

DEPARTMENT OF ELECTRICAL ENGINEERING

THIRD SEMESTER ,(EE/EEE)

SUBJECT:NETWORK THEORY

SUBJECT CODE-1303

SYLLABUS :NETWORK THEORY (3-1-0)

MODULE-I (10 HOURS)

Coupled Circuits: Self-inductance and Mutual inductance, Coefficient of coupling, dot convention, Ideal Transformer, Analysis of multi-winding coupled circuits, Analysis of single tuned and double tuned coupled circuits.

Transient study in RL, RC, and RLC networks by Laplace transform method with DC and AC excitation. Response to step, impulse and ramp inputs.

Two Port networks: Two port parameters, short circuit admittance parameter, open circuit impedance parameters, Transmission parameters, Image parameters and Hybrid parameters. Ideal two port devices, ideal transformer. Tee and Pie circuit representation, Cascade and Parallel Connections.

MODULE-II (10 HOURS)

Network Functions & Responses: Concept of complex frequency, driving point and transfer functions for one port and two port network, poles & zeros of network functions, Restriction on Pole and Zero locations of network function. Impulse response and complete response.

Time domain behavior form pole-zero plot.

Three Phase Circuits: Analysis of unbalanced loads, Neutral shift, Symmetrical components, Analysis of unbalanced system, power in terms of symmetrical components

MODULE-III (10 HOURS)

Network Synthesis: Realizability concept, Hurwitz property, positive realness, properties of positive real functions, Synthesis of R-L, R-C and L-C driving point functions, Foster and Cauer forms

MODULE-IV (10 HOURS)

Graph theory: Introduction, Linear graph of a network, Tie-set and cut-set schedule, incidence matrix, Analysis of resistive network using cut-set and tie-set, Dual of a network.

Filters: Classification of filters, Characteristics of ideal filters

BOOKS

- [1]. Mac.E Van Valkenburg, "*Network Analysis*",
- [2]. Franklin Fa-Kun. Kuo, "*Network Analysis & Synthesis*", John Wiley & Sons.

- [3]. M. L. Soni, J. C. Gupta, "*A Course in Electrical Circuits and Analysis*",
- [4]. Mac.E Van Valkenburg, "*Network Synthesis*",
- [5]. [Joseph A. Edminister](#), Mahmood Maqvi, "*Theory and Problems of Electric Circuits*", Schaum's Outline Series, TMH

MODULE- I (10 hrs)

1.Magnetic coupled circuits. (Lecture -1)

1.1.Self inductance

When current changes in a circuit, the magnetic flux linking the same circuit changes and e.m.f is induced in the circuit. This is due to the self inductance, denoted by L.

$$V = L \frac{di}{dt}$$

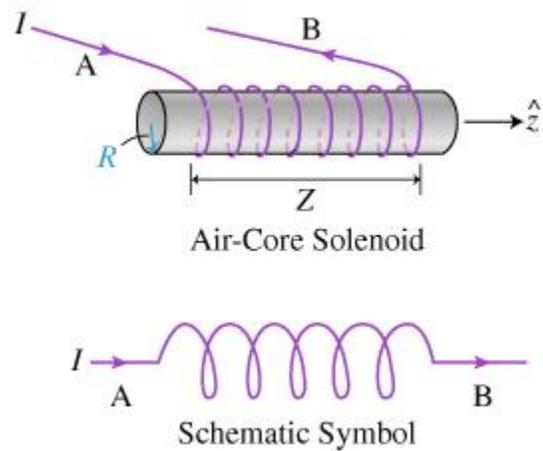
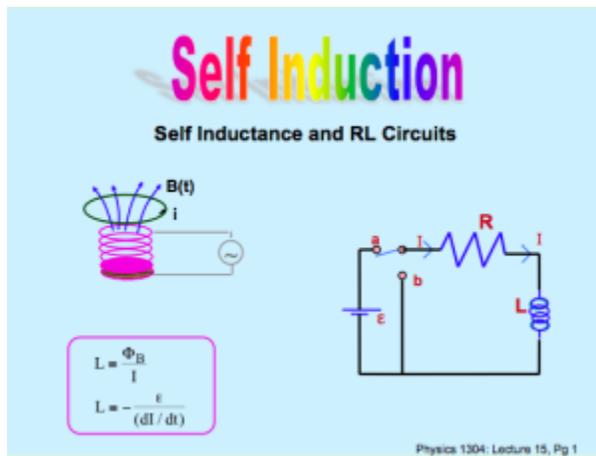


