the over speeding of rotor disks. The consideration of small plastic strains allows economies in building construction through the use of the theory of limit design. The analysis of large plastic strains is required in the mathematical treatment of the plastic forming of metals. This aspect of plasticity.

The determination of the Hitting load between elastic and plastic behaviour is also generally covered in strength of materials. However, because it is necessary to adopt a yield criterion in the theories of plasticity, this topic is covered in the chapter on Plasticity will be considered in Part Four. It is very difficult to describe, in a rigorous analytical way, the behaviour of a metal under these conditions. Therefore, certain simplifying assumptions are usually necessary to obtain a tractable mathematical solution.

Another aspect of plasticity is concerned with acquiring a better understanding of the mechanism of the plastic deformation of metals. Interest in this field is cantered on the imperfections in crystalline solids. The effect of metallurgical variables, crystal structure, and lattice imperfections on the deformation behaviour are of chief concern. This aspect of plasticity is considered in Part Two.

4.2 The Flow curve:

The stress-strain curve obtained by uniaxial loading, as in the ordinary tension test, is of fundamental interest in plasticity when the curve is plotted in terms of true stress a- and true strain e. True stress is given by the load divided by the instantaneous cross-sectional area of the specimen. True strain is discussed in the next section. The purpose of this section' is to describe typical stress-strain curves for real metals and to compare them with the theoretical flow curves for ideal materials.

The true stress-strain curve for a typical ductile metal, such as aluminium, is illustrated in Fig. 3-la. Hooke's law is followed up to some yield

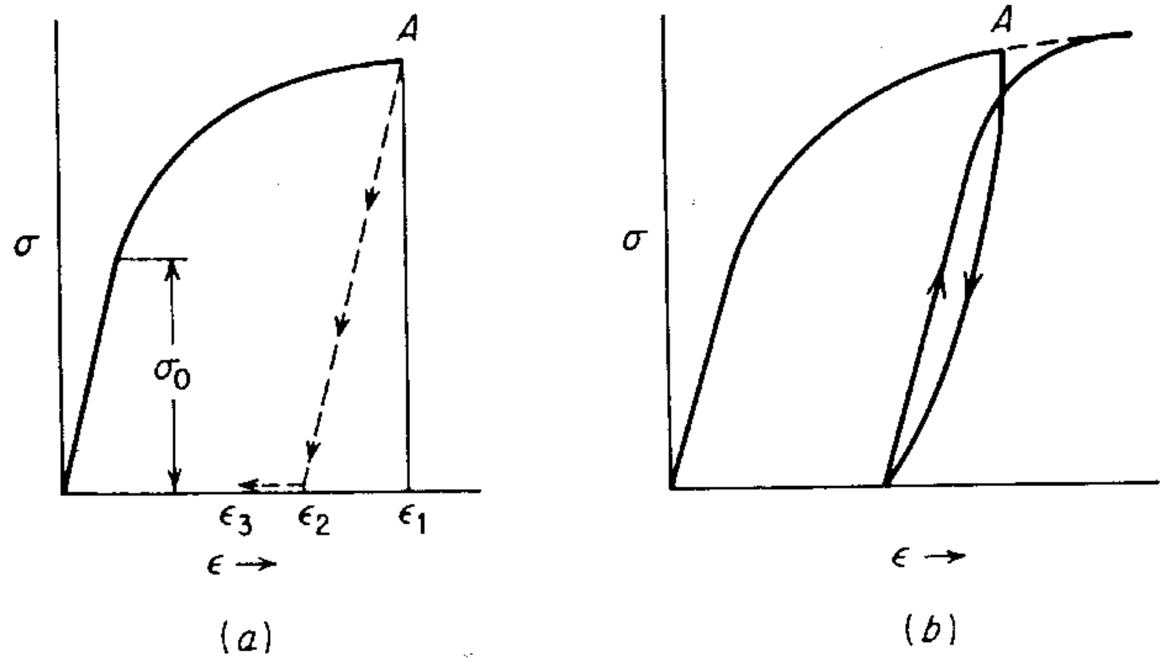


Fig 2.1 Typical true stress-strain curve for ductile curve

Stress. (The value of o-q will depend upon the accuracy with which strain is measured.) Beyond the metal deforms plastically. Most metals strain-harden in this region, so that increases in strain require higher values of stress than the initial yield stress. Generally the stress-strain curve on unloading from a plastic strain will not be exactly linear and parallel to the elastic portion of the curve (Fig. 2.1). Moreover, on reloading the curve will generally bend over as the stress approaches the original value of stress from which it was unloaded. With a little additional plastic strain, the stress-strain curve becomes a continuation of what it would have been had no unloading taken place. The hysteresis behaviour resulting from unloading and loading from a plastic strain is generally neglected in plasticity theories. A true stress-strain curve is frequently called a flow curve because it gives the stress required to cause the metal to flow plastically to any given strain. Many attempts have been made to fit mathematical equations to this curve. The most common is a power expression of the form

$$\sigma = K\varepsilon^n$$

Where K is the stress at e = 1.0 and n is the strain-hardening coefficient, is the slope of a log-log plot . This equation can be valid only from the beginning of plastic flow to the maximum load at which the specimen begins to neck down

4.3 True Strain:

Equation describes the conventional concept of unit linear strain, namely, the change in length referred to the original unit length.

$$e = \frac{\Delta L}{L_0} = \frac{1}{L_0} \int_{L_0}^{L} dL$$

This definition of strain is satisfactory for elastic strains where AL is very small. However, in plastic deformation the strains are frequently large, and during the extension the gage length changes considerably. Ludwik first proposed the definition of true strain, or natural strain, e, which obviates this difficulty. In this definition of strain the change in length is referred to the instantaneous gage length, rather than to the original gage length.

$$\varepsilon = \sum \frac{L_1 - L_0}{L_0} + \frac{L_2 - L_1}{L_1} + \frac{L_3 - L_2}{L_2} + \cdots$$

$$\varepsilon = \int_{L_0}^{L} \frac{dL}{L} = \ln \frac{L}{L_0}$$

This relationship is not valid for the principal conventional strains. The advantage of using true strain should be apparent from the following example: Consider a uniform cylinder which is extended to twice its original length. The linear strain is then e = (2Lo — Lo)/Lo = 1.0, or a strain of 100 per cent. To achieve the same amount of negative linear strain in compression, the cylinder would have to be squeezed to zero thickness. However, intuitively we should expect that the strain produced in compressing a cylinder to half its original length would be the same as, although opposite in sign to, the strain produced by extending the cylinder to twice its length. If true strain is used, equivalence is obtained for the two cases.

4.4 Yield criteria for ductile metals:

The problem of deducing mathematical relationships for predicting the conditions at which plastic yielding begins when a material is subjected to a complex state of stress is an important consideration in the field of plasticity. In uniaxial loading, plastic flow begins at the yield stress, and it is to be expected that yielding under a situation of combined stresses is related to some particular combination of the principal stresses. A yield criterion can be expressed in the general form but there is at present no theoretical way of calculating the relationship between the stress components to correlate yielding in a three-dimensional state of stress with yielding in the uniaxial tension test. The yielding criteria are therefore essentially empirical relationships. At present, there are two generally accepted theories for Predicting the onset of yielding in ductile metals.

4.5 Von Misses & Tresca yield criteria:

A somewhat better fit with experimental results is provided by the yield criterion given

$$\sigma_0 = \frac{1}{\sqrt{2}}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2}$$

According to this criterion, yielding will occur when the differences between the principal stresses expressed by the right-hand side of the equation exceed the yield stress in uniaxial tension. The development of this yield criterion is associated with the names of Von Mises, Hencky, Maxwell, and Huber. Von Mises proposed this criterion in the invariant form given primarily because it was mathematically simpler than the invariant form of the maximum-shear-stress theory. Subsequent experiments showed that Equation provides better over-all agreement with combined stress-yielding data than the maximum-shear-stress theory. A number of attempts have been made to provide physical meaning to the Von Mises yield criterion. One commonly accepted concept is that this yield criterion expresses the strain energy of distortion. On the basis of the distortion-energy concept, yielding will occur when the strain energy of distortion per unit volume exceeds the strain energy of distortion per unit volume for a specimen strained to the yield stress in uniaxial tension or compression.

The derivation on the basis of distortion energy is given below. Another common physical interpretation is that it represents the critical value of the octahedral shear stress. The total elastic strain energy per unit volume can be divided into two components, the strain energy of distortion, and the strain energy of volume change. To illustrate the resolution of total strain energy into its components. This figure illustrates the point that a general three-dimensional state of stress can be expressed in terms of a spherical or hydrostatic component of stress, a", and a stress deviator, a'. Because experiments have shown 1 that up to rather large values of hydrostatic pressure a hydrostatic state of stress has no effect on yielding, it is valid to assume that only the stress deviator can produce distortion.

4.6 Combined stress tests:

The conditions for yielding under states of stress other than uniaxial and torsion loading can be conveniently studied with thin-wall tubes. Axial tension can be combined with torsion to give various combinations of shear stress to normal stress intermediate between the values obtained separately in tension and torsion. For the combined axial tension.

$$\sigma_1 = \frac{\sigma_x}{2} + \left(\frac{\sigma_x^2}{4} + \tau_{xy}^2\right)^{1/2}$$

$$\sigma_2 = 0$$

$$\sigma_3 = \frac{\sigma_x}{2} - \left(\frac{\sigma_x^2}{4} + \tau_{xy}^2\right)^{1/2}$$

Therefore, the maximum-shear-stress criterion of yielding is given by

$$\left(\frac{\sigma_x}{\sigma_0}\right)^2 + 4\left(\frac{\tau_{xy}}{\sigma_0}\right)^2 = 1$$

and the distortion-energy theory of yielding is expressed by

$$\left(\frac{\sigma_x}{\sigma_0}\right)^2 + 3\left(\frac{\tau_{xy}}{\sigma_0}\right)^2 = 1$$

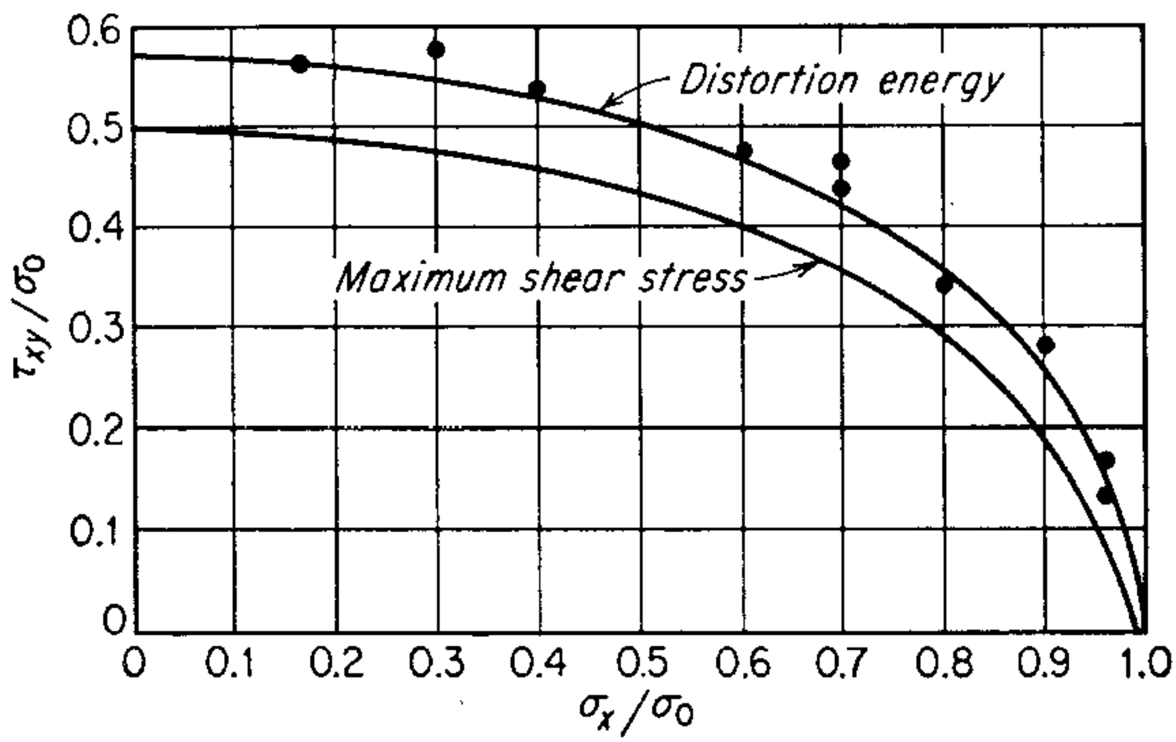


Fig 2.2 Comparison between maximum-shear-stress theory and distortion energy (Von Mise's) Theory

4.7 The yield locus:

Another type of combined stress test is to subject thin-wall tubes to axial load and internal hydrostatic pressure. Since the stress in the radial direction is negligible (0-3 = at the outer free surface), this test provides a biaxial state of stress. For the plane-stress condition, the distortion-energy theory of yielding can be expressed mathematically by

$$\sigma_1^2 + \sigma_3^2 - \sigma_1\sigma_3 = \sigma_0^2$$

4.8 Anisotropy in yielding:

A convenient way of comparing yielding criteria for a two-dimensional state of stress is with a plot such as Fig. 4. Note that the maximum shear- stress theory and the distortion-energy theory predict the same yield stress for conditions of uniaxial stress and balanced biaxial stress. The greatest divergence between the two theories occurs for a state of pure shear. It has already been shown that for this state of stress the shear-stress law predicts a yield stress which is 15 per cent lower than the value given by the distortion- energy criterion. A very sensitive method of differentiating between the two yields criteria is the procedure adopted by Lode of determining the effect of the intermediate principal stress on yielding. According to the maximum-shear-stress law, there should be no effect of the value of the intermediate stress. For the distortion-energy theory, to account for the influence of the intermediate principal stress, Lode introduced the parameter n, called Lode's stress parameter.

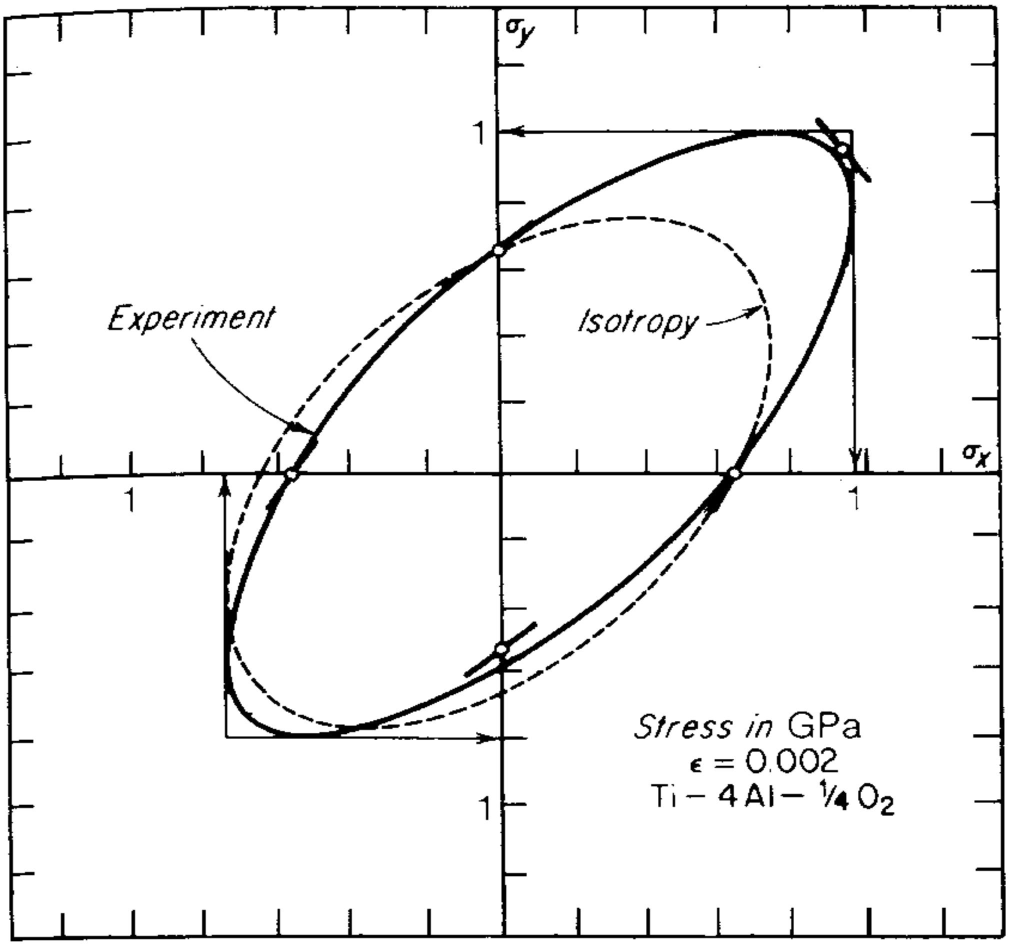
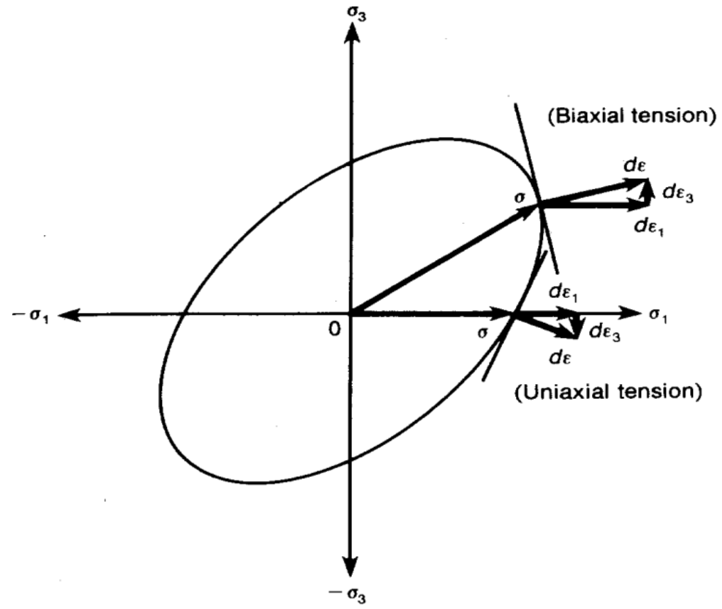


Fig 2.3. Yield locus for textured titanium-alloy sheet

$$F(\sigma_y - \sigma_z)^2 + G(\sigma_z - \sigma_x)^2 + H(\sigma_x - \sigma_y)^2 + 2L\tau_{yz}^2 + 2M\tau_{zx}^2 + 2N\tau_{xy}^2 = 1$$

4.9 Yield surface:

The octahedral stresses are a particular set of stress functions which are important in the theory of plasticity. They are the stresses acting on the faces of a three-dimensional octahedron which has the geometric property that the faces of the planes make equal angles with each of the three principal directions of stress. For such a geometric body, the angle between the normal to one of the faces and the nearest principal axis is 54°44', and the cosine of this angle is l/-\/3. The stress acting on each face of the octahedron can be resolved into a normal octahedral stress and an octahedral shear stress lying in the octahedral plane. The normal octahedral stress is equal to the hydrostatic component of the total stress.



$$\sigma = (\sigma_1 + \sigma_2 + \sigma_3)/3 = \sigma_m$$

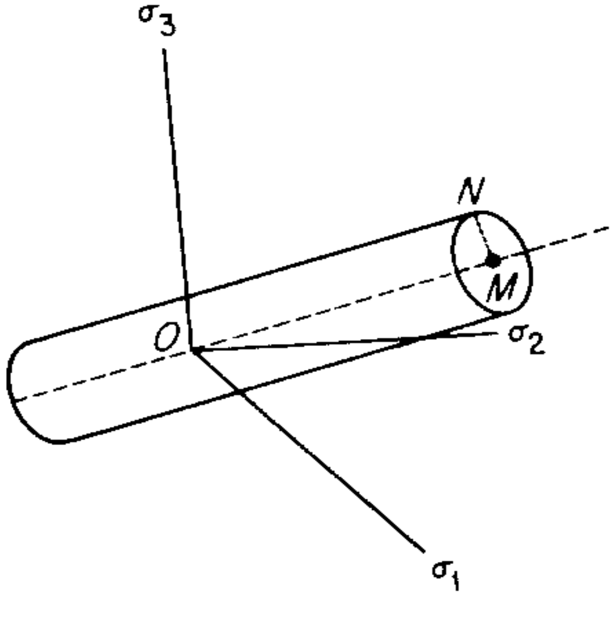


Fig 2.4 Yield surface von Mises' criterion

4.10 levy –Mises equation

Once the yield criterion is satisfied, we can no longer expect to use the equations of elasticity. We must develop a theory to predict plastic strains from the imposed stresses.
When a body is subjected to stresses of sufficient magnitude, it will plastically deform (or fracture). The natures of the stresses depend on the particular forces applied to the body and, often, the same resulting deformation may be achieved by applying forces in different ways. For instance, a ductile metallic rod may be extended (elongated) a given amount either by a single force along its axis (i.e. a tensile stress) or by the combined action of several forces acting in different directions (i.e. multi-axial loading). A simple example of the latter multi-axial loading situation to obtain the same extension in the metallic rod as that obtained in pure tension is to apply a reduced tensile stress while simultaneously compressing the rod along its length. Under such multi-axial loading, the behaviour of ductile metallic materials can be described by the Lévy-Mises equations, which relate the principal components of strain increments during plastic deformation to the principal applied stresses.
In general, there will be both plastic (non-recoverable) and elastic (recoverable) strains.
However, to a first approximation, we can ignore the elastic strain assuming that the plastic strains will dominate in a deformation processing situation. We can therefore treat the material as a rigid-plastic, i.e. a material which is perfectly rigid prior to yielding and perfectly plastic afterwards.

Since plasticity is a form of flow, we can relate the strain rate, $\frac{d\varepsilon}{dt}$ to stress σ.
Plastic flow is similar to fluid flow, except that any rate of flow (strain rate) can occur for the same yield stress.

From symmetry we can show that in an isotropic body, the principal axes of stress and strain rate coincide, i.e. it goes the way you push it.

With respect to principal axes $\frac{\dot{\varepsilon}_1}{\sigma_1'} = \frac{\dot{\varepsilon}_2}{\sigma_2'} = \frac{\dot{\varepsilon}_3}{\sigma_3'}$,

where $\dot{\varepsilon}_i = \frac{d\varepsilon}{dt} \quad (i = 1, 3)$, the normal strain rate parallel to i^{th} axis.

σ_i' = <u>deviatoric component</u> of normal stress parallel to the i^{th} axis.

and $\sigma_1' = \sigma_1 - \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$,

If we consider small intervals of time δt, and call the resultant changes in strain $\delta\varepsilon_1$, $\delta\varepsilon_2$, $\delta\varepsilon_3$, it follows that,

$$\frac{\delta\varepsilon_1}{\sigma_1 - \frac{1}{2}(\sigma_2 + \sigma_3)} = \frac{\delta\varepsilon_2}{\sigma_2 - \frac{1}{2}(\sigma_3 + \sigma_1)} = \frac{\delta\varepsilon_3}{\sigma_3 - \frac{1}{2}(\sigma_1 + \sigma_2)}$$

(the **Lévy-Mises equations**.)

As $\frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$ is an invariant of the stress tensor, it also turns out that these equations apply even if stresses and strains are not referred to principal axes, so

$$\frac{\delta\varepsilon_{11}}{\sigma_{11} - \frac{1}{2}(\sigma_{22} + \sigma_{33})} = \frac{\delta\varepsilon_{22}}{\sigma_{22} - \frac{1}{2}(\sigma_{33} + \sigma_{11})} = \frac{\delta\varepsilon_{33}}{\sigma_{33} - \frac{1}{2}(\sigma_{11} + \sigma_{22})}$$

for a general stress tensor and plastic strain increments $\delta\varepsilon_{11}$, $\delta\varepsilon_{22}$ and $\delta\varepsilon_{33}$.

The above Lévy-Mises equations describe precisely the relationships between the normal stresses (arising from any general applied stress situation with respect to a particular set of orthogonal axes) and the resulting normal plastic strains (deformation) of a body referred to the same set of orthogonal axes. In many situations, the precise stresses are not known accurately and so more empirical approaches can be very helpful in describing the deformation of a body when subjected to applied forces. A number of these approaches are considered in this TLP. However, several require further constraints, in particular the need to work in two dimensions and this introduces the concepts of **plane stress** and **plane strain**.

MODULE 3

Analysis of deformation processes- Method based on homogeneous compression slip line field theory, Upper bounds and lower bounds, Slab method of analysis.

Flow stress determination, Hot working, Cold working, Strain rate effect, Friction and lubrication, Deformation zone geometry, Workability, Residual stress.

6.1 Slip line field theory:

6.1.1 Methodology of slip line field analysis:

Slab analysis of the forming process is considered approximate due to the assumption of homogeneous deformation of material. Slip line field analysis is more accurate as it considers the non-homogeneous deformation also. This method is widely applied for forming processes such as rolling, strip drawing, slab extrusion etc. Slip line field analysis is based on the important assumptions that the deformation of material is plane strain type, no strain hardening of the material, constant shear stress at interfaces, the material is rigid plastic.

The general methodology of this analysis can be described by the following steps:

First differential equations in terms of mean stress and deviatory stress for plane strain deformation are formulated

Slip line field is constructed graphically out of orthogonal maximum and minimum shear lines.

From known stress at some point, the integral constants are determined. From this the forming load can be found.

Before we proceed to understand the methodology of the analysis a few definitions should be considered.

What are slip lines? They are planes of maximum shear, which are oriented at 45 degrees to the axes of principal stresses. Maximum and minimum slip lines are orthogonal.

What is plane strain deformation? It is a type of plastic deformation in which the material flow in one of the three principal directions is constrained. The material strain in the third direction is zero. This is possible by the application of a constraint force along the third direction. All displacements are restricted to xy plane, for example. Examples for this type of deformation include strip rolling, strip extrusion etc.

Constraint to deformation along the third axis could be introduced either through the die wall or through the rigid material adjacent to deforming material, which prevents the flow.

The basis for slip line field analysis is the fact that the general state of stress on a solid in plane strain deformation can be represented by the sum of two types of stresses, namely the mean stress and the pure shear stress.

For plane strain condition we have $\sigma_2 = \sigma_1 + \sigma_3/2$

We can write the Tresca criterion for plane strain as: $\sigma_1 - \sigma_3$ = 2k

For plane strain deformation we have the equilibrium of stresses written in differential form as:

$$\frac{\delta\sigma_x}{\delta x} + \frac{\delta\tau_{xy}}{\delta y} = 0 - - - - - - - - - - - - (3.1.1)$$

$$\frac{\delta\tau_{xy}}{\delta x} + \frac{\delta\sigma_y}{\delta y} = 0 - - - - - - - - - - - - (3.1.2)$$

These two differential equations will be transformed into two algebraic equations along a changed coordinate system, namely, along two directions of maximum shear. Then they can be solved subjected two suitable boundary conditions.

Consider the plane strain state of stress acting on x-y plane this plane. Let σ_x , σ_y , τ_{xy} are stresses acting on the plane.

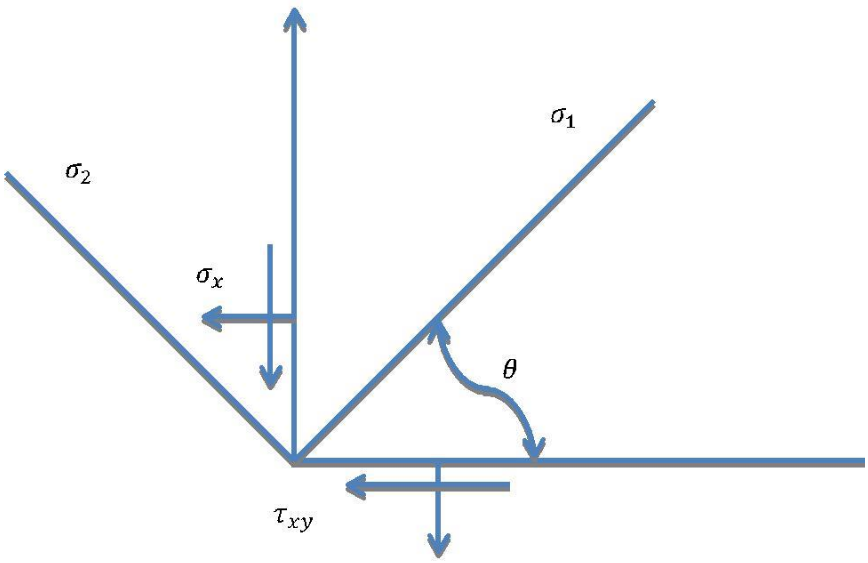


Fig 3.1 Stresses in Plane strain condition

For plane strain condition, we have:

$$\sigma_1 - \sigma_3 = 2K - (3.1.3)$$

and

$$\sigma_2 = (\sigma_1 + \sigma_3)/2 = -p(hydrostatic\ stress) - - - - - - - - (3.1.4)$$

For the stress condition we can write:

$\sigma_x = [(\sigma_1 + \sigma_3)/2 + (\sigma_1 - \sigma_3)/2] * \cos 2\theta$ — — — — — — — — — — (3.1.5)

$\sigma_y = [(\sigma_1 + \sigma_3)/2 - (\sigma_1 - \sigma_3)/2] * \cos 2\theta$ — — — — — — — — — — (3.1.6)

$\tau_{xy} = (\sigma_1 - \sigma_3)/2] * \sin 2\theta$ — — — — — — — — — — — — — — — — — —(3.1.7)

Finally we get through all the equation

$$\delta(-p)/\delta x - 2K sin2\theta\ \delta\theta/\delta x + 2K cos2\theta\ \delta\theta/\delta y = 0$$

$$\delta(-p)/\delta y + 2K cos2\theta\ \delta\theta/\delta x + 2K sin2\theta\ \delta\theta/\delta y = 0$$

6.1.2 Illustration of the slip line field analysis:

Consider the extrusion of a strip through a square die. Assume a reduction of 50%. See figure below

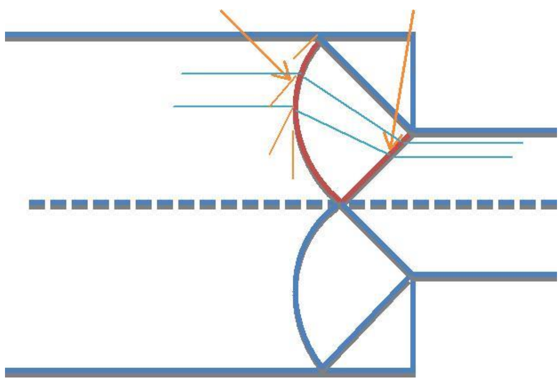


Fig.3.2. Slip lines and hodograph for axisymmetric extrusion

The slip lines are shown as radial and circular lines in the figure. The deformation field is symmetrical about centre line. Therefore we may analyse one half of the deformation region. The hodograph – velocity diagram is also shown above.

Material undergoes velocity discontinuity along . The velocity discontinuities are shown in hodograph. Similarly the velocity vectors are shown in hodograph as lines radiating from top left corner of the hodograph.

The horizontal line in hodograph represents the velocity vector of the particles before they enter the total length of the horizontal line in hodograph represents the exit velocity of the material, which is twice the initial velocity in this case – because we assume the reduction as 50%.

We need to find the punch pressure p. Stresses acting along the shear stress k and hydrostatic pressure p.

6.2 Upper bounds and lower bounds:

Slip line field analysis has limited application in forming in view of its applicability to plane strain deformation only. A more accurate and general analysis for determination of forming load is the application of limit theorems. There are two limit theorems, upper bound and lower bound. The lower bound theorem is not widely used for forming because it under estimates the forming load. Upper bound analysis overestimates the forming load. Therefore, upper bound analysis is widely used for accurately predicting forming loads. It is applicable to almost all types of forming. One should get the solution to the forming problem so that the solution should be kinematically and statically admissible. Kinematically admissible means the velocity field chosen should satisfy the expected boundary conditions for the forming process as well as satisfy the requirement of incompressibility. In upper bound we expect the kinematically admissible condition to be satisfied by the solution. Rigid body motion is assumed for the deforming material –in the form of triangular elements. This could satisfy the requirement of kinematically admissible velocity field. The velocities of various parts of the deforming material are represented in diagrams called hodographs. One has to choose a trial velocity field such that it is closer to the actual velocity field expected in the forming process.

3.2.1 Upper bound theorem:

It states that for a given set of velocity fields, the velocity field which minimizes the total energy is the nearest to the actual solution. In other words, this velocity field minimizes the function:

The upper-bound theorem can also be stated in a different way. It states that the estimate of the force obtained by equating the internal energy dissipation to external forces is equal to or greater than the correct force. We should assume a suitable flow field for the deformation. In short, the field which minimizes the energy dissipation rate, given below, is the required field:

$$\dot{E} = \int_V \bar{\sigma}\,\dot{\bar{\varepsilon}}\,dV + \int_{Sd} k|\Delta V_t|\,dS + \int_{Sf} mk|\Delta V|\,dS = \text{Forming Load X Velocity of punch/die} \text{ -------}(3.2.1)$$

The first term on right hand side is the rate of work done due to plastic straining, the second term is the rate of energy dissipated in internal velocity discontinuity and the third term represents power consumed for friction. Generally for continuous velocity field the second term can be ignored.

In a nutshell, we could say that the rate of external work done in the process is equal to internal power required for homogeneous deformation plus rate of work done in shear or redundant deformation plus rate of work done for overcoming friction. In the following example we will illustrate the methodology for determination of work done in shear deformation of a material. Subsequently, we will know how forming load could be determined applying the upper bound theorem.

3.2.2 Example: Determination of shear work done:

The general methodology of the analysis involves, first, assuming a flow filed within the deforming material that will suitable reflect the material flow. The flow field is otherwise called velocity field. Next step is to find the rate of energy consumption or the rate of work done for this flow field. Finally, the external work done is equated to the energy for the flow field. From this we can solve for the forming load.

The shear work or the rate of work in shearing a material can be determined easily from an assumed flow field for the forming process. Fist we must assume a suitable velocity field. We must draw the velocity vector diagram, called hodograph for the assumed flow field, which should be kinematically admissible. Once drawn, we can determine the shear work done or energy dissipated in shear. Equating the energy dissipated in shear to the rate of external work done, one can determine the forming force (under ideal condition). This analysis is based on the assumptions: deformation is homogeneous without work hardening, there is no friction or there is sticking friction at interface and the flow is two dimensional.

Let us consider a simple example to illustrate this approach. Consider a rigid element of the deforming material, pqrs, which is along the x axis. See figure below. Let this element have a velocity of V1. Let this element pass through a plane yy. After passing through the plane the element changes direction, attains a velocity of V2. It also gets distorted to a new shape, p’q’r’s’. The plane yy can be called shear plane, as it causes the shear of the material.

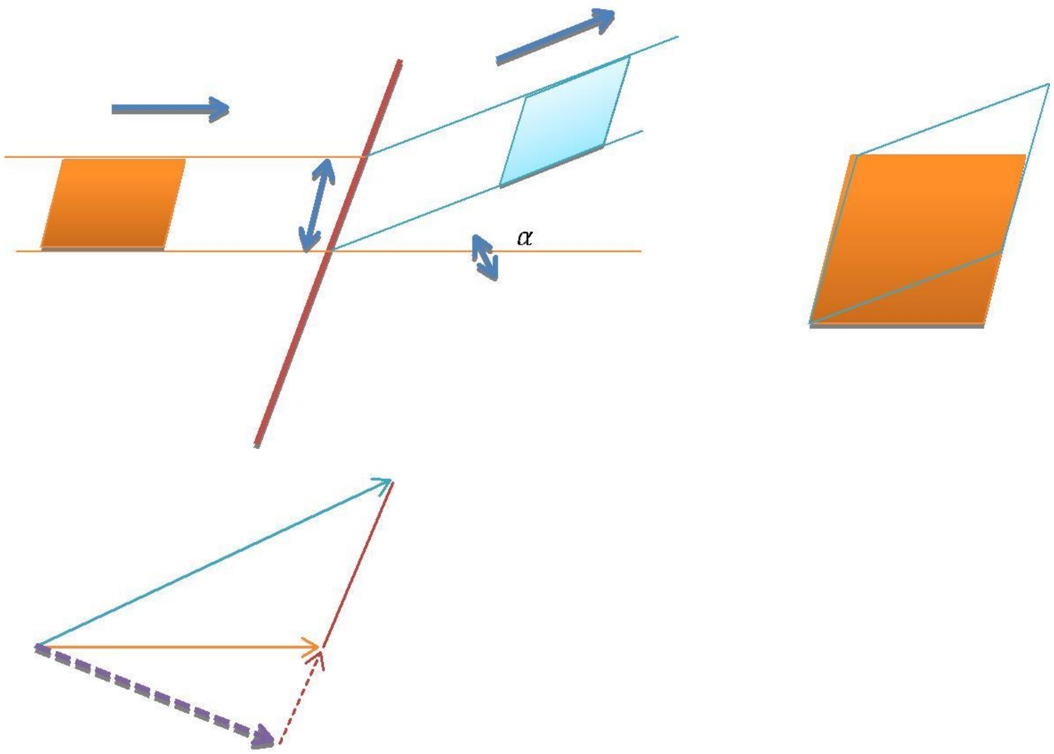


Fig. 3.3 Shear deformation of a plane element through the shear plane y-y’ and hodograph

3.3 Forming of slab-line method:

Forming of materials is a complex process, involving either biaxial or tri-axial state of stress on the material being formed. Analysis of the forming process therefore is highly involved. Prediction of forming load in a particular process is rather empirical. However, fairly accurate methods have been developed in order to predict the forming process and process parameters. Some of the early methods of forming analysis include slab analysis; slip line field analysis, upper bound analysis etc. With the availability of high speed computers, we can depend on finite element method for accurate predictions of forming loads. Numerous metals forming software has been developed based on finite element procedures for complex shapes with more realistic boundary conditions. In this lecture we will discuss the simple slab method of forming analysis, with a typical example.

Slab method is a simple analytical procedure based on principles of mechanics. We can assume a simple relation between forming load and material flow stress in the form: F = k A, where k is an empirically determined constant which takes into account friction, redundant deformation etc. The general methodology involved in slab method can be stated as follows: First the material under deformation is sliced into infinitesimally small portions. Then force balance is made on the small element. From force balance a differential equation in terms of the forming stress, geometric parameters of the billet and friction coefficient is formulated. This differential equation is solved with suitable boundary conditions. The solution gives us the required forming stress. This method may involve some simplifying assumptions. Hence this method may be considered approximate. Moreover, it may not be easy to apply this method for more complex forming processes, such as impression die forging. Slab method is developed with the assumption that the material flow is homogeneous during forming.

3.3.1 Slab method- Upsetting a ring:

Let us try to understand the slab method of forming analysis with the help of a simple example. Sliding or Columbic friction often occurs at the material tool interface. As a result of friction the forming load is enhanced. The flow of material is also non-uniform due to friction. Another type of friction condition, namely, shear friction or sticking friction could be convenient to consider in the analysis. In shear friction model, we assume the frictional shear stress to be proportional to shear yield strength of the material. Thus we have: $\tau = mk$, where m is friction factor and k is shear yield strength. The following assumptions are the basis of the slab analysis:

1. The reference axes are in the directions of the applied stresses
2. Friction does not cause non-uniform deformation. Therefore material is assumed to deform homogeneously – a plane remains a plane after deformation.

Consider the homogeneous deformation of a ring shaped specimen subjected to upsetting force. Let us assume shear friction at tool-material interface. The ring compression process is widely used for finding the coefficient of friction for given condition of friction. Consider an elemental portion of the ring specimen and the various stresses on this element. The following diagram shows the stresses acting on the elemental part of the ring.

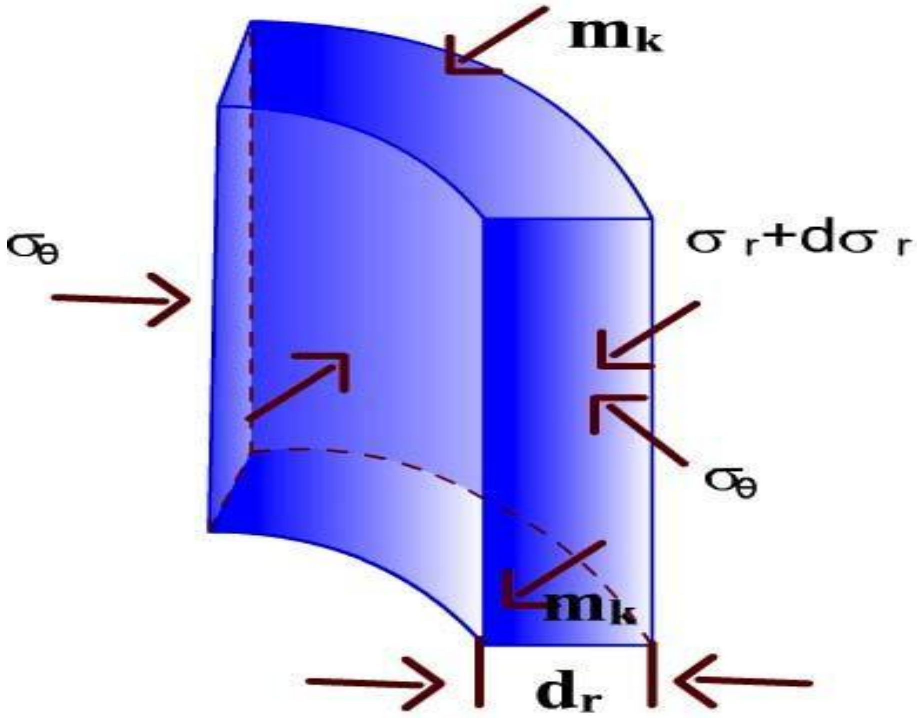


Fig. 3.4 Stresses acting on elemental ring subjected to upsetting

Consider a small sector of an elemental ring of radius r, radial thickness dr, height h and the angle of the sector as d . The ring is subjected to upset force F, which is to be determined.

The various stresses acting on the sector are:

Radial stresses   The corresponding forces are: rd h and   (r+dr)d h

Hoop stress   The corresponding force is given by: dr h sin

Frictional shear stress mk The corresponding force is: mk r drd

The arc length of the sector element is given by: rd

We can also note that for uniform deformation of the ring,

There exists a neutral radius in the ring, such that the material deformation happens towards the axis for radii less than the neutral radius. There is a decrease in diameter of the ring.

For radii greater than the neutral radius, the material flow is away from the axis-axially outward. This condition exists because of friction. Therefore, the friction force is observed to act axially outward within the neutral section. It acts radially inward in sections beyond the neutral section.

For continuity of the stresses, we can take the neutral section radius as:
Also note that k =   according to von Mises yield criterion
One can get the average upset force, F from the local stress as followed:

$$F = (1 + 1/2\sqrt{3} * m/h\,(R_0 - R_i))\sigma A$$

In the above equation the bracketed term represents the factor which accounts for friction effect during the forming. The limitation of uniform deformation assumption in slab method is overcome in another method of analysis called slip line field analysis, which is discussed in the next lecture.

The upset force is found to vary linearly with the friction factor m, as observed from the above equation. Further, we also note that the forming force required increases with reduction in height of the ring. Rings of smaller height require greater forming force as compared to rings of larger height. This is expected because the redundant deformation zone extends towards centre for rings of smaller height.

Ring compression test is a simple test for determination of friction factor or the coefficient of friction. It can also be used for studying the lubrication characteristics of different lubricants.

3.4 Flow stress:

Flow stress is the stress required to sustain a certain plastic strain on the material. Flow stress can be determined form simple uniaxial tensile test, homogeneous compression test, plane strain compression test or torsion test.In forming of materials, we are concerned with flow stress of material being formed, as this affects the ability of material to undergo deformation. Factors such as strain rate, temperature, affect the flow stress of materials. A simple power law expression for flow stress of a material which does not show anisotropy can be expressed as: $\boldsymbol{\sigma = k\varepsilon^n}$ where n is known as strain hardening exponent.

Higher strain hardening exponent values enhance the flow stress. Similarly, flow stress is enhanced with increase in strain rate during a plastic deformation process. Effect of strain rate on flow stress becomes more pronounced at higher temperatures. AT higher temperatures [hot working], strain hardening may not have effect on flow stress. However, during cold working effect of strain on flow stress cannot be neglected. In such cases, average flow stress can be determined between two given strains. In hot forming, temperatures of working are above recrystallization temperature.

Therefore, the grains of the metal get elongated along direction normal to applied force, giving raise to anisotropy. During recovery process, locked up dislocations get released. Residual stresses are reduced. Recrystallization of new grains can happen when the metal gets heated above recrystallization temperature. Secondary grain growth may follow primary recrystallization. In hot working, metal may get softened after hot deformation process. Recrystallized grain size affects the flow stress of material. A general expression for flow stress, encompassing temperature, strain, strain rate, recrystallization has been given in the form:

$$\sigma = \frac{2}{\sqrt{3}(1-m)} K\varepsilon^{n}\dot{\varepsilon}^{m}\exp\left(-\beta T\right)$$

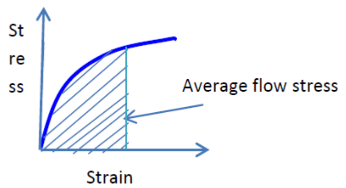
n is strain hardening exponent, m is strain rate sensitivity exponent, T is temperature.