FIG.1

1.2.Mutual Inductance

The total magnetic flux linkage in a linear inductor made of a coil is proportional to the current passing through it; that is,

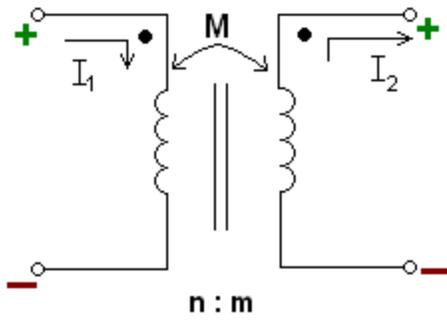


Fig. 2

$$\lambda = Li$$

. By Faraday's law, the voltage across the inductor is equal to the time derivative of the total influx linkage; given by,

$$L \frac{di}{dt} = N \frac{d\phi}{dt}$$

1.3. Coupling Coefficient

A coil containing N turns with magnetic flux ϕ linking each turn has total magnetic flux linkage $\lambda = N\phi$

. By Faraday's law, the induced emf (voltage) in the coil is

$$e = - \left(\frac{d\lambda}{dt} \right) = -N \left(\frac{d\phi}{dt} \right)$$

. A negative sign is frequently included in this equation to signal that the voltage polarity is established according to Lenz's law. By definition of self-inductance this voltage is also given by $L di = dt \mathcal{P}$; hence,

The unit of flux(ϕ) being the weber, where $1 \text{ Wb} = 1 \text{ V s}$, it follows from the above relation that $1 \text{ H} = 1 \text{ Wb/A}$. Throughout this book it has been assumed that ϕ and i are proportional to each other, making

$$L = (N\phi) / I = \text{constant}$$

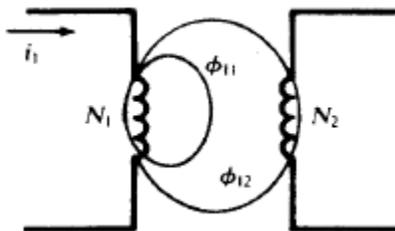


Fig.3

In Fig.3 , the total flux resulting from current i_1 through the turns N_1 consists of leakage flux,

ϕ_{12} , and coupling or linking flux, ϕ_{21} . The induced emf in the coupled coil is given by $N_2(d\phi_{12}/dt)$. This same voltage can be written using the mutual inductance M:

$$M = N_1 \frac{d\phi_{21}}{di_2}$$

$$e = M \frac{di_1}{dt} = N_2 \frac{d\phi_{12}}{dt}$$

or

$$M = N_2 \frac{d\phi_{12}}{di_1}$$

Also, as the coupling is bilateral,

$$M = N_1 \frac{d\phi_{21}}{di_2}$$

$$M^2 = \left(N_2 \frac{d\phi_{12}}{dt} \right) \left(N_1 \frac{d\phi_{21}}{di_2} \right)$$

$$= \left(N_2 \frac{(k\phi_1)}{di_1} \right) \left(N_1 \frac{(k\phi_2)}{di_2} \right)$$

$$= k^2 \left(N_1 \frac{d\phi_1}{di_1} \right) \left(N_2 \frac{d\phi_2}{di_2} \right)$$

$$= k^2 L_1 L_2$$

Hence, mutual inductance, M is given by

$$M = k\sqrt{L_1 L_2}$$

And the mutual reactance X_M is given by

$$X_M = k\sqrt{X_1 X_2}$$

The coupling coefficient, k, is defined as the ratio of linking flux to total flux:

$$k = \frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2}$$

1.4. Series connection of coupled circuit (lecture 2)

When inductors are connected together in series so that the magnetic field of one links with the other, the effect of mutual inductance either increases or decreases the total inductance depending upon the amount of magnetic coupling. The effect of this mutual inductance depends upon the distance apart of the coils and their orientation to each other.