Materials are subjected to complex states of stresses during forming. Stress required for forming, yield or flow stress therefore depends on several factors, such as strain, strain rate, temperature etc.

From the uniaxial tensile test, one can understand material behaviour considerably. Form the tensile test data, we can determine flow stress, though this method has limitations due to localized deformation called necking. Flow curve is the stress-strain curve for a material in the plastic range. It describes material behaviour in metal forming. From flow curve, we can determine the flow stress as: $\sigma = k\varepsilon^{n}$

The average flow stress is given as: $\sigma_{av} = \frac{K\varepsilon^{n}}{1+n}$

where ε is maximum strain during deformation process and n is strain hardening exponent.



Knowing the final strain in the forming process, one can calculate flow stress using above equation.

3.4.1 Work done in deformation processing:

The plastic strain energy during deformation of a material is defined as energy stored in the material when it gets plastically deformed. The work done during the deformation is stored as strain energy. For plastic deformation under uniaxial deformation, it is given by the expression: dW = Force X change in length of the specimen = Stress X Initial Area X Strain X Initial length This can be written as:

Strain energy per unit volume = stress X strain increment du = σdε

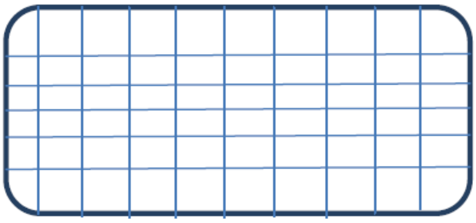
This upon integration between zero strain and a finite plastic strain ε, gives:

$$u = \frac{K\varepsilon^{n+1}}{1+n} = \sigma_{av}\varepsilon$$

For tri-axial stress, the plastic work per unit volume is given as:

$$du = \sigma_1 d\varepsilon_1 + \sigma_2 d\varepsilon_2 + \sigma_3 d\varepsilon_3$$

This energy represents the minimum energy required for deformation without friction, redundant deformation etc. In reality, deformations happen with friction at work piece-tool interface. Further, there is in- homogeneous deformation due to friction. Such inhomogeneous deformation leads to additional shear deformation. This is called redundant deformation because shearing is not a part of the desirable shape change of the material. Work is involved in shearing material. This work is known as redundant work.



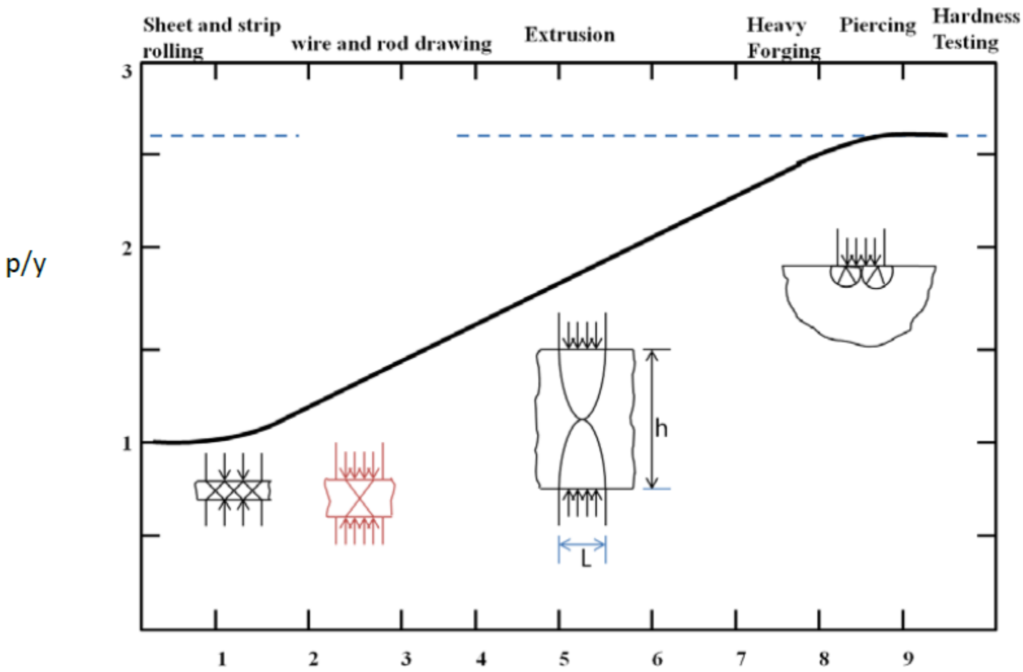
3.4.2 Deformation zone geometry:

Bulk deformation of materials happen generally within converging die shapes. The small region of the die through which the metal is subjected to plastic deformation is called deformation zone. Geometry or shape of the deformation zone affects the redundant deformation work, and hence the total forming force.

Deformation zone geometry also affects the residual stress, internal defects during forming. Deformation zone geometry is defined by the parameter Δ. This parameter is defined as ratio of thickness or height of deformation zone to contact length of deformation zone.

$\Delta = h / L$ For extrusion it can be shown that $h = (h_o + h_f)/2$ and $L = (h_o - h_f)/2\sin\alpha$. This factor increases with increasing die angle and decreases with reduction. Redundant strain is expressed by a parameter ϕ, which is a function of Δ.

$\phi = 1 + \Delta/4$ for plane strain deformation As deformation zone geometry increases, redundant strain increases. This leads to increase in redundant work.



As observed from the above diagram, the deformation pressure increases with increase in deformation zone geometry, due to increase in redundant work. Also smaller h/L value, lower is forming pressure – in the absence of friction. However, effect of friction increases in smaller h/L cases. High Δ leads to high residual stress in the formed part. Friction in forming is considered in the next section. The total energy required during forming can be now written as: $u_{total} = u_{ideal} + u_{friction} + u_{redundant}$

Forming efficiency is now defined as $\eta = u_{ideal} / u_{total}$. Rolling process has a high efficiency of 75 to 95% due to its low redundant deformation and smaller deformation zone geometry. Whereas, processes with high deformation zone geometry, such as forging and extrusion have low efficiency, 30 to 65%.

3.5 Friction in metal forming:

Surface asperities on two surfaces in contact get interlocked with each other. When a surface tends to slide against another stationary surface, say a die surface, there is a shear stress induced at interface which opposes the flow of material. This condition is called sliding friction. Condition of sliding friction or sticking friction can arise at the interface between the work piece and die/tool in forming operations. Sliding friction arises due to surface shear stress opposing the metal flow. Friction is undesirable as it increases the deformation work required, leads to non-homogeneous deformation of material, causes tool wear, causes residual stress in the product and may lead to cracking of surface.

Coefficients of friction in forming processes are quiet high. If coefficient of friction becomes very high it leads to a situation called sticking friction.

In this case, the surface shear stress exceeds shear yield strength of material, the two surfaces adhere to each other. Metal beneath the surface undergoes shear deformation.

Theoretical forming pressure without friction is: $p = \sigma_f$[flow stress]. With sliding friction, having coefficient of Coulomb friction $\mu = \tau/p$, the forming pressure increases exponentially along the interface, as given below for a disc under forging:

$P = \sigma_f(\exp^{2\mu/h(a-r)})$, where h is height of billet, a is radius of the cylindrical billet.

As a result of friction, forming pressure rises exponentially, forming a friction hill in the lateral direction. Friction factor m is sometimes used in place of coefficient of friction.

$m = \tau / k$, where k is shear yield strength.

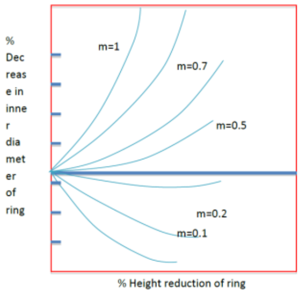
m is independent of normal pressure at interface and it is easy to measure.

Use of Coulomb coefficient of friction is sometimes misleading, as we find from the definition of coefficient of friction that μ decreases with increasing pressure, which is not correct. Therefore, m is preferred in analysis of friction– especially in hot working.

Lubrication is necessary in order to reduce friction in metal forming. For cold forming operations, fats, fatty acids, mineral oils, soap emulsions are generally used. For hot forming, glass, graphite, mineral oils can be used as lubricants. The coefficient of friction μ or friction factor m can be measured using the ring compression test.

In this test, a ring of OD:ID:Height = 6:3:1 is subjected to axial compression. With incremental load applied, the change in inner diameter of the ring is taken to be a measure of friction factor. If there is no friction, the inner diameter of the ring increases. With friction, there Is decrease in inner diameter of the ring.

The test can be repeated for different types of lubricants – under varying μ. With the calibration curves drawn between change in inner diameter and height reduction, one can find out the friction factor for given condition of interfacial friction. There is no need to measure the deformation force in the test.



3.6 Effect of temperature in metal forming:

Forming process requires stress above flow stress of the material being deformed. The effect of external work done on work piece during forming is converted into heat. About 5 to 10% of the work is stored within as internal energy. Friction can also result in heating and increase in internal energy of work piece. Assuming frictionless deformation, the temperature increase during metal forming operation can be written as:

$\Delta T = u_{plastic} / \rho C_p$, where $u_{plastic}$ is plastic work done per unit volume of work piece. C_p is specific heat and ρ is density.

With friction,

$\Delta T = \lambda u_{total} / \rho Cp$, where λ is fraction of deformation work converted to heat. Normally, λ = 0.95 to 0.98.

Temperature rise is calculated using stress-strain curve, as the plastic work is calculated as the area under stress-strain curve for plastic flow. For slow deformations, the temperature rise of the work piece may be small as the heat generated gets dissipated through the die, surrounding air, etc. However, adiabatic condition may prevail under large deformation speeds, resulting in large rise in temperature of the work piece. This may cause incipient melting. Therefore, strain rate also influences the temperature rise during working. For low carbon steel, the temperature rise for a true strain of 1 has been estimated to be 553 K. This is without heat lost from the billet.

3.6.1 Cold and hot forming:

Cold forming is carried out at a temperature lower than recrystallization temperature of the work piece material. Hot working is a process carried out at temperatures above recrystallization temperature, namely, 0.6 T_m. High strain rates – 0.5 to 500 s^{-1} are involved in hot working. No strain hardening takes place in hot working. Processes of recrystallization, recovery and grain growth dominate in hot working. Energy required for hot working is low, as flow stress decreases with increase in temperature. Large strains (ε = 2 to 4) are possible in hot forming because of recovery process. Due to oxidation on surface during hot working, poor surface finish and poor dimensional tolerances are inherent defects. Die wall chilling may result in non-uniform material flow. Upper limit for hot working is hot shortness, in which the metal becomes brittle above a certain temperature due to grain boundary melting or melting of low melting phase such as sulphur in steel.

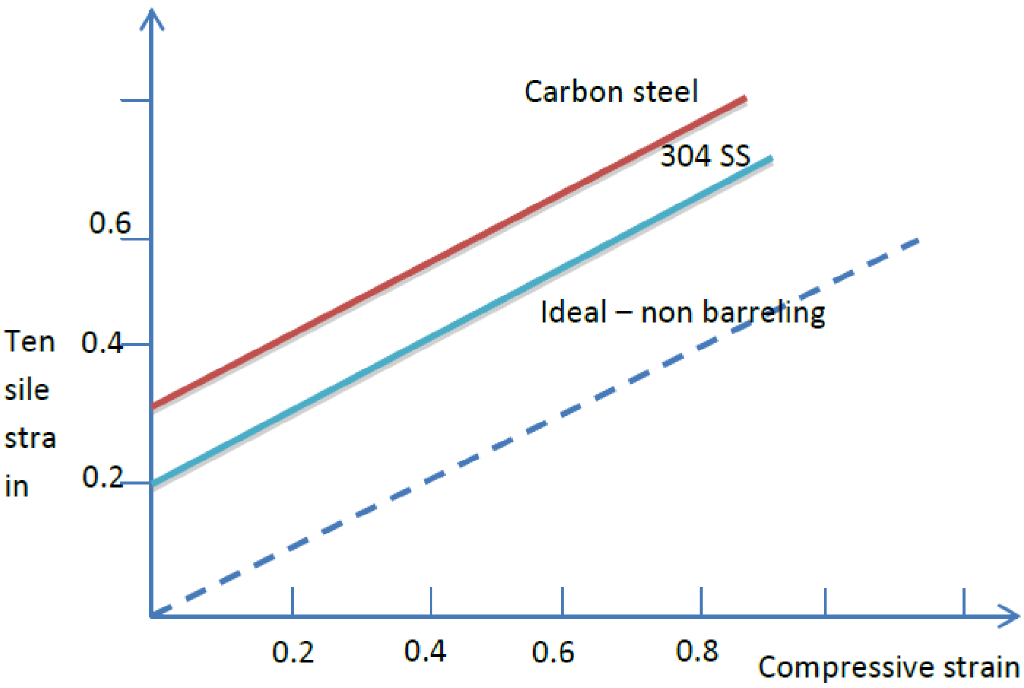
During hot working, material softening happens due to two mechanisms – dynamic recovery or dynamic recrystallization. In dynamic recovery, dislocation cross-slipping, climbing occurs. This mechanism is predominant in high stacking fault energy metals, with low activation energy for creep. On the other hand for metals with low stacking fault energy, like copper, nickel, the dynamic recrystallization is predominant mechanism of softening. During hot working static recovery can happen in between the working phases, thereby softening the metal. Rapid cooling after hot working may bypass this static recovery, thereby retaining the high strength of the metal. Strain induced precipitation or phase transformation can increase the flow stress, reduce ductility. Age hard enable (Al) alloys are subjected to freezing temperatures before forming, to suppress precipitation during forming. Thermo mechanical treatments can be used for achieving optimum levels of strength and hardness. Cold working leads to work hardening. The formed material may have to be annealed to relieve internal stresses and reduce hardness and strength after cold forming. However, if high strength and hardness are desirable, normally cold worked structure is retained. Cold working has high dimensional accuracy. Working on a metal at temperatures above room temperature but below recrystallization temperature is called warm working. Warm working may have the advantages of reduced working pressures, reduced levels of residual stresses and oxidation, improved surface finish and dimensional accuracy.

3.7 Workability:

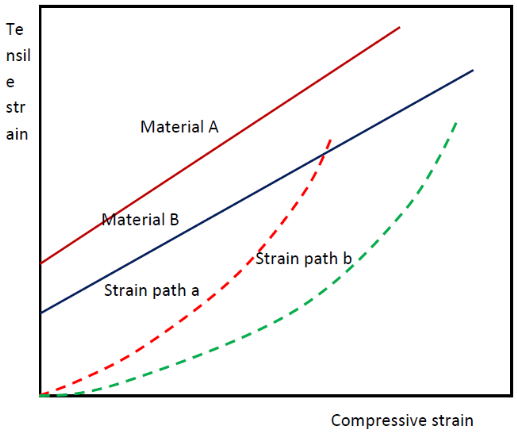
Materials differ in their ability to undergo plastic deformation. The extent of plastic deformation in a material is dependent on the materials grain structure, nature of bonding, presence of defects like dislocation and external factors such as temperature. Workability is the ease with which a material can be subjected to plastic deformation to achieve the desired shape without crack formation. In case of ductile materials the limit of forming is dictated by the beginning of necking. Once necking starts, due to localized deformation, further deformation of the work piece to finished shape becomes impossible. Therefore, in most of materials, the starting of necking is considered as the limit of working or forming. Workability is dependent on material characteristics and external factors such as tool and die geometry, friction, strain rate etc. The other criterion for workability may be the formation of cracks on the surface or within the material during the forming process. Cracks on external surface may form due to excessive tensile loads or friction. Internal cracks may form due to the presence of voids, second phase particles etc. Necking during tensile deformation may result in formation of voids, which may grow in size during loading. Cracks result due to excessive growth of voids and their coalescence. In compressive loading, generally surface cracks are formed due to excessive tensile stresses induced on the bulged surfaces. Bulging is a non-uniform deformation during compressive loading of billets.

A generalized fracture criterion may serve as a way of establishing workability of ductile materials. Combinations of stress and strain in ductile materials can lead to fracture unless the tensile stress induced reaches a critical value. More easily, tensile and compressive strains are correlated with each other in order to arrive at a criterion for workability. The simple upset test serves as effective technique for developing the workability limits for materials. By varying the diameter to height ratio of cylindrical billets, which are subjected to simple upset test, one can develop fracture criteria.

The following graph is developed from simple upset test:



In the figure shown above, the broken line having a slope of ½ represents fracture limit for an ideal material subjected to upsetting without bulging. Any combination of tensile and compressive strains which lead to fracture is represented by a point located above this line. Similar fracture criteria lines for stainless steel and carbon steel are also shown. Any combination of strains represented by points below the limit line will not cause fracture. Note that the tensile strains at fracture are found out from bend test. H.A.Khun developed a workability diagram which includes the process factor in addition to the strain limit factor (a material factor) for fracture.



The strain paths, represented in figure as dashed lines, are obtained by drawing grid lines on surface of a model which is subjected to upset test. The solid lines represent fracture limit. For material B with strain path a as chosen mode of deformation, fracture is sure to take place at the strain represented as the point of intersection between the fracture limit line and strain path line. If on the other hand the strain path b is chosen for either material, the fracture is not likely to occur within the working limits of the forming (upsetting).

MODULE-4

Syllabus: Analysis of metal forming processes (only limited portion), Forging: Load calculation in plane strain forging, Rolling: Forces & geometrical relationship in rolling, Rolling load and torque in cold rolling, Von-Karman work equation, Extrusion: Analysis of extrusion process, extrusion pressure, Drawing: Drawing load.

4.1 Introduction and classification of forging processes

4.1.1 Introduction:

Bulk deformation processes involve shaping of materials to finished products which have small surface area to thickness or surface area to volume ratio. Sheet metal forming produces parts having large surface area to thickness ratio. In sheet metal forming thickness variations are not desirable. Examples for sheet metal forming are: beverage cans, automobile body etc. Bulk forming processes may be primary processes such as rolling of ingot to blooms or billets, in which the cast metal is formed into semi-finished raw material. In secondary forming, the raw materials, such as blooms, billets are converted into finished parts such as gears, wheels, spanners etc. Rolling, forging, extrusion and drawing are bulk forming processes. The present module describes the salient aspects of forging process.

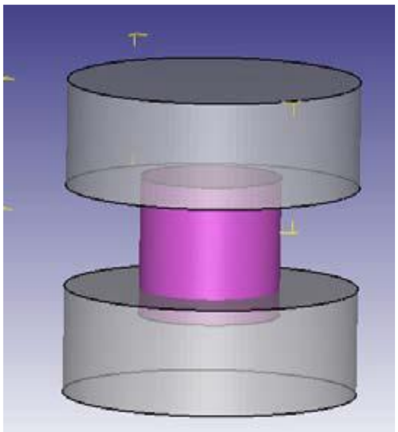
4.1.2 Forging:

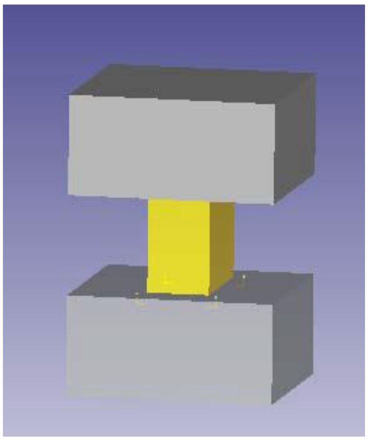
In ancient times, people employed forging for making coins, jewellery, weapons, Forging is a deformation processing of materials through compressive stress. It is carried out either hot or cold. Hot forging is done at temperatures above recrystallization temperatures, typically 0.6 Tm, or above, where Tm is melting temperature. Warm forging is done in the temperature range: 0.3 Tm to 0.5 Tm. Cold forging has advantages such as good surface finish, high strength and greater accuracy. Hot forging requires lower loads, because flow stress gets reduced at higher temperatures. Strain rates in hot working may be high – 0.5 to 500 s-1. Strains in hot forging are also high – true strains of 2 to 4 are common. Typical applications of forging include bolts, disks, gears, turbine disk, crank shaft, connecting rod, valve bodies, small components for hydraulic circuits etc. Forging has several advantages. Closer dimensional accuracies achieved require very little machining after forging. Material saving is the result. Higher strength, greater productivity, favourable grain orientation, high degree of surface finish are other merits. However, complex die making is costly.

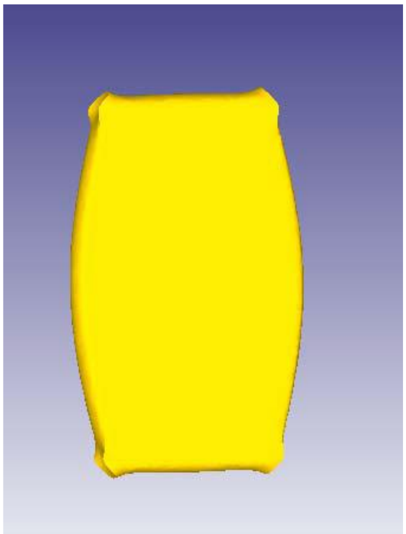
4.1.3 Types of forging:

In forging the material is deformed applying either impact load or gradual load. Based on the type of loading, forging is classified as hammer forging or press forging. Hammer forging involves impact load, while press forging involves gradual loads. Based on the nature of material flow and constraint on flow by the die/punch, forging is classified as open die forging, impression die forging and flash less forging.

Open die forging: In this, the work piece is compressed between two platens. There is no constraint to material flow in lateral direction. Upsetting is an open die forging in which the billet is subjected to lateral flow by the flat die and punch. Due to friction the material flow across the thickness is non-uniform. Material adjacent to the die gets restrained from flowing, whereas, the material at centre flows freely. This causes a phenomenon called barrelling in upset forging.







Impression die forging both die and punch have impressions, shapes which are imparted onto the work piece. There is more constrained flow in this process. Moreover, the excess metal flows out of the cavity, forming flash. Flash less forging – in this the work piece is totally constrained to move within die cavity. No excess material and hence no flash forms. Flash less forging involves high level of accuracy. Design of shape of die cavity, finished product volume is important.