Mutually connected inductors in series can be classed as either “Aiding” or “Opposing” the total inductance. If the magnetic flux produced by the current flows through the coils in the same direction then the coils are said to be **Cumulatively Coupled**. If the current flows through the coils in opposite directions then the coils are said to be **Differentially Coupled** as shown below.

1.4.1. Cumulatively Coupled Series Inductors

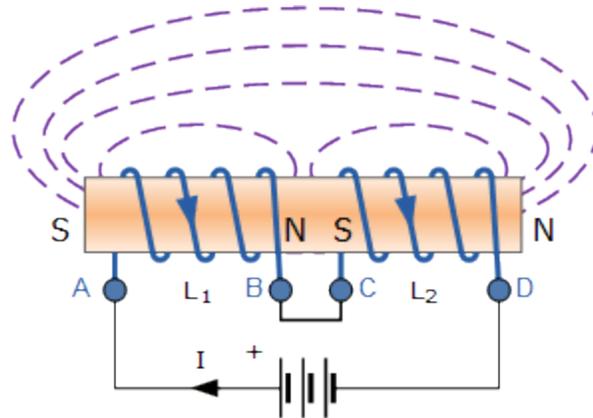


Fig.4

While the current flowing between points A and D through the two cumulatively coupled coils is in the same direction, the equation above for the voltage drops across each of the coils needs to be modified to take into account the interaction between the two coils due to the effect of mutual inductance. The self inductance of each individual coil, L_1 and L_2 respectively will be the same as before but with the addition of M denoting the mutual inductance.

Then the total emf induced into the cumulatively coupled coils is given as:

$$L_T \frac{di}{dt} = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + 2 \left(M \frac{di}{dt} \right)$$

Where: $2M$ represents the influence of coil L_1 on L_2 and likewise coil L_2 on L_1 .

By dividing through the above equation by di/dt we can reduce it to give a final expression for calculating the total inductance of a circuit when the inductors are cumulatively connected and this is given as:

$$L_{\text{total}} = L_1 + L_2 + 2M$$

If one of the coils is reversed so that the same current flows through each coil but in opposite directions, the mutual inductance, M that exists between the two coils will have a cancelling effect on each coil as shown below.

1.4.2. Differentially Coupled Series Inductors

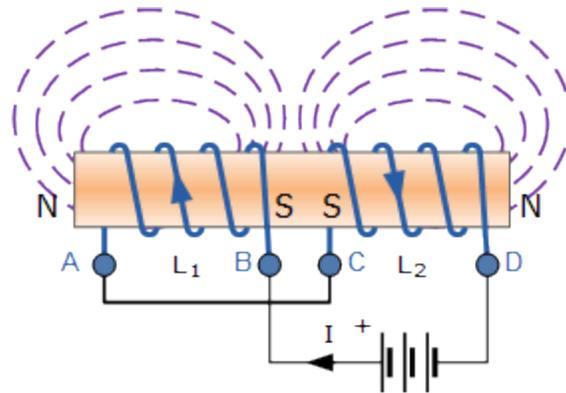


Fig.5

The emf that is induced into coil 1 by the effect of the mutual inductance of coil 2 is in opposition to the self-induced emf in coil 1 as now the same current passes through each coil in opposite directions. To take account of this cancelling effect a minus sign is used with M when the magnetic field of the two coils are differentially connected giving us the final equation for calculating the total inductance of a circuit when the inductors are differentially connected as:

$$L_{\text{total}} = L_1 + L_2 - 2M$$

Then the final equation for inductively coupled inductors in series is given as:

$$L_T = L_1 + L_2 \pm 2M$$

Inductors in Series Example No2

Two inductors of 10mH respectively are connected together in a series combination so that their magnetic fields aid each other giving cumulative coupling. Their mutual inductance is given as 5mH. Calculate the total inductance of the series combination.

$$L_T = L_1 + L_2 + 2M$$

$$L_T = 10\text{mH} + 10\text{mH} + 2(5\text{mH})$$

$$L_T = 30\text{mH}$$

Inductors in Series Example No3

Two coils connected in series have a self-inductance of 20mH and 60mH respectively. The total inductance of the combination was found to be 100mH. Determine the amount of mutual inductance that exists between the two coils assuming that they are aiding each other.