4.1.3.1 Open die forging:

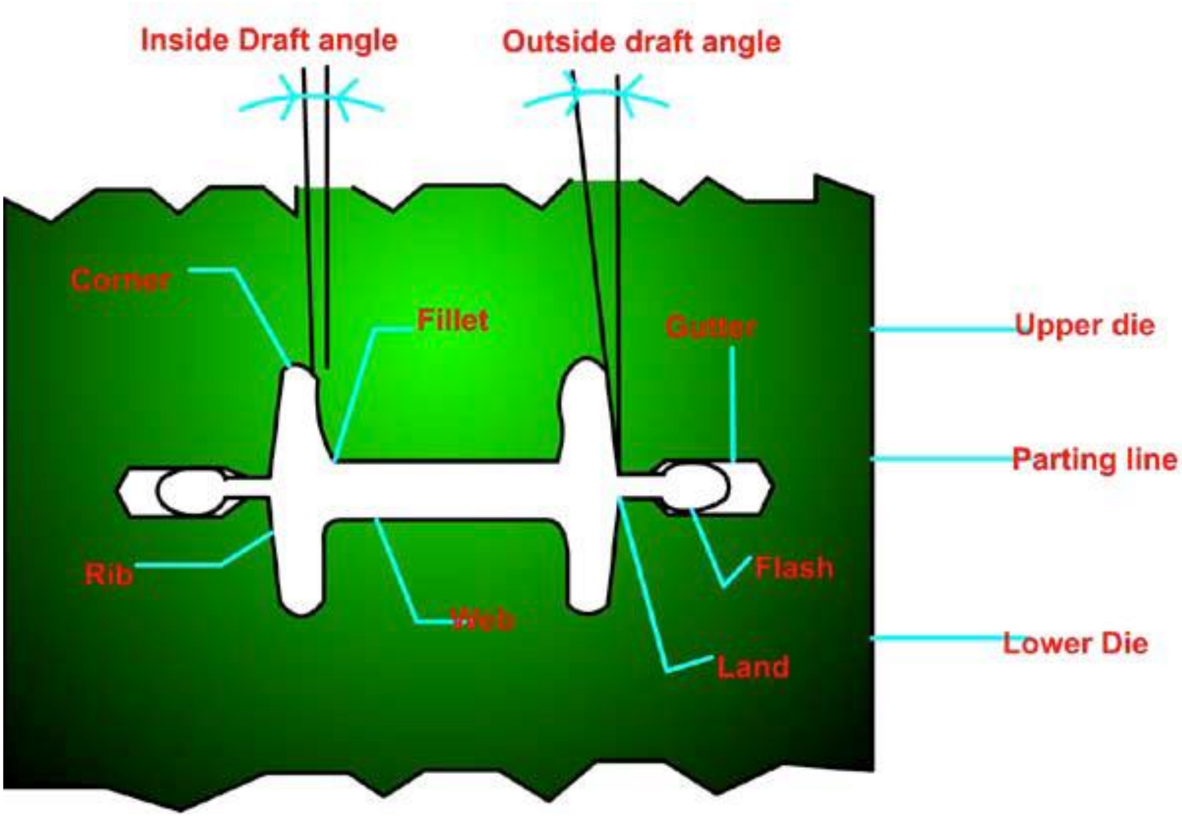
In open die forging a cylindrical billet is subjected to upsetting between a pair of flat dies or platens. Under frictionless homogeneous deformation, the height of the cylinder is reduced and its diameter is increased. Forging of shafts, disks, rings etc are performed using open die forging technique. Square cast ingots are converted into round shape by this process. Open die forging is classified into three main types, namely, cogging, fullering and edging. Fullering and Edging operations are done to reduce the cross section using convex shaped or concave shaped dies. Material gets distributed and hence gets elongated and reduction in thickness happens. Cogging operation involves sequence of compressions on cast ingots to reduce thickness and lengthen them into blooms or billets. Flat or contoured dies are used. Swaging is carried out using a pair of concave dies to obtain bars of smaller diameter.

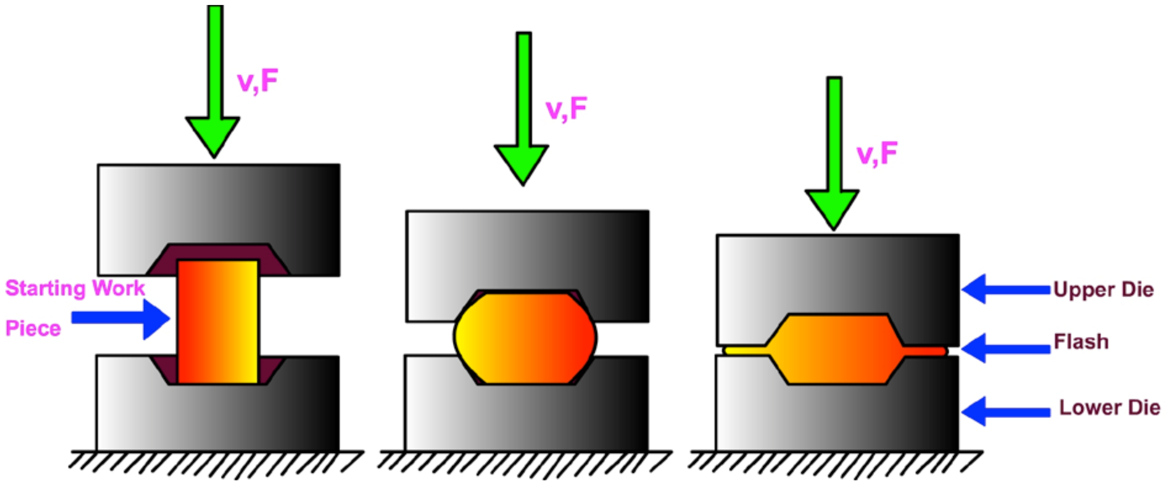
4.1.3.2 Closed die forging:

It is also known as impression die forging. Impressions are made in a pair of dies. These impressions are transferred to the work piece during deformation. A small gap between the dies called flash gutter is provided so that the excess metal can flow into the gutter and form a flash. Flash has got a very important role during deformation of the work piece inside the die cavity. Due to high length to thickness ratio of the flash gutter, friction in the gap is very high. Due to this the material in the flash gap is subjected to high pressure. There is high resistance to flow. This in turn promotes effective filling of the die cavity. In hot forging, the flash cools faster as a result of it being smaller in size. This enhances the resistance of the flash material to deformation resistance. As a result of this, the bulk of work piece is forced to deform and fill the die cavity more effectively – even intricate parts of the die cavity are filled.

Flash is subsequently trimmed off in order to obtain the required dimensions on the forged part. Often multiple steps are required in closed die forging. Flash is to be properly designed so that the metal could flow and fill the intricate parts of the die cavity. A thin flash with larger width requires higher forging loads. Before getting forged to intermediate shape inside the primary die set called blocking die, the billet is fullered and edged. This is called preforming. Subsequently, it is forged to final shape and dimensions in the finishing die. Closer dimensional accuracy is possible in closed die forging. However, higher forging loads are required. Parts with wider and thinner ribs, or webs are difficult to forge as they require higher forming loads. Impression dies are usually provided with taper called draft of 5^0 in order to facilitate easy removal of the finished part. Die preheating may be required to prevent the die chilling effect which may increase the flow stress on the periphery of the billet. As a result, incomplete filling or cracking of the preform may occur.

Dimensional tolerances in impression die forging may be as close as ±0.5% of the dimensions of the forged part. In case of hot forging, dimensional accuracy is less. Some of the factors such as die surface finish, draft allowance, accuracy of die impression dimensions, die wear, lubrication etc control the quality of finished product.





Forging load for impression die forging:

Predicting the forging load for impression die forging is rather empirical due to the complexities of material flow involved. One empirical relation for forging load, given by Schey is as followed: F = C1YfAf, where C1is a shape factor or constraint factor which depends on the complexity of the forging process. Yf is the flow stress of material at the given strain,Af is the projected area of the forging.

Typical values of C1:

Simple upsetting	1.25 to 2.5
flash less forging (Coining)	5 to 8
Complex forging with flash	8 to 12

From the above equation, one can determine the capacity of forging press, as the force predicted by the empirical equation is the highest.

4.1.3.3 Precision die forging:

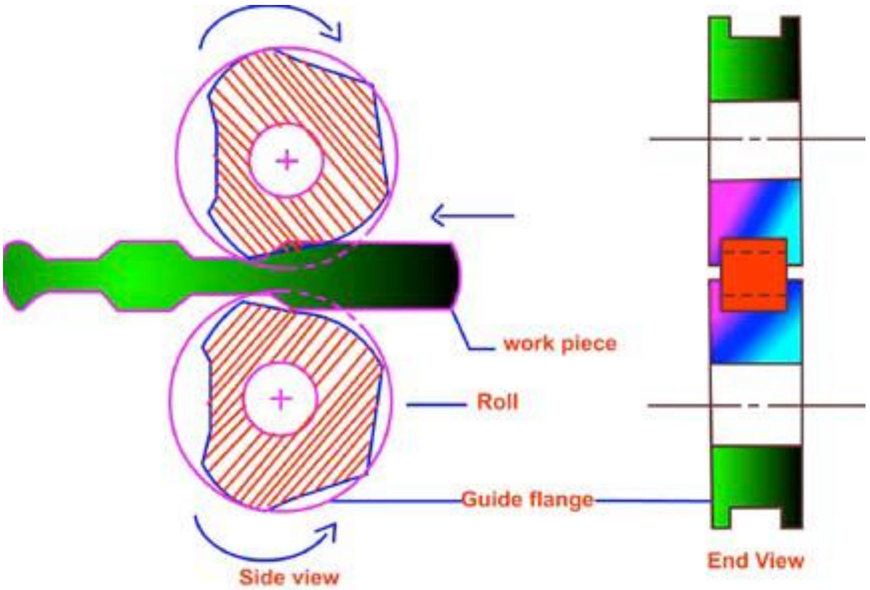
Near-net-shape forming is possible through precision die forging, in which high dimensional accuracy, elimination of after-machining and complex shapes of parts are achieved through precision dies and higher forging loads are achieved. Alloys of aluminium, titanium, magnesium are commonly precision forged. Ferrous materials are difficult to precision-forge because of die wear, higher temperatures of forging, excessive forging loads requirement.

4.1.3.4 flash less forging

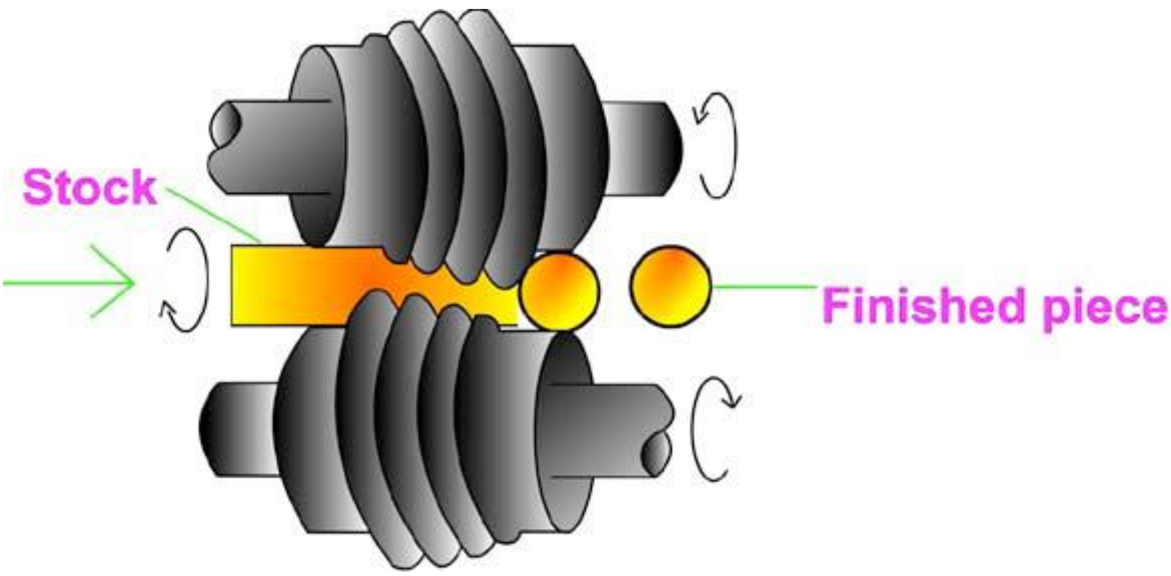
It is a closed die forging process in which the work volume is equal to die cavity volume, with no allowance for flash. Excess material or inadequate material will lead to defective part. If billet size is less then under filling takes place. Over sized billet leads to die damage or damage to the press. A variant of closed die forging is **isothermal forging**. In this process, the die is heated up to the same temperature of the billet. This helps in avoiding die chilling effect on work piece and lowering of flow stress. This process is suitable for complex parts to be mass-produced. **Coining** is a special type of closed die forging. Complex impressions are imparted to both surfaces of the blank from the die. Forging loads involved are very high – as high as 6 times the normal loads. Minting of coins is an example of this process. Coining, when used for improving surface finish of products is called sizing. Piercing: It is a process in which a punch makes deep indentations to produce cavity on work piece. Work piece may be kept inside a die or may be free. Higher forming loads are required. Heading: Heads of bolts, nails are made by heading, which is an upsetting process. Special types of machines are used for heading.

4.1.3.5 Roll forging:

In this process, the bar stock is reduced in cross-section or undergoes change in cross-section when it is passed through a pair of grooved rolls made of die steel. This process serves as the initial processing step for forging of parts such as connecting rod, crank shaft etc. Finished products like tapered shafts, leaf springs can also be made.

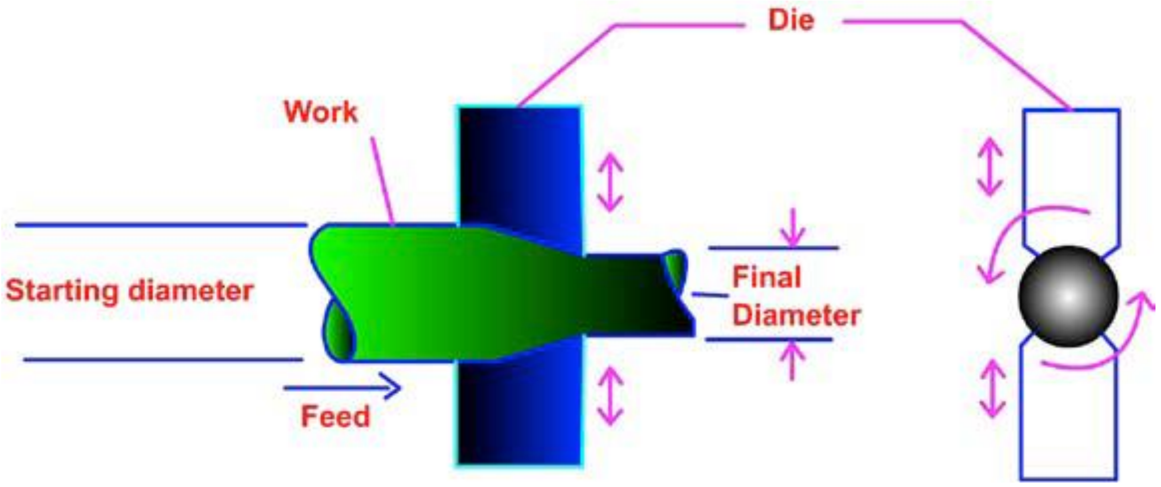


A particular type of roll forging called **skew rolling** is used for making spherical balls for ball bearings. In this process, the cylindrical bar stock is fed through the gap between a pair of grooved rollers which are rotating. Continuous rotation of the rolls and the stock gives raise to formation of a spherical shaped blank, which is subsequently finished to required dimensions.



4.1.3.6 Rotary forging:

In this process the punch is given orbital rocking motion while pressing the work piece. As a result of this the area of contact between work and punch is reduced. Therefore lower forging loads are sufficient. The final part is formed in several smaller steps. Example of parts produced by this process include bevel gears, wheels, bearing rings.

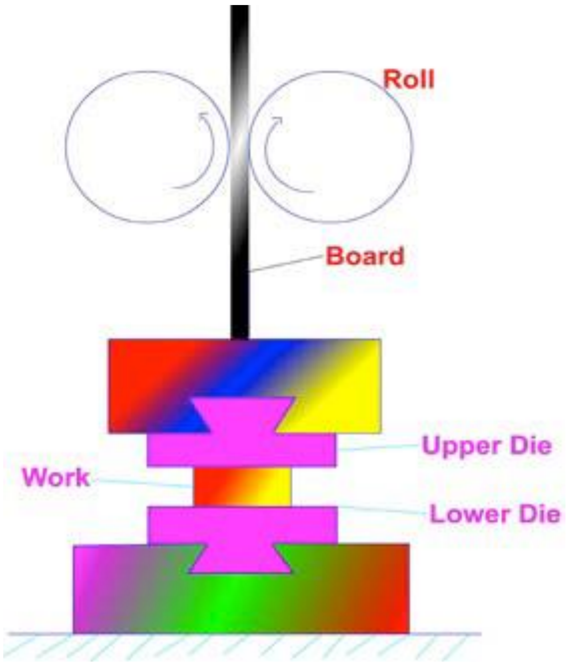


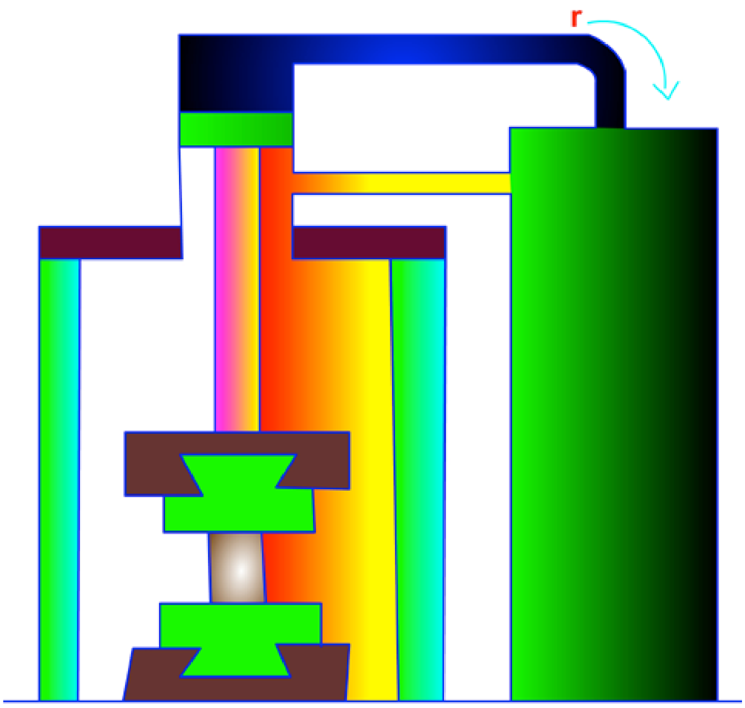
Hubbing: It is a pressing operation in which a hardened steel block, with one end machined to the form, is pressed against a soft metal. This process is used for making mould cavities. Hardened steel form is called hub. Hubbing is advantageous because it is easy for machining the positive form than machining the negative cavity.

4.2. Forging Equipment and General analysis of forging

4.2.1 Forging equipment:

Forging presses apply the required force gradually. Presses are of hydraulic type, mechanical or screw type. Eccentrics, knuckles or cranks are used in these presses for converting rotary motion into linear motion of the ram. The stroke of ram decides the energy available at the end of stroke. Hydraulic presses use hydraulic power. They are power driven machines. They are usually slow in operation. Screw presses operate based on friction wheel and screw. Both presses operate at slower ram speeds and can provide constant ram force. Presses give a squeezing type of action on the work piece. They are suitable for forging and long stroke operations. Hydraulic presses are suitable for extrusion type operations as full load is available at all times. In power hammers, the total energy available for forging is equal to the kinetic energy of the ram plus the hydraulic pressure energy. In case of flywheel operated presses, the energy available is dependent on the moment of inertia of flywheel as well as its rotational speed.





Hydraulic Press

Forging hammers provide impact loads. Gravity hammers provide the forging load by the falling weight of the ram. One half of the die is fixed on the ram and the other half is fixed on machine table. They are suitable for impression die forging, where a single blow or a few blows will deform the metal inside the cavity. Board hammers operate by frictional rising of the board with ram. Power hammers use pneumatic or steam power additionally to accelerate the ram. Total energy available at ram end is the sum of kinetic energy of the ram and the power of the air or steam used.

4.2.2 Analysis of forging:

A number of methods are available for the analysis of metal forming processes. Slab method is based on mechanics approach, in which we consider the static equilibrium of forces on the billet. In another method, the velocity field of the deforming material is found first. From kinematically admissible velocity field, the work done during the process is formulated. The formulated work equation is then solved. This approach is known as upper bound analysis. In this section we analyse the open die forging processes – upsetting of plane strip and circular disc in order to determine the forging force, using slab method. First we ignore friction and write down the theoretical equation for the forging load. Then we consider the effect of friction.

Considering a cylindrical billet of initial height ho, the strain rate in upset forging can be expressed as: €= -v/h where h is the instantaneous height and v is the velocity of the ram. As the height of the billet gets reduced the strain rate increases to very high values. The true height strain of the billet can be found from the formulae

€=ln h_0/h_f -------------------- 4.2.1

where ho is initial height and hf is final deformed height of billet. Neglecting friction at interface between the billet and die, the ideal forging force at the die-work interface is given by:

F = Y A --------------4.2.2

A is area of billet at any instant. Y is yield stress of the material of billet. Applying volume constancy principle we have:

A h = Ao ho

Therefore,

F = Y Ao ho / h --------------------4.2.3

Here, Y can be taken to be the flow stress of the material at a given strain. Work done during the deformation is given as:

W = = Ao ho$\int_0^e \sigma d€$ ----------------4.2.4

The average flow stress is given by: $\frac{k\varepsilon \mathbf{n}}{n+1}$ -------------------------------4.2.5

Therefore, work done is given by **W =Ȳ εVolume = εAoho**--------4.2.6

And the forging load is **F = Ȳ**A ---4.2.7

The area of the forged disc keeps increasing as forging proceeds. As a result the force required increases. Flow stress also increases due to work hardening. This also leads to the application of greater forging load with continued deformation. Friction at work-tool interface makes the flow of metal nonhomogeneous. Metal in contact with the die surface is subjected to maximum restraint due to friction shear stress. Flow here is the least. Whereas, at the central section the restraint being the lowest, material flow is the maximum here. This kind of non-uniform flow results in bulging of the lateral surface of the disc. This is called barrelling. In case of rectangular billets, there will be double barrelling. In case of hot forging, the material in contact with the dies gets cooler and hence offers more resistance to deformation. The central section is offering least resistance to flow. Further, the coefficient of friction in hot forming is high. All these result in barrelling. Due to barrelling, the forging load required is higher than that predicted by the theoretical equation above.

We can write the forging force for non-homogeneous upsetting as:
F = **ϒAkf, --------------------------4.2.8**

Where kf is a forging shape factor, given by**:**

kf= 1 +$\frac{0.4\mu d}{h}$

Example: Cold upset forging of a cylindrical billet of initial height 60 mm and initial diameter 30 mm, results in a final reduced height of 40 mm. The material of the billet has flow stress given by the expression=300 ε $^{0.2}$MPa. The coefficient of friction between the billet and die surfaces can be assumed to be 0.1. What is the forging force required at the reduced height?

Solution:
We may use the approximate expression, equation 8, for solving this problem.
F = **ϒAkf**

F is forging force, **ϒ** is average flow stress, A is area of billet. Kf is a factor which accounts for friction and is given by:

kf= 1 +$\frac{0.4\mu d}{h}$

Applying the principle of volume constancy, $A_o h_o$ = $A_f h_f$⍰A_f= $A_o h_o / h_f$⍰df = 51.97 mm True strain = ln(h_o/h_f) = 0.405

Average flow stress = $\frac{k\varepsilon\mathbf{n}}{n+1}$= 208.65 MPa
K_f = 1.052

F = 275.68 kN

4.3 Analysis of plane strain upset forging of rectangular billet
There are different methods of analysis of bulk deformation processing, like slab analysis, slip line field line, upper bound analysis, FEM analysis. The outcome of all these analyses is the forming load. In this section we focus on slab method, which is the simplest type of analysis for forming load.

Upsetting of rectangular plate-analysis
Consider a rectangular billet of height ho, width (x axis) 2a and unit depth (z axis). Let this billet be subjected to plane strain upsetting. Plane strain condition here means there is no normal and shear strain along the z direction – depth direction. The slab undergoes strain only along the y axis-height direction and along the x direction – width direction.
We can make a force balance on a small elemental strip of width dx, height h and unit depth.

Assumptions: compressive stresses are positive. Sliding Columbic friction Coefficient of friction is low The height of the billet is small so that the forging pressure is constant over the height of the billet.
Assume that σx and σy are principal stresses [Though σy can not be assumed as principal stress as a shear stress is also acting on the plane on which the normal stress is acting].

Here σyis the forging stress necessary at any height h of the billet.
Force balance on the element gives:
Assuming the dimension of the billet perpendicular to the plane of the paper,

$(\sigma_x+d\sigma_x)$h+2$\mu\sigma_y$dx-σ_xh=0---4.2.10

We have to eliminate σ_x because there are two unknowns in the above equation.
For eliminating σ_x we can apply the von Mises yield criterion for plane strain. According to this criterion, we have:

σ_y- σ_x=2Y/√3=Y'

From this we have dσ_y=$d\sigma_x$

The force balance equation now becomes:

$\frac{d\sigma_y}{\sigma_y}$=-2$\boldsymbol{\mu}$dx/h---4.2.11

Upon integration, we get: $\sigma_y = Ae^{\frac{-2\mu x}{h}}$ ----------------------4.2.12

To solve the constant A, we need a boundary condition.

At x = a, σ_x = 0 [free surface]
From the yield criterion we have: At x=a, σ_y = Y'

Substituting this in equation 12 and simplifying we get,
P= σ_y = Y'[$e^{\frac{2\mu(a-x)}{h}}$] --4.2.13

Equation 13 can also be written as:

P = Y'[$e^{\frac{l\mu(1-\frac{2X}{L})}{h}}$]--4.2.14

Where L = 2a width of the billet From the above equation we find that as L/h increases, the forging pressure increases – resistance to compressive deformation increases. This fact is utilized in closed die forging where the deformation resistance of flash, being high [due to high L/h] the die filling is effective. Note: Y' is plane strain yield strength of the material If the material is work hardening type of material, we have to replace Y' with Y'f which is the flow stress of the material.

Average forging pressure:

The average forging pressure is given as:

p=$\int_0^a pdx$--4.2.15

Substituting for p from equation 4.2.13, we get:

$$\bar{p} = \frac{\bar{Y}}{\frac{2\mu a}{h}}(e^{\frac{2\mu a}{h}} - 1)$$

We can get approximate expression for average forging load by expanding exponential function as infinite series. We get:

$$\bar{p} = Y'(1 + \frac{\mu a}{h})$$

---4.2.16

Note that the forge pressure is a function of instantaneous height of billet. As height gets reduced, after successive plastic flow, forging pressure increases. If the rectangular billet is subjected to plane stress compression – stress acting along the height axis and the length axis, there will be material flow in the width direction. It is found that the extent of flow along width direction is several times greater than the flow along longitudinal direction. Because of lower friction along width, material flows freely along width direction. If a rectangular block is compressed, due to friction and non-uniform flow, bulging and barrelling take place. Bulging refers to the non-uniform flow considered on the plane of the loading, while barrelling refers to the non-uniform deformation along the height of the specimen. The reason for bulging and barrelling is the material flow along the diagonal direction is rather sluggish, compared to the other directions.

Sticking friction:

The frictional shear stress – μp increases towards the axis as the forging pressure p increases. However, the maximum frictional shear stress can not exceed the shear yield strength of the material.
When the limiting condition of τ = k, we can say sticking exists at the interface. Generally, we can relate the friction shear stress with shear yield strength by the relation: τ = mk m is friction factor, which can not exceed 1. Under sticking friction the friction shear stress and shear yield strength are related as: τ = k ---------------------------------4.1.17

where k is shear yield strength. For sticking friction, the limit of friction shear stress is the shear yield strength of the material□□m=1. In general, with τ = mk, the forging pressure is given by:

$$P = Y'\frac{m}{h}(a-x) + Y'$$

---4.1.18

As per the above equation, with sticking friction [m = 1], one can write the forging pressure as:

$$P = Y'\frac{(a-x)}{h} + Y'$$

---4.1.19

Example: A rectangular block of height 40 mm, width 100 mm and depth 30 mm is subjected to upset forging under sliding friction condition, with a friction coefficient of 0.2. The material of the billet has flow stress expressed as: $Y = 300\varepsilon^{0.2}$. Calculate the forging load required at the height reduction of 30%, assuming plane strain compression.

Solution: Due to plane strain assumption, the depth side of the block remains without deformation. We can use the solution obtained for plane strain compression. The average forging pressure is given by:

$$\bar{p} = \frac{\bar{Y}}{\frac{2\mu a}{h}}\left(e^{\frac{2\mu a}{h}} - 1\right)$$

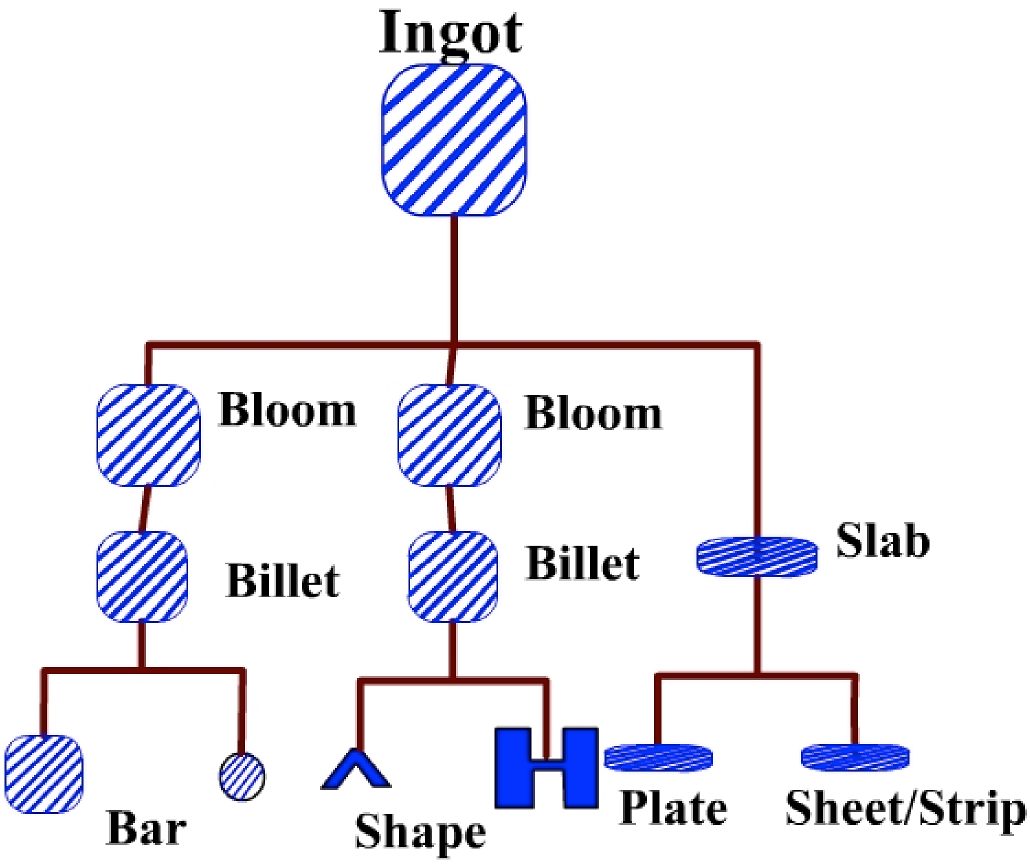
Given: ao = 50 mm, ho = 40 mm, hf = (1-0.3)ho = 28 mm, depth = w = 30 mm.
To find width after the deformation, we can use volume constancy.
2aoho = 2ah □□a = 71.43 mm
True strain = ln(ho/hf) = 0.357

Average flow stress = $\frac{k\varepsilon^n}{n+1}$ = 203.4 MPa
Average forging pressure = 199.41X1.773 = 353.59 MPa.
Average Forging load = 353.59 X 71.43 X 30 = 757. 7 kN(For one half of the bar)
Total forging load = 2X757.7kN.

4.4 ROLLING - INTRODUCTORY CONCEPTS:

4.4.1 Introduction

Rolling is one of the most important industrial metal forming operations. Hot Rolling is employed for breaking the ingots down into wrought products such as into blooms and billets, which are subsequently rolled to other products like plates, sheets etc. Rolling is the plastic deformation of materials caused by compressive force applied through a set of rolls. The cross section of the work piece is reduced by the process. The material gets squeezed between a pair of rolls, as a result of which the thickness gets reduced and the length gets increased. Mostly, rolling is done at high temperature, called hot rolling because of requirement of large deformations. Hot rolling results in residual stress-free product. However, scaling is a major problem, due to which dimensional accuracy is not maintained. Cold rolling of sheets, foils etc is gaining importance, due to high accuracy and lack of oxide scaling. Cold rolling also strengthens the product due to work hardening. Steel ingot is the cast metal with porosity and blowholes. The ingot is soaked at the hot rolling temperature of 1200^{o}C and then rolled into blooms or billets or slabs. Bloom is has a square cross section, with area more than 230 cm2. A slab, also from ingot, has rectangular cross-section, with area of at least 100 cm2 and width at least three times the thickness. A billet is rolled out of bloom, has at least 40 mm X 40 mm cross-section. Blooms are used for rolling structural products such as I-sections, channels, rails etc. Billets are rolled into bars, rods. Bars and rods are raw materials for extrusion, drawing, forging, machining etc. Slabs are meant for rolling sheets, strips, plates etc.



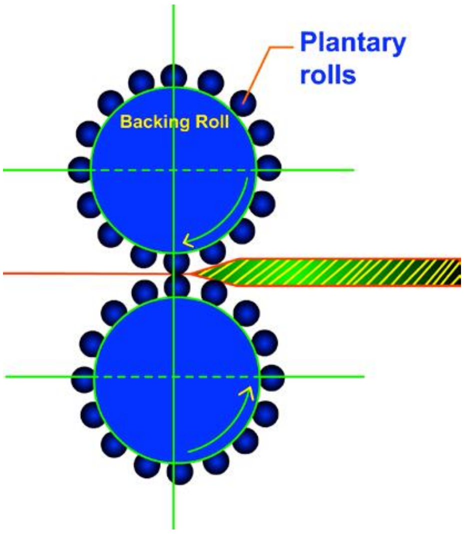
Rolling sequence for fabrication of bars,shapes and flat products from blooms,billets and slabs

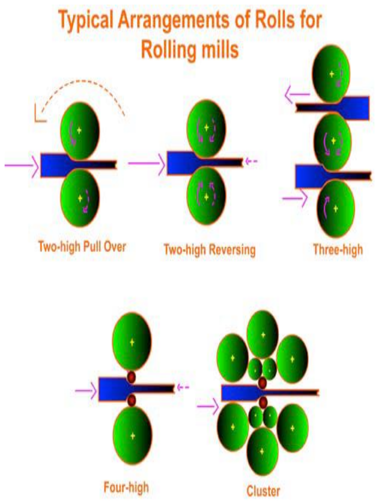
Plates have thickness greater than 6 mm whereas strips and sheets have less than 6 mm thickness.
Sheets have greater width and strip has lower width – less than 600 mm.

4.4.2 Rolling mills:

Rolling mill consists of rolls, bearings to support the rolls, gear box, motor, speed control devices, hydraulic systems etc. The basic type of rolling mill is two high rolling mill. In this mill, two opposing rolls are used. The direction of rotation of the rolls can be changed in case of reversing mills, so that the work can be fed into the rolls from either direction. Such mills increase the productivity. Non reversing mills have rolls rotating in same direction. Therefore, the work piece cannot be fed from the other side. Typical roll diameters may be 1.4 m.

A three high rolling mill has three rolls. First rolling in one direction takes place along one direction. Next the work is reversed in direction and fed through the next pair of roll. This improves the productivity. Rolling power is directly proportional to roll diameter. Smaller dia rolls can therefore reduce power input. Strength of small diameter rolls are poor. Therefore, rolls may bend. As a result, largerdia backup rolls are used for supporting the smaller rolls. Four high rolling mill is one such mill. Thin sections can be rolled using smaller diameter rolls. Cluster mill and Sendzimir mill are used for rolling thin strips of high strength materials and foils [0.0025 mm thick]. The work roll in these mills may be as small as 6 mm diameter – made of tungsten carbide. Several rolling mills arranged in succession so as to increase productivity is called rolling stand. In such arrangement, anuncoiler and windup reels are used. They help in exerting back tension and front tension.





Planetary mill has a pair of large heavy rolls, surrounded by a number of smaller rolls around their circumference. In this mill, a slab can be reduced to strip directly in one pass. Feeder rolls may be needed in order to feed the work piece into the rolls. Merchant mill is specifically used for rolling bars. Hot rolling is usually done with two high reversing mill in order to breakdown ingots into blooms and billets. For increased productivity, universal mill has two vertical rolls which can control the width of the work simultaneously.

Non ferrous materials are cold rolled into sheets from hot rolled strips. Four high tandem mills are generally used for aluminium and copper alloys. In order to achieve up to 90% reduction in thickness in cold rolling, a series of rolling mills may be used to share the total reduction. One important application of cold rolling is the removal of yield point from mild steel sheets using skin pass rolling [temper rolling]. In this the steel sheet is given a light reduction of 0.5 to 1.5% . Such a process eliminates yield point elongation. If yield elongation of steel occurs during sheet metal operation, such as deep drawing, the surface of the sheet metal becomes rough due to formation of Luder bands, also called stretcher strains. Flatness of rolled sheets can be increased by roller levelling. In this process, the sheet is passed between a pair of rolls which are driven by individual motors and are slightly offset. Rolls should have high stiffness, hardness and strength. Cast iron, cast steel and forged steel are also used as rolls.

4.4.3 Grain structure in rolling:
When the wrought or cast product gets hot rolled, the grain structure, which is coarse grained, becomes finer in size, but elongated along the direction of rolling. This type of textured grain structure results in directional property [anisotropy] for the rolled product. In order to refine the grains, heat treatment is performed immediately after rolling, which results in recrystallization after rolling.

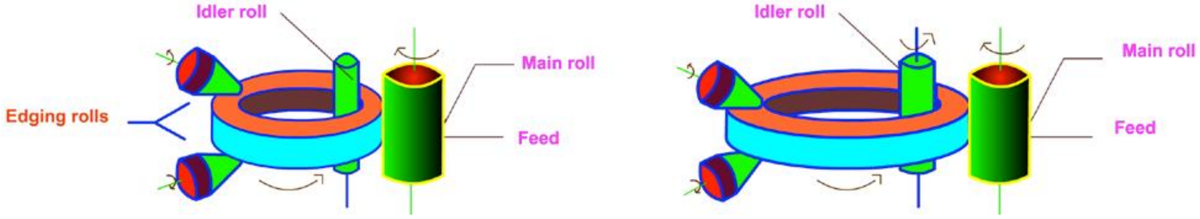
4.4.4 Special rolling processes:
Bulk deformation processes such as shape rolling, thread rolling, roll piercing, ring rolling also use pair of rolls. Some of such important processes are discussed briefly below: Thread and gear rolling:
Threads on cylindrical work pieces can be cold formed using a pair of flat dies or cylindrical rolls under reciprocating or rotary motion. Screws, bolts and other externally threaded fasteners are produced by thread rolling. Thread rolling is a high productivity process involving no loss of material. Due to grain flow in thread rolling strength is increased. Surface finish of rolled threads is very good. Gears can also be produced by the thread rolling process. Compressive stresses introduced during the process is favourable for fatigue applications. Auto power transmission gears are made by thread rolling.

Shape rolling:

Structural sections such as I-sections, rails, channels can be rolled using set of shaped rolls. Blooms are usually taken as raw materials for shape rolling. Multiple steps are required in shape rolling.

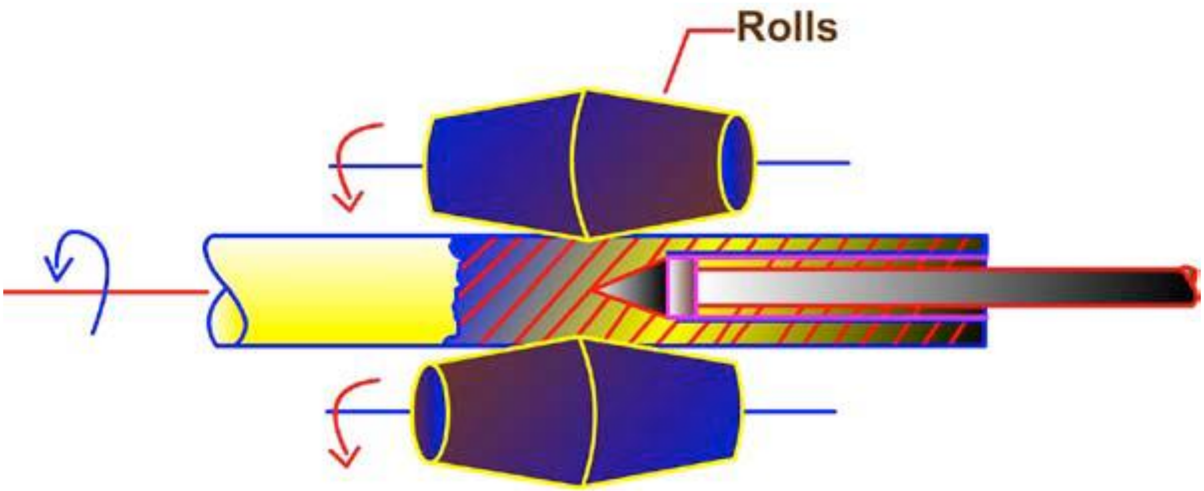
Ring rolling:

Smaller diameter, thicker ring can be enlarged to larger diameter, thinner section by ring rolling. In this process, two circular rolls, one of which is idler roll and the other is driven roll are used. A pair of edging rollers are used for maintaining the height constant. The ring is rotated and the rings are moved closer to each other, thereby reducing the thickness of ring and increasing its diameter. Rings of different cross-sections can be produced. The major merits of this process are high productivity, material saving, dimensional accuracy and grain flow which is advantageous. Large rings for turbines, roller bearing races, flanges and rings for pipes are some of the applications of this process.



Tube piercing:

Rotary tube piercing is used for producing long thick walled tubes. Cavity forms at the centre due to tensile stress, in a round rod when subjected to external compressive stress – especially cyclic compressive stress.



The Mannesmann process makes use of a tube piercing in rotary mode. A pair of skewed rolls are used for drawing the work piece inside the rolls. The roll axes are oriented at 6 degrees with reference to axis of work piece. A mandrel is used for expanding the central hole, and sizing the inner diameter. Pilger mill uses reciprocating motion of both work and mandrel to produce tubes. Work is periodically rotated additionally.

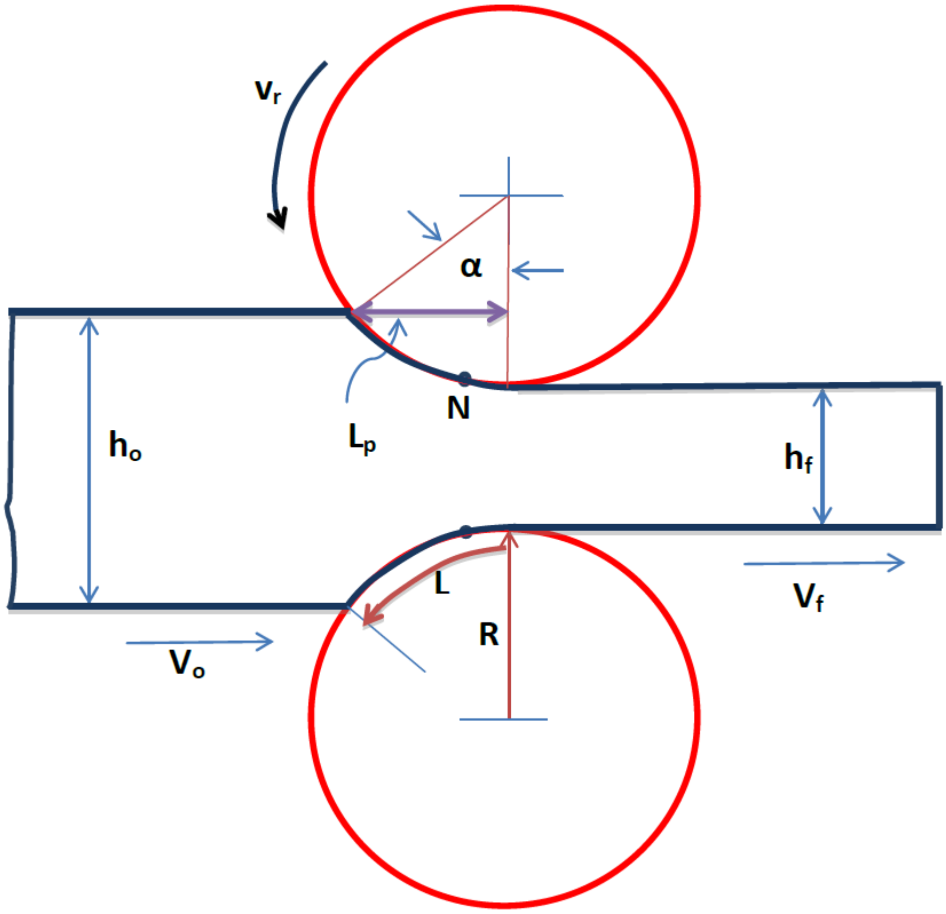
4.4.5 Geometric Relations:
Consider the rolling of a strip of initial thickness ho, between a pair of rolls of radius R. The rolls are rotating in same direction. The strip is reduced in thickness to hf with width of the strip assumed to remain constant during rolling – because width is much larger than thickness. Flat rolling is a plane strain compression process.