$$L_T = L_1 + L_2 \pm 2M$$

$$100 = 20 + 60 + 2M$$

$$2M = 100 - 20 - 60$$

$$\therefore M = \frac{20}{2} = 10\text{mH}$$

1.4.DOT RULE (lecture 3)

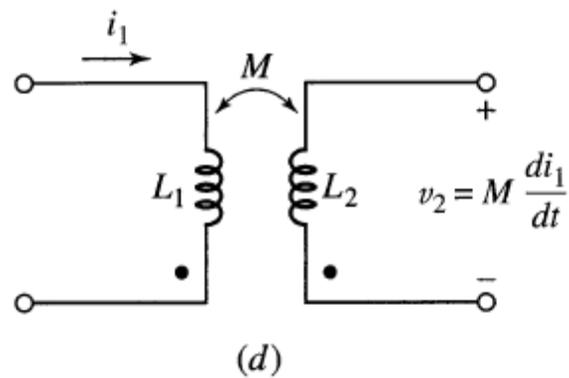
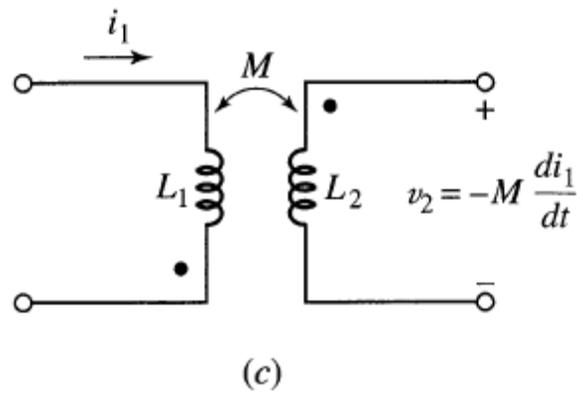
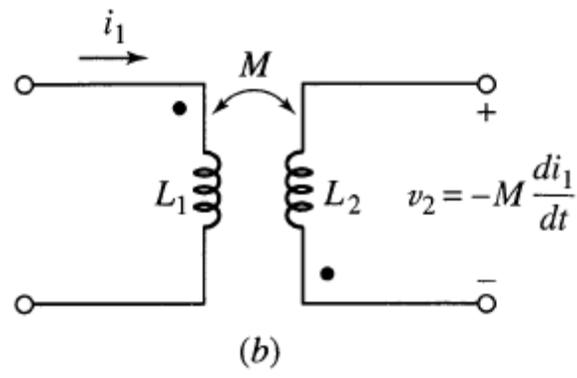
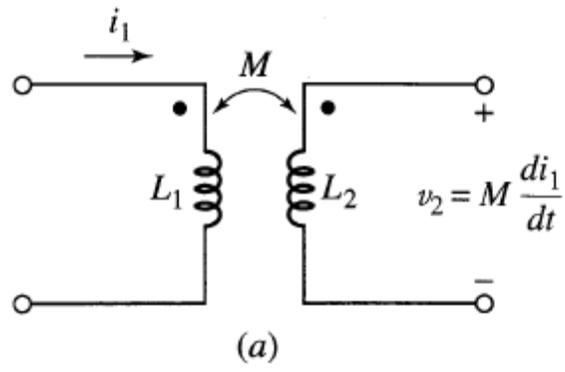