Draft refers to reduction in thickness.
Draft = ho-hf
Reduction (r) is the ratio of thickness reduction
R = (ho-hf)/ ho = 1 - hf/ ho --------------------------------------(4.4.1)

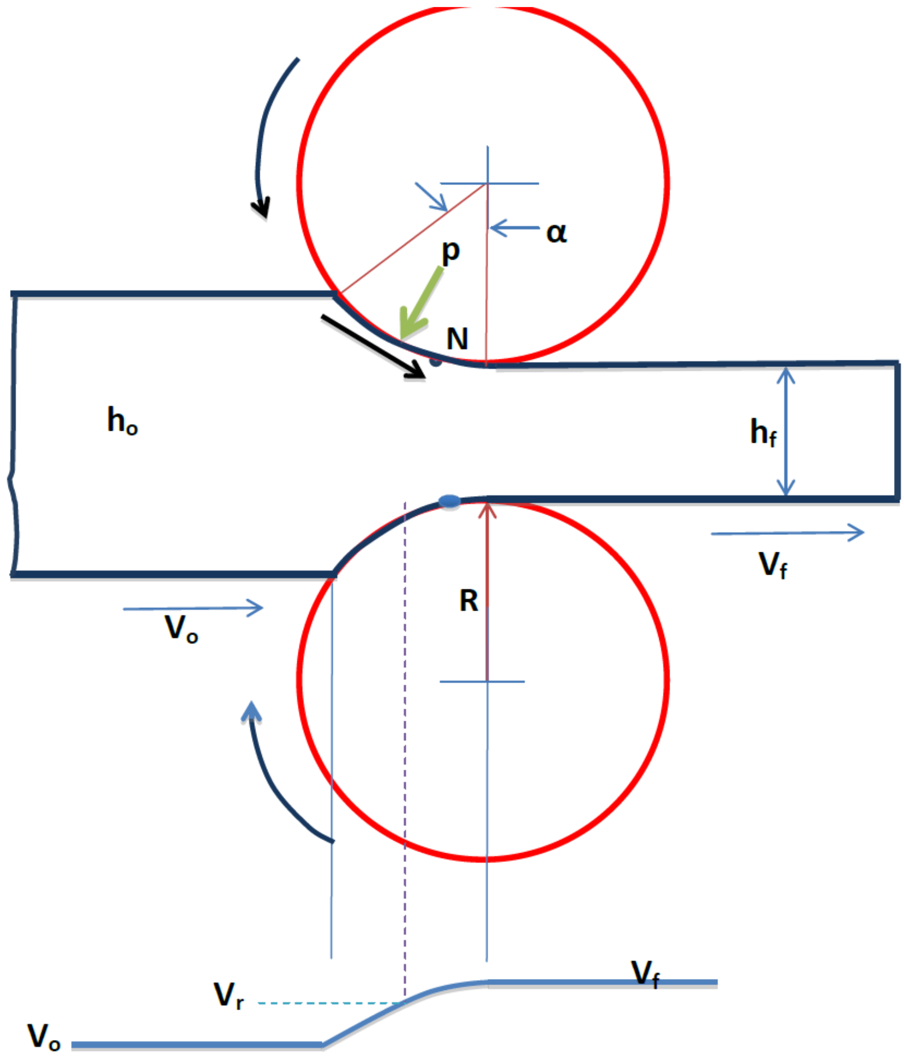
If the change in width of the strip is taken into consideration, we can find the final width by applying the volume constancy principle. Volume of material before rolling = volume after rolling.

That is, holowo = hfLfwf---(4.4.2)

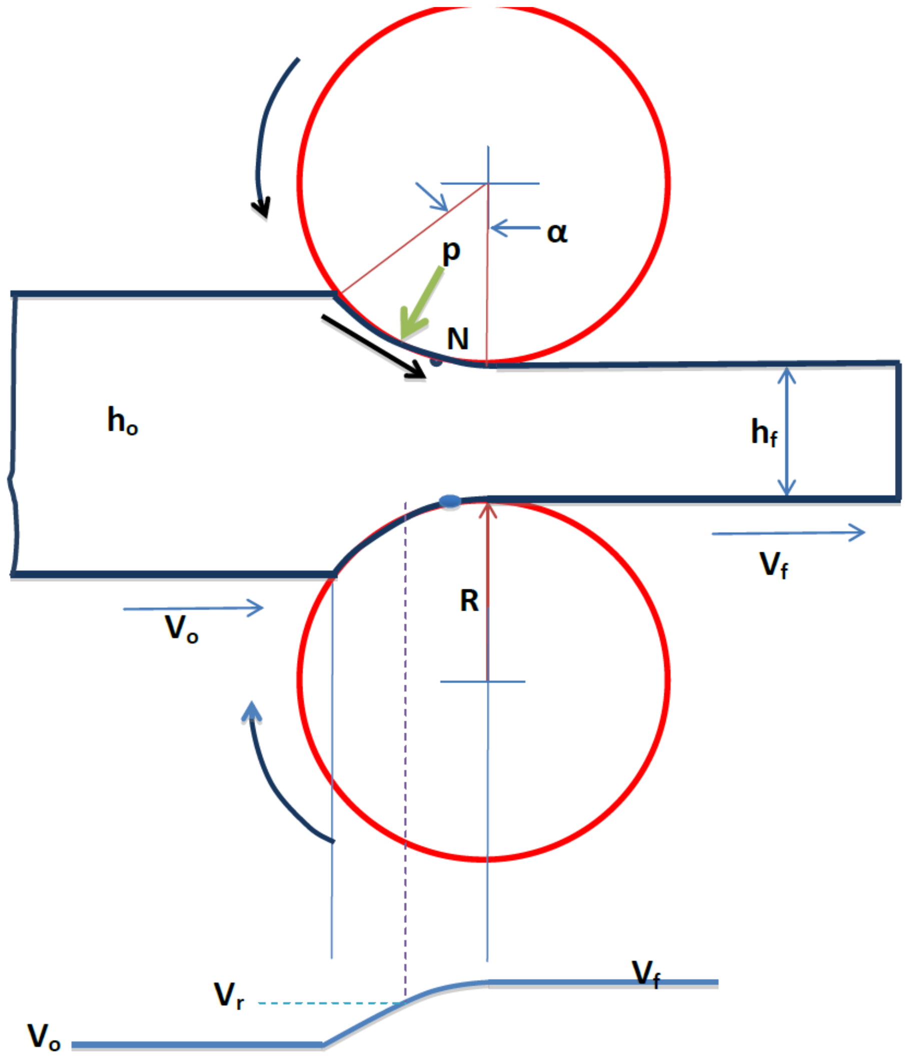
L is length of the strip.



Flat rolling-terminology:
R – roll radius
vf - velocity of strip at roll exit
L – contact arc length
N – neutral point
Lp – projected arc length
ho – strip initial thickness
hf– strip final thickness
α – angle of bite
vr – velocity of roll
vo – velocity of strip at entrance to roll



$V_f > V_r > V_o$



$V_f > V_r > V_o$

From the diagram above, we note that the velocity of the strip increases from Vo to Vf as it passes through the rolls. This velocity increase takes place in order to satisfy the principle of volume constancy of the billet during the deformation process.

i.e. $h_o \cancel{w} V_o = h_f \cancel{w} V_f \rightarrow \dfrac{V_f}{V_o} = \dfrac{h_o}{h_f}$ -----------------------------(4.4.3)

w is width of the strip, which is assumed to be constant during rolling. From equation 4.4.3 we find that the strip velocity increases during rolling, as it passes between the rolls. At some section the velocity of rolls and strip velocity are equal. This point is called neutral point. Ahead of neutral point, the strip is trailing behind the rolls. Beyond the neutral point the strip leads the rolls. Frictional shear stress τ acts tangential to the rolls at any section along the arc of contact between rolls and strip.

However, the direction of τ reverses at the neutral point. Between the entry section of the roll gap and the neutral section, the direction of friction is the same as the direction of motion of the strip – into the roll gap. Therefore, the friction aids in pulling the strip into the rolls in this part of the travel. The direction of friction reverses after the neutral point, as the velocity of strip is higher than the velocity of the rolls. Friction force opposes the forward motion of the strip in sections beyond the neutral section. However, the magnitude of the friction acting ahead of neutral section is greater than that beyond the neutral section. Therefore, the net friction is acting along the direction of the strip movement, thereby aiding the pulling of the strip into the roll gap.

The forward slip is defined as the difference in velocity between the strip at exit and roll divided by roll velocity. i.e. FS = (Vf–Vr)/Vr ----------------------------------(4.4.4)

At roll exit the forward slip is positive, meaning that the work piece moves faster than roll here. The projected arc length [Lp], which is the length of the straight line got by projecting the arc of contact onto a horizontal line or plane.

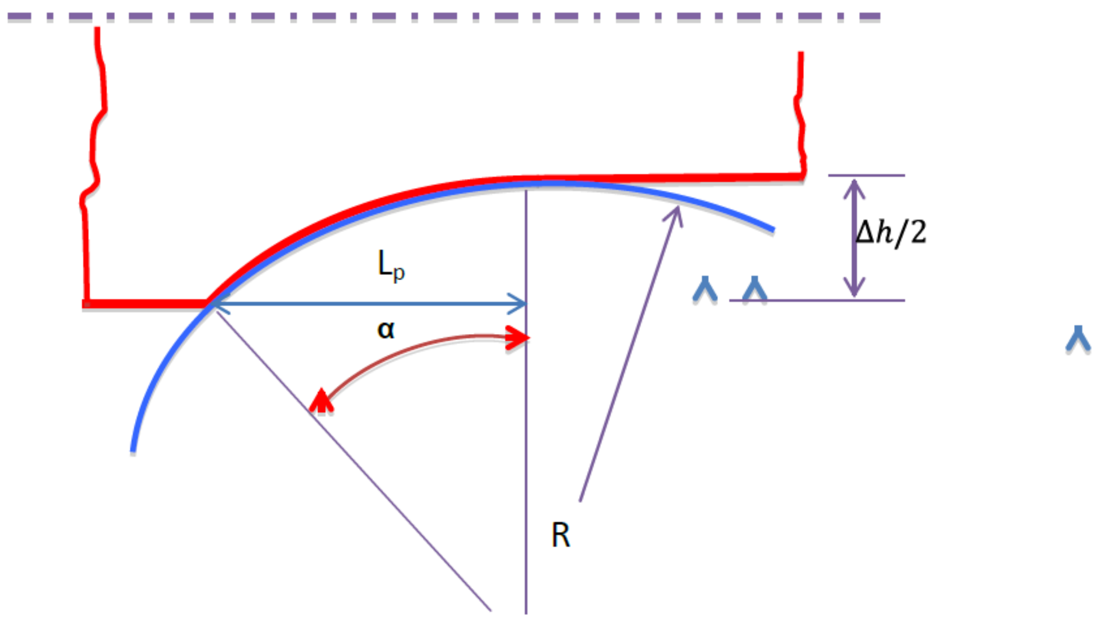
From the geometry of the arc of contact, we can getLp as below:

$$L_p^2 = R^2 - (R - \Delta h)^2$$

Ignoring power of small quantity

$$L_p = \sqrt{R\,\Delta h} \qquad \text{---(4.4.5)}$$

Where Δh is the draft and is= ho – hf



4..4.6 Limiting condition for friction between roll and work:

The rill exerts a normal pressure p on the work. This pressure may be imagined to be the pressure exerted by the work piece on the rolls to separating them.

Due to the roll pressure a tangential friction shear stress is induced at the interface contact between roll and work piece. This friction stress can be written as:

$\tau = \mu p$ Sliding friction is assumed between roll and work.

At the entry section, if the forces acting on the strip are balanced, we get:
p sinα = μ p cosα → area over which both forces are acting is the same
If the work piece is to be pulled into the rolls at entry section,

the following condition is to be satisfied
μ p cosα ≥ p sinα → $\mu \geq$ tanα → tan$\alpha_{max} = \mu$ ------------------------**(4.4.6)**

Or the minimum condition for work to be pulled into the rolls can be written as:
μ = tanα
If the tangent of angle of bite exceeds the coefficient of friction, the work piece will not be drawn into the roll gap **α = 0** indicates rolling**.** From geometry of the roll-strip contact, we can write:

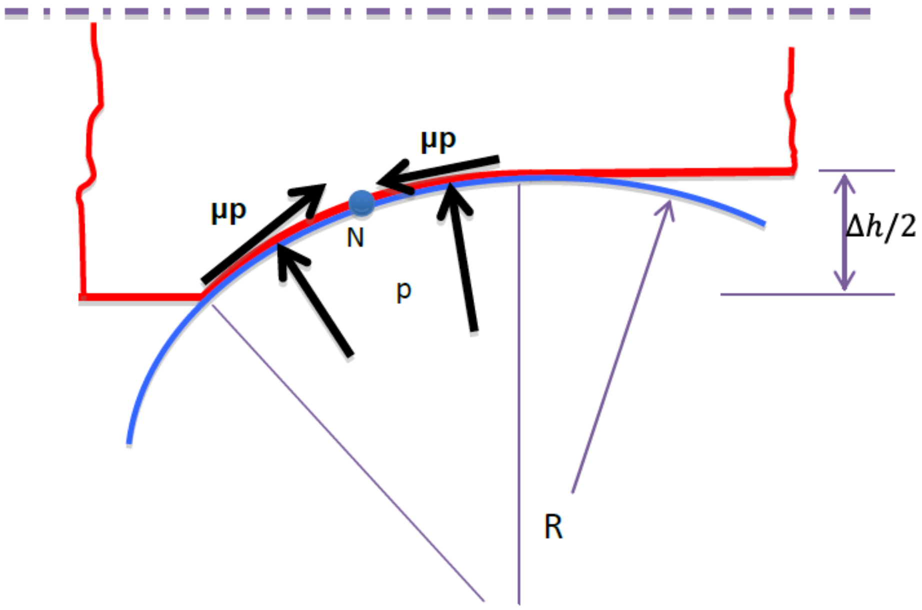
$$\tan\alpha_{max} = L_p/(R - \Delta h/2) \approx \sqrt{R\Delta h}/R = \sqrt{\Delta h/R} = \mu \quad \text{--------------------(4.4.7)}$$

We can infer from the above equation that for the same angle of bite [same friction condition], a larger roll will enable thicker slab to be drawn into the roll gap. This is because for large radius roll the arc length is larger, and hence Lp is larger.

From equation 4.4.7 above we find:

$$\Delta h_{max} = \mu^2 R \quad \text{---------------------------------(4.4.8)}$$

From equation 8 we can conclude that decreasing the roll radius reduces the maximum achievable reduction in thickness of strip. We can also conclude that higher coefficient of friction can allow larger thickness of the strip to be drawn into the roll throat. Longitudinal grooves are made on the roll surface in order to increase friction. This enables the breakdown of large thickness ingots during hot rolling.



Example: What is the maximum possible reduction that could be achieved on a strip of250 mm thick, if it is cold rolled using rolls of diameter 600 mm with a coefficient of friction value of 0.09. What is the corresponding thickness if the rolling is carried out hot with =0.5?
Solution: We know that the maximum reduction is given by: **Δhmax = µ2R (Equation 8)** For cold rolling,
Maximum reduction = 4.86 mm
For hot rolling, maximum reduction = 150 mm, which is almost 30 times the reduction in cold rolling.

4.4.7 Simplified analysis:

The parameters which influence the rolling process are: roll diameter, friction, material flow stress, temperature of working etc. Without considering friction, we can get the rolling load, approximately, from the material flow stress and the area of contact between roll and strip.

Roll pressure $p = \overline{Y_f}$ ---------------------------------(4.4.9)

Where is $\overline{Y_f}$ average flow stress in plane strain compression

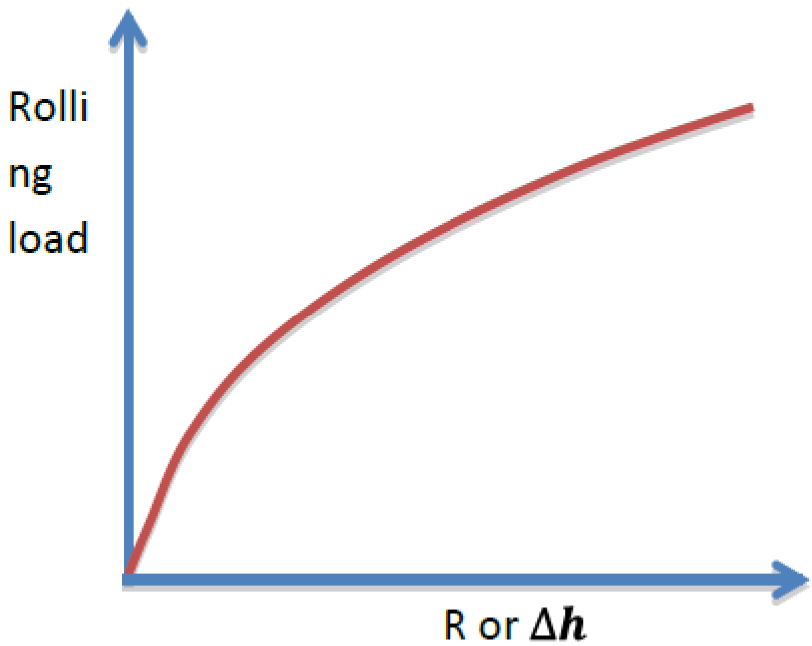
It is given as:

$$\overline{Y_f} = \frac{2}{\sqrt{3}} Y$$ -------------------------------(4.4.10)

Rolling load F is now written as:

$$F = \overline{Y_f}\, Lp\, w_m = \overline{Y_f}\sqrt{R\Delta h}\, w_m$$ ------------------------(4.4.11)

w_m is the average width of the strip We have assumed that the area over which the roll force is acting is the projected area of the arc of contact. Moreover, the above equation is for a single roll.
As we see from the above equation, the roll force increases with increase in roll radius or increase in reduction of thickness of the strip (Δh).



Alternatively, we can write the average flow stress based on true strain during rolling. For a material which obeys power law relation between plastic stress and strain, in the form:

$$\sigma = k\, \varepsilon^n$$

average flow stress, is given by:

$$Y'_f = \frac{K\varepsilon^n}{1+n}$$ -------------------------------------(4.4.12)

The true strain in rolling is given as:

$$\varepsilon = ln\frac{h_o}{h_f}$$ -

---------------------------(4.4.13)

Now, roll force

$$F = Y'_f L_p w_m$$

--------------------------------(4.4.14)

The above equation is based on the assumption that the material work hardens. In cold rolling, the work material gets work hardened considerably. Therefore, the above equation is more appropriate for cold rolling. The mean flow stress is determined from plane strain compression test, which is discussed in earlier module. It is assumed that the rolls do not undergo elastic deformation.

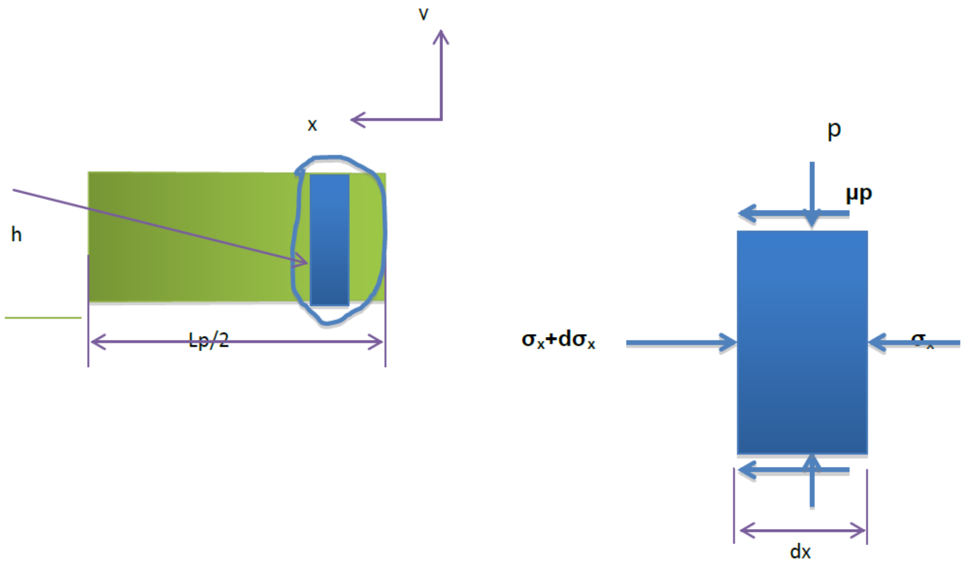
4.4.8 Slab analysis of strip rolling with friction – another approximate method:

Consider the rolling of a strip of initial thickness ho. The interface between the roll and work has sliding friction with constant coefficient of friction. We assume that the roll pressure is constant over the arc of contact.

The strain on the work material is plane strain – no strain in width direction. Further, we assume that there is no elastic deformation of work and also, the deformation of work is homogeneous. To apply the slab analysis to the rolling processes, we assume that the rolling is plane strain compression process.

Further, the contact surface between roll and work piece is equal to the projected area of the arc of contact. Further, we approximate the deformation zone as a rectangular shape, instead of conical shape and apply the analysis for plane strain compression.

Assume that the deformation volume of the work piece is in the form of rectangular prism of width Lp, height $\bar{h} = (h_o+h_f)/2$ and depth unity, as shown in figure



Consider an elemental strip of width dx, height h and depth of unity. The various stresses acting on the element are as shown in figure. p is roll pressure, **μp** is the shear stress due to friction, σx is normal stress acting on the outward face of the element, σx + dσx is the stress acting on inner face of the element. Writing the force balance along the x axis,

$$-(\sigma_x + d\sigma_x)h + \sigma_x h = 2\mu p dx$$

$$d\sigma_x h = -2\mu p dx$$ ---(4.4.15)

Applying the Tresca yield criterion, assuming that p and σ_x are principal stresses,

$$\sigma_x + p = Y' \rightarrow \sigma_x = -p + Y',$$ ---(4.4.16)

where Y' is plane strain flow stress which is given by:

$$Y' = \frac{2}{\sqrt{3}} Y$$

Note: we have taken p as negative here
And also we have:

$$d\sigma_x = dp$$ ---(4.4.17)

Substituting equation into equation we have:

$$\frac{dp}{p} = \frac{2\mu dx}{h}$$

--(4.4.18)

On integration we get:

$$\ln p = \frac{2\mu x}{h} + A$$ --

---(4.4.19)

To solve for the constant A, we can apply the boundary condition:

AT x = 0, σx =0 and from Tresca criterion, we have:

p = Y' at x=0

Applying this in equation 20 we get: A = ln(Y')

Substituting the expression for A in equation we get:

$$\ln(p/Y') = \frac{2\mu x}{h} \quad \text{Or} \quad \frac{p}{Y'} = e^{\frac{2\mu x}{h}}$$

---(4.4.20)

To get average pressure we can write:

$$\bar{p} = \frac{2}{L_p}\int_0^{\frac{L_p}{2}} p dx, \text{ Because: } \frac{\bar{p}L_p}{2} = \int_0^{\frac{L_p}{2}} p dx$$

Substituting for p from equation 21 and integrating we get:

$$\frac{\bar{p}}{Y'} = \frac{\bar{h}}{\mu L_p}\left(e^{\frac{\mu L_p}{\bar{h}}} - 1\right)$$

--(4.4.21)

The above equation gives the approximate average rolling pressure for plane strain rolling process, neglecting the curvature of the strip as it passes between the rolls.

The rolling load can be determined from the equation 4.4.21 by noting that the area of contact is taken as projected length of contact multiplied by the depth of the work piece.

From the above equation we understand that the rolling load increases with reduction in the height h of the work or increasing in rill diameter. Below a certain minimum height of the strip(below a critical thinning), the rolling load increases to very high value, because the resistance of the sheet increases to very high values. As a result, we may not be able to roll the sheet. Instead the sheet just gets pushed in between rolls, without appreciable reduction in thickness. In order to roll thin sheets, we can use rolls of smaller diameter, backed up by large diameter rolls. Also we understand that the length of arc of contact decreases with roll radius. Please note that as the coefficient of friction increases, the rolling load also increases.

Example: A 35 mm thick steel slab is hot rolled using a 900 mm roll. There is a reduction of 40% on the thickness. The coefficient of friction is 0.5. The material flow stress increases from 200 MPa at the entrance of the rolls to 280 MPa at the exit. What is the rolling load calculated by the approximate method of analysis? Assume a constant width of 800 mm for the slab. Roll flattening can be ignored.

Solution: Equation 22 gives the average rolling pressure

$$\frac{\bar{p}}{Y'} = \frac{\bar{h}}{\mu L_p}(e^{\frac{\mu L_p}{\bar{h}}} - 1)$$

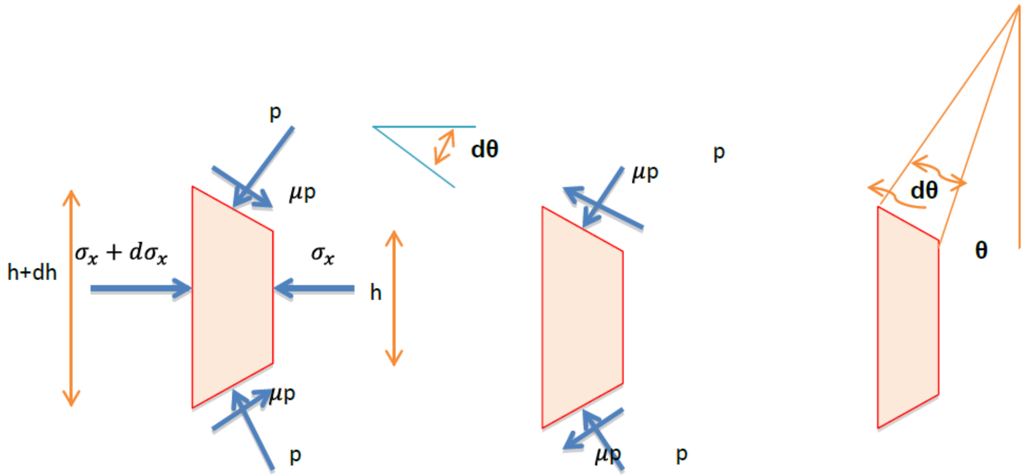
Let us take Y' as average of the flow stress at exit and entry.
Y' = 240 MPa (ho-hf)/ho = 0.4
Therefore, hf = 21 mm
$\bar{h}$= (21+35)/2 = 28 mm

Lp = projected arc length = $\sqrt{R\Delta h}$ = 112.25 mm
Average roll pressure = 205.73 MPa
Rolling force = 205.73X112.25X800 = 18.5 MN

4.4.9 Rolling of strip – more accurate slab analysis:

The previous lecture considered an approximate analysis of the strip rolling. However, the deformation zone in rolling process is very complex and is curved. Therefore, we have to consider the various states of stresses acting, considering the curvature of the deformation zone. In cold rolling the work material is likely to undergo strain hardening as it comes out of the rolls.

In the present lecture we consider the analysis considering various stresses acting on an elemental strip. Slab method of analysis is applied in order to obtain the rolling load in terms of the geometry of the deformation zone and roll diameter. We assume that rolls are not undergoing any elastic deformation. Consider an elemental strip within the deformation zone, as shown below: We assume that the rolling is plane strain process, as there is little spread of material along the width of the strip. Further, the friction coefficient remains constant through the rolling process.



Element before neutral section *After neutral section*

The element makes an angle of with the roll centre.
Consider the element at an angle of dθ from the line joining centres of the rolls The following forces act on the element:

Normal roll pressure force: pR dθ

Tangential friction force: pR dθ

The compressive forces:

$\sigma_x h$ and $[\sigma_x + d\sigma_x]$[h+dh]

The normal and tangential forces can be resolved along the direction of rolling – x axis:

pR $d\theta sin\theta$ and μpR $d\theta cos\theta$

Making a force balance on the element shown above:

$$[\sigma_x + d\sigma_x][h+dh] - \sigma_x h - 2pR\, d\theta sin\theta \pm 2\mu pR\, d\theta cos\theta = 0$$
---------(4.4.22)

Ignoring the products of small quantities, dividing by dθ and simplifying, we get:

$$\frac{d(\sigma_x h)}{d\theta} = 2pR(sin\theta \pm \mu cos\theta)$$

--(4.4.23)

This equation is called von Karman equation.

In cold rolling, under low friction conditions angle is small [6 degrees]. We can approximately take; sinθ=θ and cosθ = 1.

These approximates were proposed by Bland and Ford. Now the above equation becomes:

$$\frac{d(\sigma_x h)}{d\theta} = 2pR(\theta \pm \mu)$$

--(4.4.24)

From von Mises yield criterion applied to plane strain we have:

$$\sigma_1 - \sigma_3 = \frac{2}{\sqrt{3}}Y$$ ---(4.4.25)

$$p - \sigma_x = \frac{2}{\sqrt{3}}Y = Y'$$ ---(4.4.26)

Substituting this in the above equation,

$$\frac{d(p - Y')h)}{d\theta} = 2pR(\theta \pm \mu)$$ ---------------28

Or $$Y'h\frac{d(\frac{p}{Y'})}{d\theta} - (p/Y' - 1)\frac{dY'h}{d\theta} = 2pR(\theta \pm \mu)$$ ---(4.4.27)

The second term on left hand side can be ignored because, Y'h is constant. That is, when h increases, Y' decreases and vice versa.

Now we have:

$$Y'h\,\frac{d(\frac{p}{Y'})}{d\theta} = 2pR(\theta \pm \mu)$$

---------------------------------(4.4.28)

$$\frac{\frac{d}{d\theta}\left(\frac{p}{Y'}\right)}{\frac{p}{Y'}} = \frac{2R}{h}(\theta \pm \mu)$$

--(4.4.29)

we can approximately write:

$h = h_f + R\theta^2$

Substituting this in 4.4.29 and integrating we get the general solution to the above differential equation as:

$p = AY' \frac{h}{R} e^{\pm\mu H}$----------32

where $H = 2\sqrt{\frac{R}{h_f}} \tan^{-1}[\sqrt{\frac{R}{h_f}}\theta]$

--(4.4.30)

Applying the boundary conditions: At entry θ=α, and H = Ho

AT exit θ=0, and H = 0 We get the roll pressure as:

$p = Y' \frac{h}{h_o} e^{\mu(Ho-H)}$

$p = Y' \frac{h}{h_f} e^{\mu H}$

---(4.4.31)

From the above expressions we note that the local rolling pressure depends on the angular position of the section and the height of the work, h. It is also dependent on R/hf is equivalent of a/h in forging.

As this ratio increases, the rolling pressure also increases. The total rolling force P can be evaluated by integrating the local rolling force over the arc of contact.

$P = Rb\int_0^{\alpha} p d\theta,$

where b is width of the strip

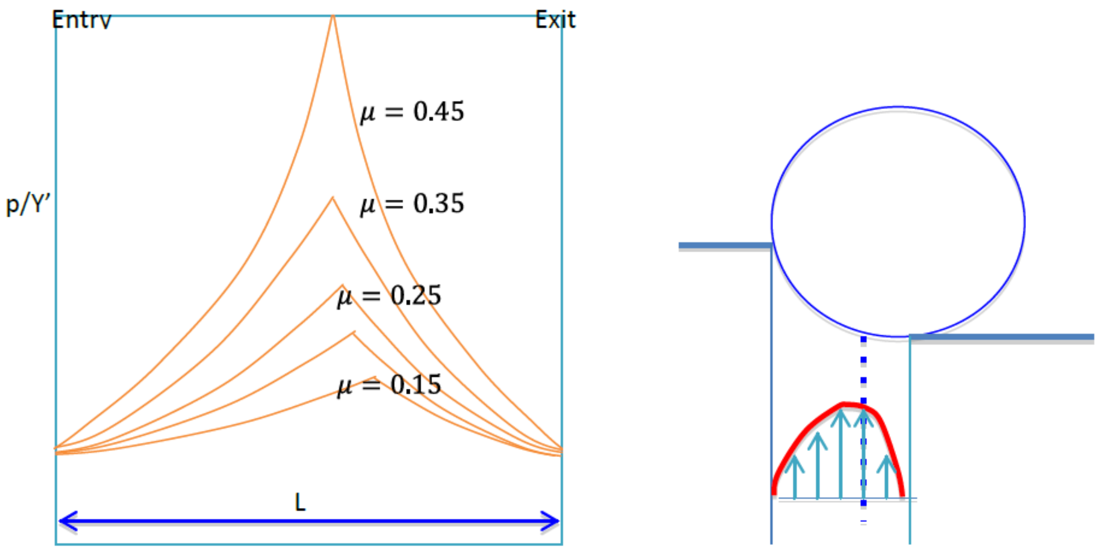


Fig. 1.1.2: Actual variation of roll pressure

4.4.10 Determination of neutral point:

The neutral point can be determined by equating the roll pressure before neutral point to that after neutral point. Equating the equations 33 and 34 and solving for Hn,

$$H_n = \frac{1}{2}\left(H_o - \frac{1}{\mu} ln \frac{h_o}{h_f}\right)$$ --(4.4.32)

Substituting this in equation 32A and solving for θ,

$$\theta_n = \sqrt{\frac{h_f}{R}} \tan\left[\sqrt{\frac{h_f}{R}} \frac{H_n}{2}\right]$$ --(4.4.33)

Example: Determine the rolling power required to roll low carbon steel strip, 250 mm wide, 12 mm thick, if the final thickness is 9 mm. Assume sliding friction between the rolls and work, with a coefficient of friction 0.12. The 250 mm radius rolls rotate at a speed of 300 rpm. Take k= 550 MPa, n = 0.26 for steel.

Solution: We can take the average roll force for sliding friction condition as:

$$F = Lw\bar{Y}'(1+\frac{\mu L}{2h_{av}})$$

True strain = = ln(ho/hf) = 0.287

The average flow stress of the material = $k\varepsilon^n/1 + n$ = 315.73 MPa

Plane strain flow stress Y' =2/√3 Average flow stress = 364.58 MPa

hav = (12+9)/2 = 10.5 mm

L = $\sqrt{R\Delta h}$ = 27.39 mm

Rolling load F = 2.9MN

Roll torque = FXL/2

Power = πnT= 1.25 MW

4.4.11 Roll torque and power:

Roll torque can be estimated from the rolling force. Torque is equal to force multiplied by the radius at which the force acts. We can assume that the roll force is acting perpendicular to the strip at a radius equal to one half of the projected arc length of contact. For each roll, the torque is:

T = FL/2 Roll power is given by:

Power = 2πNT

Torque can be more accurately determined from:

$$T = \int_0^{\theta_n} pwR^2 d\theta - \int_{\theta_n}^{\alpha} pwR^2 d\theta$$

Here the minus sign is due to the fact that the friction force acts against the rolling direction beyond the neutral section. Total roll torque consists of the rolling torque plus the torque required to overcome friction in roll bearings plus torque at motor shaft plus torque for overcoming friction in transmission system. Roll power is applied in order to deform the work material, to overcome friction in rotating parts etc.

4.5 Types of extrusion and extrusion equipment:

4.5.1 Introduction:

Extrusion is a compressive deformation process in which a block of metal is squeezed through an orifice or die opening in order to obtain a reduction in diameter and increase in length of the metal block. The resultant product will have the desired cross-section. Extrusion involves forming of axisymmetric parts. Dies of circular on non-circular cross-section are used for extrusion. Generally, extrusion involves greater forming forces. Large hydrostatic stress in extrusion helps in the process by enhancing the ductility of the material. Metals like aluminium, which are easily workable, can be extruded at room temperature. Other difficult to work metals are usually hot extruded or warm extruded. Both circular and non circular parts can be obtained by extrusion. Channels, angles, rods, window frames, door frames, tubes, aluminium fins are some of the extruded parts. Difficult to form materials such as stainless steels, nickel alloys are extruded due to its inherent advantage, namely, no surface cracking due to reaction between the billet and the extrusion container. Extrusion results in better grain structure, better accuracy and surface finish of the components. Less wastage of material in extrusion is another attractive feature of extrusion. Lead pipes were extruded in late 1700's in England. Later on lead sheathing of electric cables was done by extrusion.

4.5.2 Types of extrusion:

Extrusion ratio: It is the ratio of area of cross-section of the billet to the area of cross-section of the extrude. $R = Ao/Af$ another parameter used in extrusion is shape factor, ratio of perimeter to the cross-section of the part. An extruded rod has the lowest shape factor.

Extrusion is classified in general into four types. They are: Direct extrusion, indirect extrusion, impact extrusion and hydrostatic extrusion. In extrusion process, the billet is placed in a container, pushed through the die opening using a ram and dummy block. Both ram and billet move. Direct extrusion: Direct extrusion, also called forward extrusion, is a process in which is the billet moves along the same direction as the ram and punch do. Sliding of billet is against stationary container wall.Friction between the container and billet is high. As a result, greater forces are required. A dummy block of slightly lower diameter than the billet diameter is used in order to prevent oxidation of the billet in hot extrusion. Hollow sections like tubes can be extruded by direct method, by using hollow billet and a mandrel attached to the dummy block.

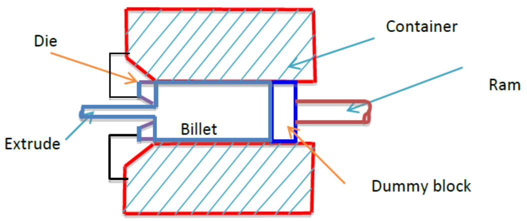
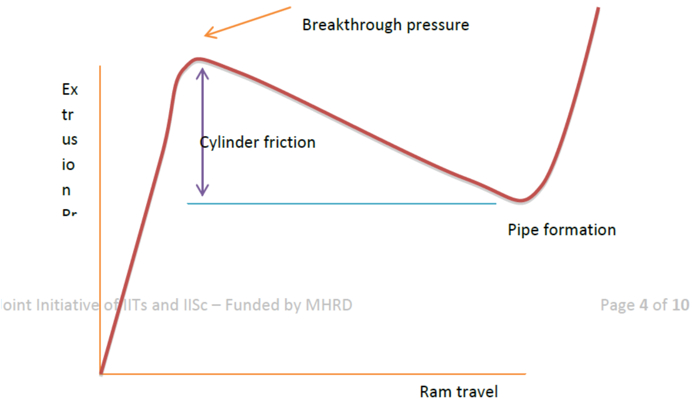


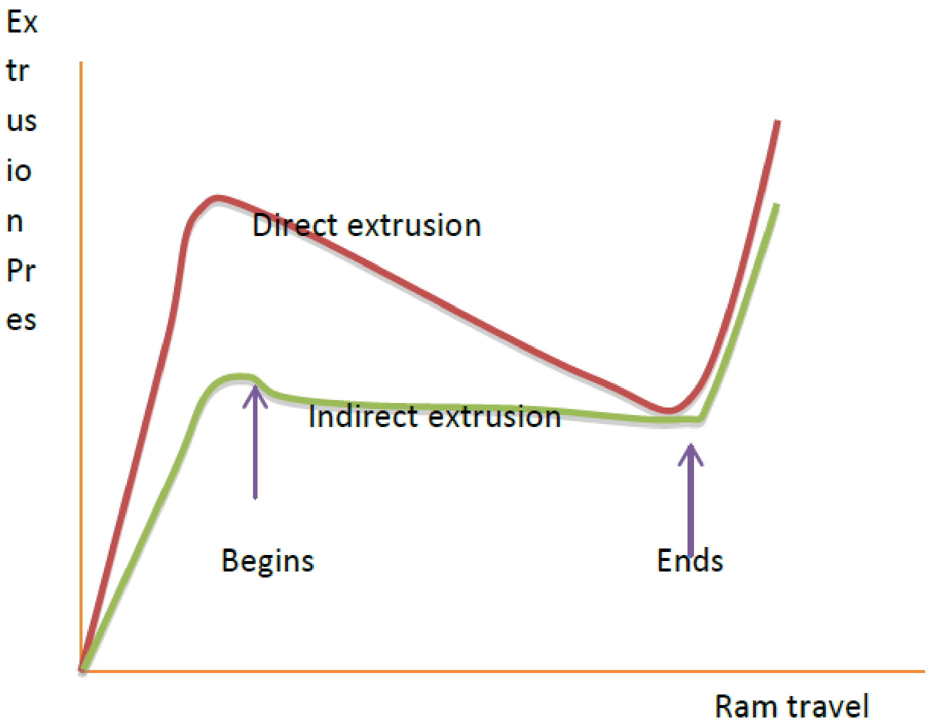
Fig. 1.2.1: Direct extrusion process



Extrusion force, which is the force required for extrusion, in direct extrusion, varies with ram travel as shown in figure above. Initially the billet gets compressed to the size of container, before getting extruded. Also, initially static friction exists between billet and container. As a result the extrusion pressure or force increases steeply as shown. Once the billet starts getting extruded, it length inside the container is reduced. Friction between billet and container now starts reducing. Therefore, extrusion pressure reduces. The highest pressure at which extrusion starts is called breakthrough pressure. At the end of the extrusion, the small amount of material left in the container gets pulled into the die, making the billet hollow at centre. This is called pipe. Beyond pipe formation, the extrusion pressure rapidly increases, as the small size billet present offers higher resistance. As the length of the billet is increased, the corresponding extrusion pressure is also higher because of friction between container and billet.

Therefore, billet lengths beyond 5 times the diameter are not preferred in direct extrusion. Direct extrusion can be employed for extruding solid circular or non-circular sections, hollow sections such as tubes or cups.

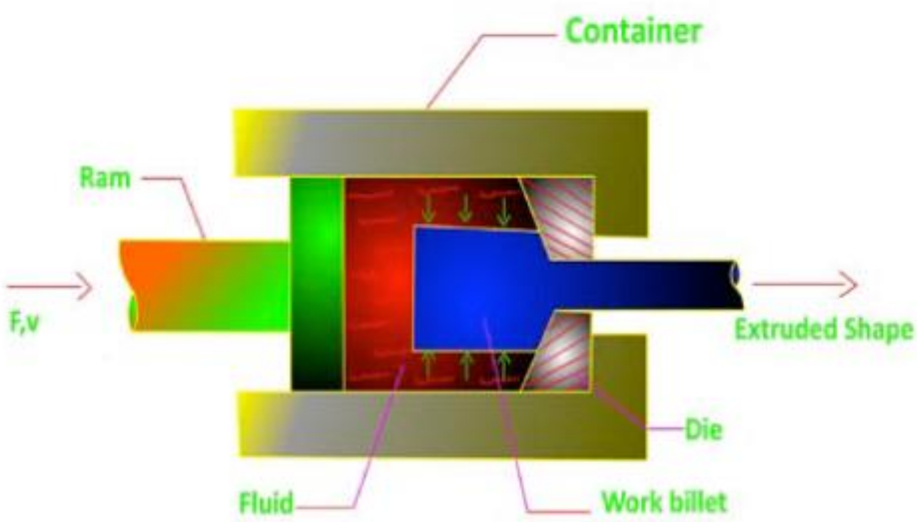
Indirect extrusion:



Indirect extrusion (backward extrusion) is a process in which punch moves opposite to that of the billet. Here there is no relative motion between container and billet. Hence, there is less friction and hence reduced forces are required for indirect extrusion. For extruding solid pieces, hollow punch is required. In hollow extrusion, the material gets forced through the annular space between the solid punch and the container. The variation of extrusion pressure in indirect extrusion is shown above. As seen, extrusion pressure for indirect extrusion is lower than that for direct extrusion. Many components are manufactured by combining direct and indirect extrusions. Indirect extrusion can not be used for extruding long extrudes.

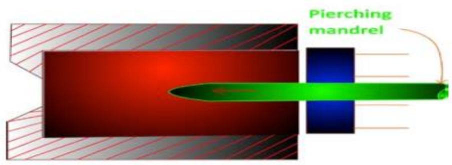
Hydrostatic extrusion:

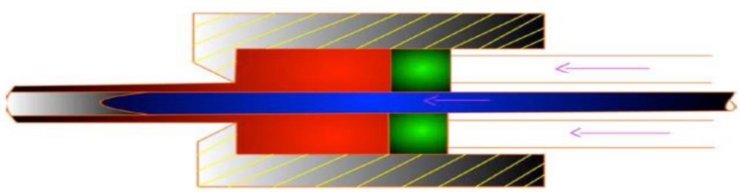
In hydrostatic extrusion the container is filled with a fluid. Extrusion pressure is transmitted through the fluid to the billet. Friction is eliminated in this process because of there is no contact between billet and container wall. Brittle materials can be extruded by this process. Highly brittle materials can be extruded into a pressure chamber. Greater reductions are possible by this method. Pressure involved in the process may be as high as 1700 MPa. Pressure is limited by the strength of the container, punch and dies materials. Vegetable oils such as castor oil are used. Normally this process is carried out at room temperature. A couple of disadvantages of the process are: leakage of pressurized oil and uncontrolled speed of extrusion at exit, due to release of stored energy by the oil. This may result in shock in the machinery. This problem is overcome by making the punch come into contact with the billet and reducing the quantity of oil through less clearance between billet and container. Hydrostatic extrusion is employed for making aluminium or copper wires-especially for reducing their diameters. Ceramics can be extruded by this process. Cladding is another application of the process. Extrusion ratios from 20 (for steels) to as high as 200 (for aluminium) can be achieved in this process.



Impact extrusion:
Hollow sections such as cups, toothpaste containers are made by impact extrusion. It is a variation of indirect extrusion. The punch is made to strike the slug at high speed by impact load. Tubes of small wall thickness can be produced. Usually metals like copper, aluminium, lead are impact extruded.

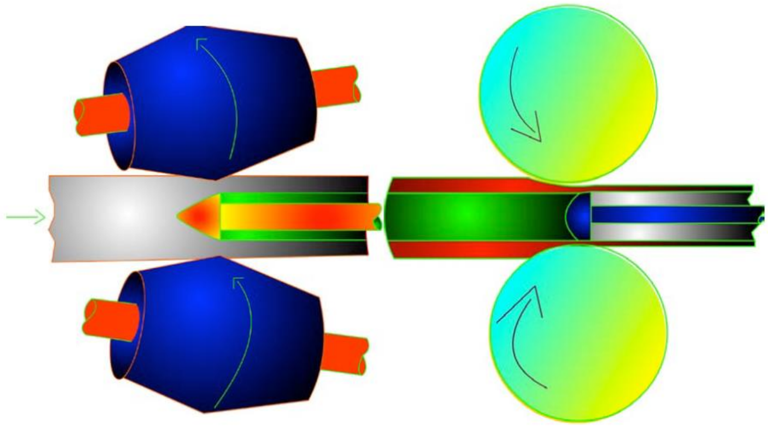
Tube extrusion: Employing hollow billet and a mandrel at the end of the ram, hollow sections such as tubes can be extruded to closer tolerances. The mandrel extends up to the entrance of the die. Clearance between the mandrel and die wall decides the wall thickness of the tube.





Tubes can also be made using solid billet and using a piercing mandrel to produce the hollow. The piercing mandrel is made to move independently with the help of hydraulic press. It moves along with the ram coaxially. First the ram upsets the billet, keeping the mandrel withdrawn. Next the mandrel pierces the billet and ejects a plug of material from central. Then the ram and mandrel together are moved in and extrude the billet.

Plug rolling and Mannesmann processes are also the other methods of producing seamless tubes.



Port hole extrusion is another method of producing tubes and hollow sections in aluminium, magnesium etc. In this method, a die with a number of ports and a central mandrel supported by a bridge is used. The billet is squeezed through the ports and flows in separate streams. After the die section the extruded streams are joined together by welding in the welding chamber.

4.5.3 Cold and hot extrusion:

Cold extrusion could produce parts with good surface finish, high strength due to strain hardening, improved accuracy, and high rate of production. However, the process requires higher pressure and tools are subjected to higher stresses. Proper lubrication is necessary for preventing seizure of tool and work piece.

Phosphate coated billets are lubricated with soap. Hot extrusion can be employed for higher extrusion ratios. Inhomogeneous deformation can occur due to die wall chilling of the billet. Metal may get oxidized. The oxide layer can increase friction as well as the material flow. Glass is used as lubricant for hot extrusion. Molybdenum disulphide or graphite is the solid lubricants used in hot extrusion. Canned extrusion using thin walled cans made of copper or tin is usually used for extruding highly reactive metals and metal powders.

4.5.4Extrusion presses:

Hydraulic presses of vertical or horizontal type are used for extrusion. Vertical presses are of capacity ranging from 3 to 20 MN. Horizontal presses occupy less space, but the billets get non uniformly cooled. Horizontal presses up to 50 MN capacity are being used. Tubular extrusions are mostly done in vertical presses, while horizontal presses are used for bar extrusion.

4.5.5 Material flow in extrusion:

Extrusion pressure depends on the nature of material flow and redundant deformation during extrusion. Metal flow can be studied by etching square grid on the cross-section of one half of the billet. The billet is cut across along the length and after making the grid; the two halves are joined by brazing or simply placed together inside the die. After extrusion the two halves of the billet are separated and the grid lines are inspected for shape change. The nature of material flow pattern depends on the friction at the interface, temperature variations in the billet etc. A frictionless condition at die-billet interface will result in homogeneous deformation, without formation of dead metal or shear zone. In case of high friction, a region of no-flow or dead metal zone forms in the corner zones. Near the die exit, shear zones are formed, especially on the periphery of the billet. Shear deformation-redundant deformation leads to enhanced extrusion load. Material flow through shear zone may result in defective extrudes. During hot extrusion, the metal near die wall gets chilled out resulting in enhancement of resistance to flow. The material at the centre flows freely. This kind of variation across the billet results in dead metal zone near wall of the die. Under high friction conditions, the shear zone may extend back into the central part of the billet. Larger dead metal zone may form due to high friction, leading to defects in extruded parts. Redundant work will be higher. Redundant work refers to the work done in the redundant shear deformation due to friction. As friction increases, metal flow becomes highly non-homogeneous, enhancing the shear deformation.

4.5.6 Extrusion pressure for ideal extrusion:

Consider a cylindrical billet of initial diameter do, length lo, being subjected to axisymmetric extrusion without friction and redundant deformation.

Let the final diameter be df. The ideal plastic work done per unit volume is given by:

$$w = \int \sigma d\varepsilon$$

Assuming that the stress is equal to average flow stress of the material in compression, we can write,

$$w = \widetilde{Y}' \int d\varepsilon = \overline{Y}'\varepsilon \quad \text{---(4.5.1)}$$

We consider the average flow stress here because the material undergoes strain hardening during extrusion. Therefore its flow stress increases from entrance to exit.

We know that

$$: \overline{Y}' = k\frac{\varepsilon^n}{1+n}$$

And the strain during extrusion is given as:

$$\varepsilon = \ln(A_o/A_f), \quad \text{---(4.5.2)}$$

because, AoLo = AfLf for constancy of volume

Extrusion ratio R is defined as R = Ao/Af

Therefore, w = lnR The total work done during extrusion is given by:

wX

Volume = ALlnR --(4.5.3)

Also we can write work done = Pressure X Area X Displacement = pAL From the above expressions for work, we get the extrusion pressure as:

p = lnR ---(4.5.4)

If redundant work due to friction is assumed, the extrusion pressure is expected to be higher than that predicted by equation (4.5.4).

Extrusion force = $pAo = p\frac{\pi}{4}d_o^2$

We define the extrusion efficiency as the ratio of ideal work of deformation to actual work of deformation.

And

$$\eta = \frac{w_{ideal}}{w_{actual}} \text{and} \qquad W_{actual} = W_{ideal} + W_{friction} + W_{redundant}$$

$$\text{Or,} \quad p_{actual} = \overline{Y}'\ln R/\eta \qquad \text{---------2.5}$$

Where n is the extrusion efficiency.

If one has to consider the friction between the container and the billet alone, then the total extrusion pressure can be taken to be the sum of the die pressure and the pressure required to overcome friction in the container.

p = die pressure + friction pressure

The friction pressure is given by:

$p_f = \frac{4\tau L}{D}$, (assuming sliding friction)

where L is length of billet in container and D is diameter of billet and is interface shear stress. Note: As seen from equation 4, as the extrusion ratio increases, the extrusion force also increases.

Example: A certain material has a strength coefficient of 400 MPa and a strain hardening exponent of 0.16 . A billet of this material has a diameter of 30 mm and a length of 80 mm. This billet is extruded to a ratio of 4. Assuming square die, estimate the extrusion force required, ignoring friction. Use the following formula for extrusion pressure:

$$p = \bar{Y}\left[(a+b\ln R)+\frac{2L}{D_o}\right], \text{ where } a = 0.8,\ b=1.5$$

Solution:

R = 4 (given)

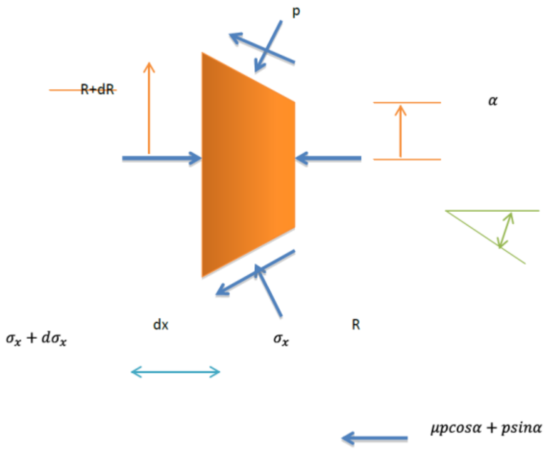
$\bar{Y} = k\varepsilon^n$ = 421.64 MPa.

p = 3461 MPa

Extrusion force = 2.45 MN

4.5.7 Direct extrusion - More accurate analysis - Slab analysis:

Slab analysis can be used for determining the extrusion pressure during direct extrusion. In order to simplify the analysis, we may assume that the die angle is small. Consider axi-symmetric extrusion. Consider an elemental cylinder of thickness dx within the deformation zone of extrusion. The radius of the cylinder reduces from R+dR to R as a result of extrusion. We can make a force balance on the cylinder, assuming that constant sliding friction exists between the billet and die wall. The semi-cone angle of the die is considered as . Let Ro and Rf be the initial and final radii of the billet.



The slant surface area is given as:

$\frac{\pi(R+dR)^2-\pi R^2}{sin\alpha} = \frac{2\pi R dR}{sin\alpha}$ (neglecting small terms) -

The force balance equation for the elemental slab along the direction of extrusion may be written as:

$$(\sigma_x + d\sigma_x)\pi(R + dR)^2 - \sigma_x \pi R^2 = (\mu p cos\alpha + p sin\alpha)\frac{2\pi R dR}{sin\alpha}$$

Simplifying, neglecting the terms involving square and product of small quantities, we get:

$$d\sigma_x + 2\sigma_x \frac{dR}{R} = 2p(1 + \mu cot\alpha)\frac{dR}{R}$$

Applying the Tresca yield criterion:
Note: Treat applied stress as positive and induced stress as negative.

That is: p is negative and the applied stress σ_x will have anegative induced stress.Therefore the yield criterion is written in the form:

$-\sigma_x + p = Y$ □□ $p = Y + \sigma_x$
Substituting for p into equation 8 above, integrating we get:

$$\frac{1}{B}\ln[(B)\sigma_x + (1 + B)y] = 2lnR + c$$

---------------------------------------(4.5.6)

applying the boundary condition:

At R = Rf, $\sigma_x = 0$, Solving for c and simplifying, we get:

$$\sigma_x = Y[\frac{1+B}{B}][(\frac{R}{R_f})^{2B} - 1]$$

--(4.5.7)

where, B = cotα, Y is yield strength of material in compression., is extrusion pressure.
Equation 10 gives the extrusion pressure at any location x along the deformation zone. The punch pressure to be applied can be obtained by substituting R = Ri
in equation 10
And also noting that the punch pressure = - σ_x

i.e p_{ext} = $Y[\frac{1+B}{B}][1-\left(\frac{R_o}{R_f}\right)^{2B}]$

Extrusion force at punch now becomes: F = Aopext

Example: For an alloy of aluminium the flow stress at a temperature of 420° C is given by the expression: $\sigma = C\dot{\varepsilon}^m$ where C = 200 MPa and m = 0.11. This alloy is hot extruded from an initial diameter of 180 mm to a final diameter of 60 mm. Length of billet is 400 mm. The speed of extrusion is 60 mm/s. Assuming square die and poor lubrication determine the extrusion force. Consider the friction in the container also.

Solution:
Assuming the semi-die angle as 45°
Extrusion pressure can be found from the expression:

p_{ext} = $Y[\frac{1+B}{B}][1-\left(\frac{R_o}{R_f}\right)^{2B}]$

B = $\mu cot\alpha$ = 0.15
Assuming =0.15
R = 9
Average Strain rate can be calculated from:
$6vlnR/D_o$ = 4.39
Y = 235.34 MPa
p = 703.67 MPa
Friction pressure in container = 4kL/Do (Assuming sticking friction)

K = Y/$\sqrt{3}$ = 235.34/1.732 = 135.87

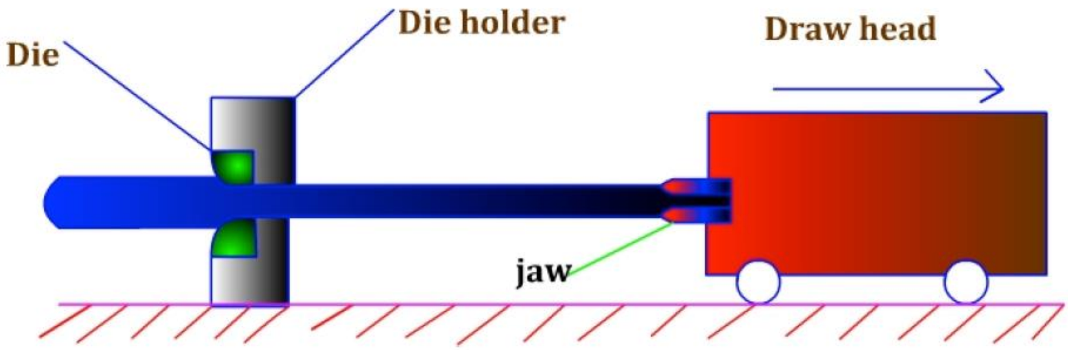
Pf = 1207 MPa
Extrusion force = 48.6 MN

4.6. Wire and bar drawing - Basic concepts:

4.6.1 Wire drawing - introduction :
Bar or wire drawing is a deformation process in which the work piece in the form of cylindrical bar or rod is pulled through a converging die. The stress applied is tensile. However, the material is subjected to compressive stress within the die thereby deforming plastically. A bar or rod is drawn down in order to reduce its diameter. In general, drawing results in reduction in area of cross-section. Drawn rods are used as raw materials for making bolts etc. Wire drawing is used for producing wires e.g. electrical wires, cables, strings, welding electrodes, fencing etc. Basic difference between bar drawing and wire drawing is the size of bar stock used for bar drawing is large. Wire is a drawn product having less than 5 mm. For wire drawing smaller diameter bar stock is used.
Wire drawing is usually done in multiple steps, using 4 to 12 dies, because the length of the wire drawn is very large-several meters. Bar drawing is done in single draft. Draft is the difference between initial and final diameter. Wire drawing is a continuous process. A draw bench is used for drawing of rods, bars and tubes because rods and bars can not be coiled. The rod or bar is pointed

by swaging operation and fed into the drawing die. The tip of the bar is clamped into the jaws of the draw head and the drawing operation is carried out continuously.

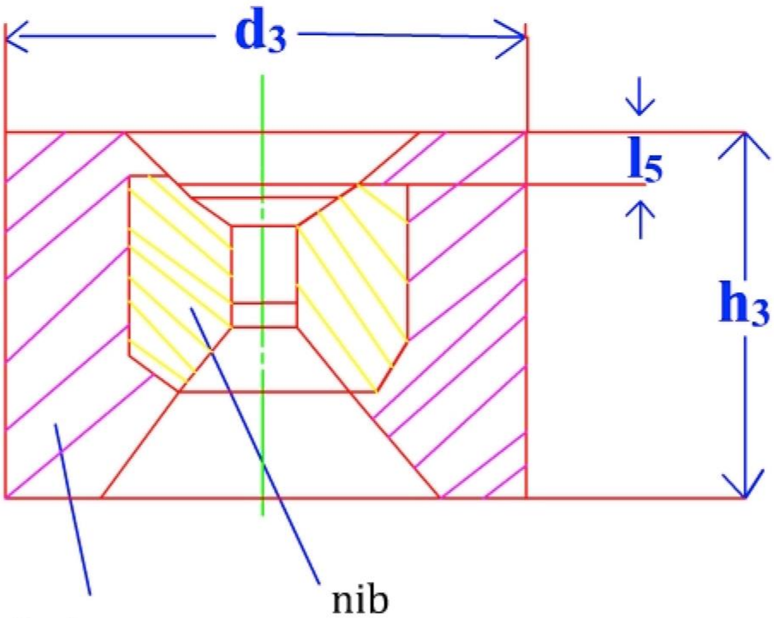
The draw head is moved using chain drive or hydraulic power pack. Draw speeds can be as high as 1500 mm/s. In wire drawing a series of dies are used in tandem. The drawn wire is wound on capstan between each pair of dies. Usually drawing is done cold. Maximum reduction in cross-sectional area per pass of drawing is restricted to 45%. Beyond this reduction, tensile stress may increase and surface finish may become poor. Due to large stress involved in drawing, the drawn wire gets strain hardened. Therefore, intermediate annealing is required before next stage of drawing.



The raw material for wire drawing is usually a hot rolled rod. The rod is coiled and fed into the die after subjected to acid pickling to remove oxides. Before drawing, the rod is lubricated. In order to retain the lubricant of the surface, oxalate or sulphate coating is given to the rod. Soap solution or oil is used as lubricant. The rod is dipped into lubricant bath before fed into the die.

A bull block is used on the other end in order to wind the drawn wire. Wire drawing is completed with multiple draw head and bull blocks, with maximum reduction in each step limited to 35 to 40%. After each step of reduction, the wire diameter is reduced. Velocity of the wire and length of the wire, therefore will increase successively. This requires that the bull block be rotated at higher speeds after each reduction. A stepped cone can be used if reduction in number of blocks is to be reduced.
Drawing speeds can be as high as 30 m/s. Intermediate annealing is required before next step of drawing in order to improve the ductility of the wire. Patenting is a heat treatment process adopted for high carbon steels (musical wires) in order to obtain optimum strength and ductility. In this process the wire is dipped in molten lead bath kept at 315°C. This will ensure the formation of pearlitic structure in the drawn wire, thereby improving its strength. Wet drawing involves dipping the wire inside a lubricant bath before the next stage.

4.6.2 Drawing die:
Die for drawing may be made of tool steel, tungsten carbide or diamond. For drawing fine wires, diamond die is used. Normally the die is made as an insert (called nib) into alloy steel casing. The cross-section of a drawing die assembly is shown below:



The entrance of the die assembly has bell assembly so as to facilitate the entry of lubricant along with the wire. Reduction in diameter takes place in approach angle section. Back relief provides space for expansion of the drawn wire. The bearing region causes frictional drag on the wire, which helps in movement of the wire inside the die. The steel casing helps hold the die.

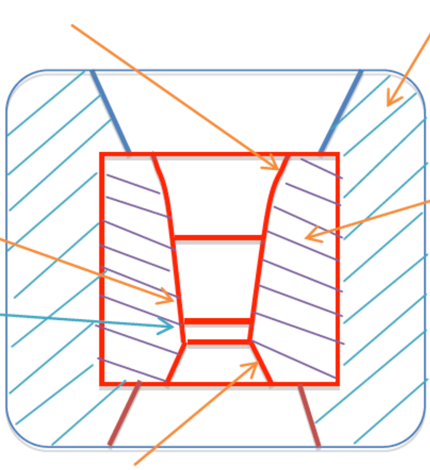


Fig. 1.2.2: Cross-section of a drawing die assembly

4.6.3 Typical drawing processes:

Drawing of bars could be carried out using a draw bench, as shown in figure. Hydraulically-operated or motor-driven carriages are used fro drawing the bar through the die. Multiple bars could be drawn in the draw bench, using several drawing dies on the same machine. Drawing of wires is usually accomplished using single or multiple drafts as shown in figures below:

Continuous drawing of wires is done through a series of drawing dies, with intermediate winding drums. These drums are run by electric motors. They wind the drawn wire before feeding it to the next reduction stage. The drum applies mild tension on the wire, which is being drawn. Multiple steps of drawing, also called tandem drawing are required as the reduction of diameter achieved per pass is usually limited.

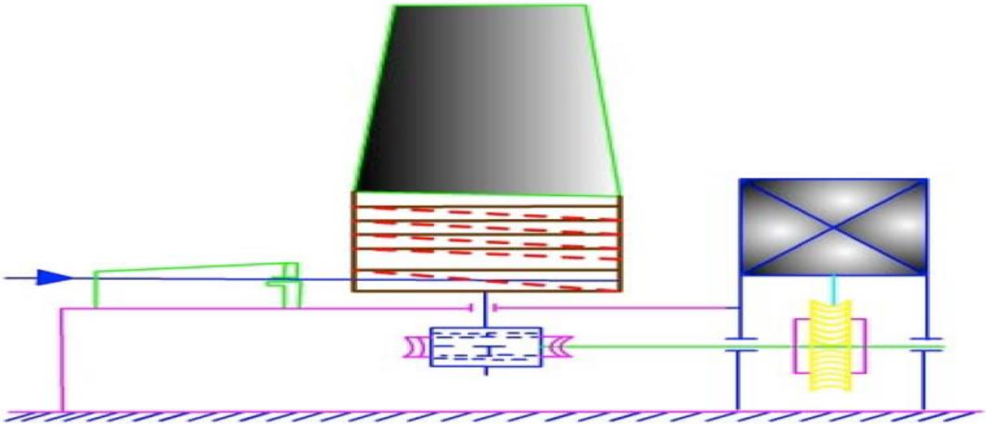


Fig. 1.3.1: Single draft drawing

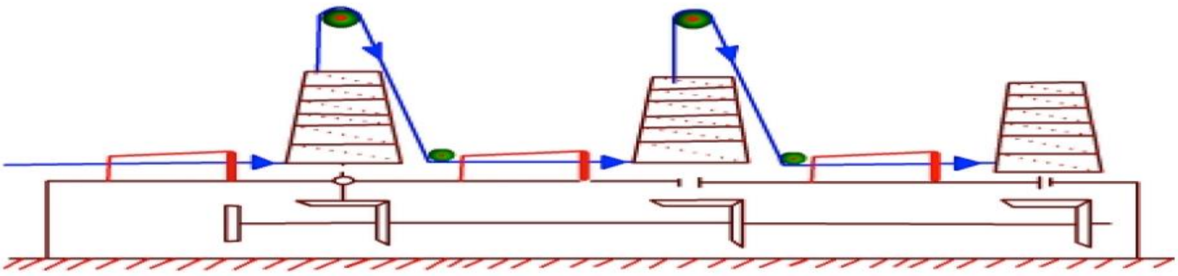


Fig. 1.3.2: Tandem Drawing