Fig.6

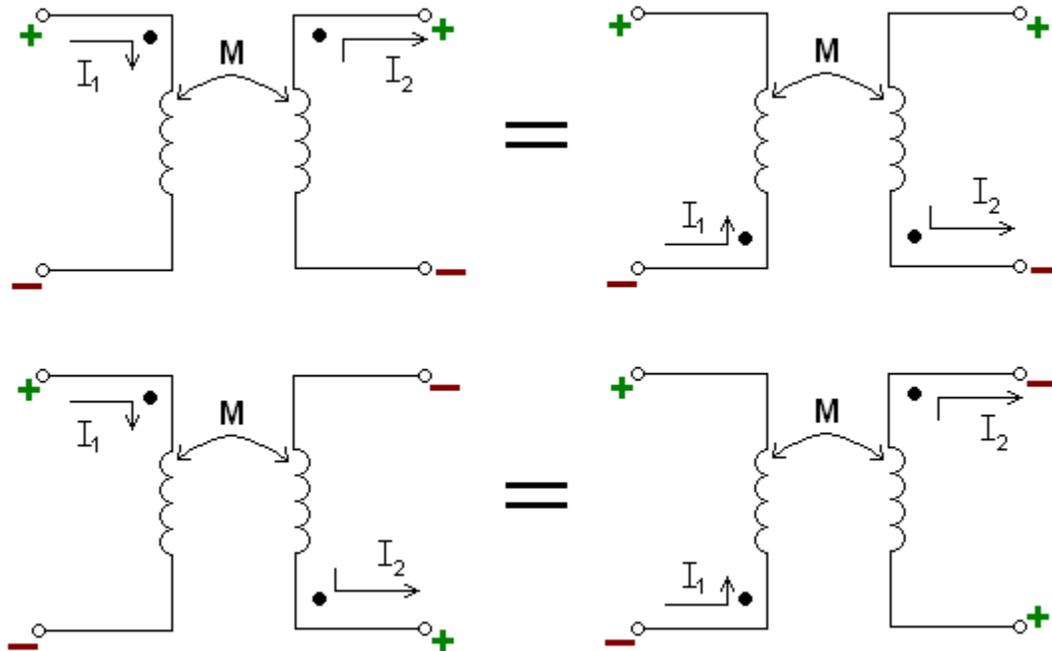


Fig.7

The sign on a voltage of mutual inductance can be determined if the winding sense is shown on the circuit diagram, as in Fig. To simplify the problem of obtaining the correct sign, the coils are marked with dots at the terminals which are instantaneously of the same polarity. To assign the dots to a pair of coupled coils, select a current direction in one coil and place a dot at the terminal where this current enters the winding. Determine the corresponding flux by application of the right-hand rule [see Fig. 14-7(a)]. The flux of the other winding, according to Lenz's law, opposes the first flux. Use the right-hand rule to find the natural current direction corresponding to this second flux [see Fig. 14-7(b)]. Now place a dot at the terminal of the second winding where the natural current leaves the winding. This terminal is positive simultaneously with the terminal of the first coil where the initial current entered. With the instantaneous polarity of the coupled coils given by the dots, the pictorial representation of the

core with its winding sense is no longer needed, and the coupled coils may be illustrated as in Fig. 14-7(c). The following dot rule may now be used:

(1) when the assumed currents both enter or both leave a pair of coupled coils by the dotted terminals, the signs on the M-terms will be the same as the signs on the L-terms; but

(2) if one current enters by a dotted terminal while the other leaves by a dotted terminal, the signs on the M-terms will be opposite to the signs on the L-terms

1.5. CONDUCTIVELY COUPLED EQUIVALENT CIRCUITS

From the mesh current equations written for magnetically coupled coils, a conductively coupled equivalent circuit can be constructed. Consider the sinusoidal steady-state circuit of Fig. 14-9(a), with the mesh currents as shown. The corresponding equations in matrix form are

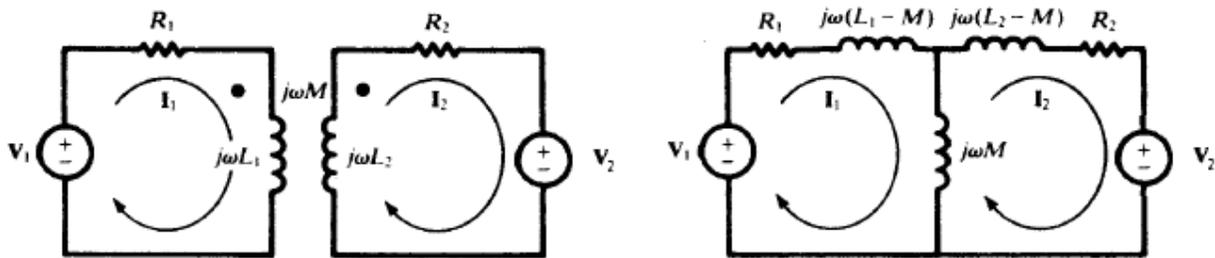


Fig.8

$$\begin{bmatrix} R_1 + j\omega L_1 & -j\omega M \\ -j\omega M & R_2 + j\omega M_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

1.6. Single tuned coupled circuit

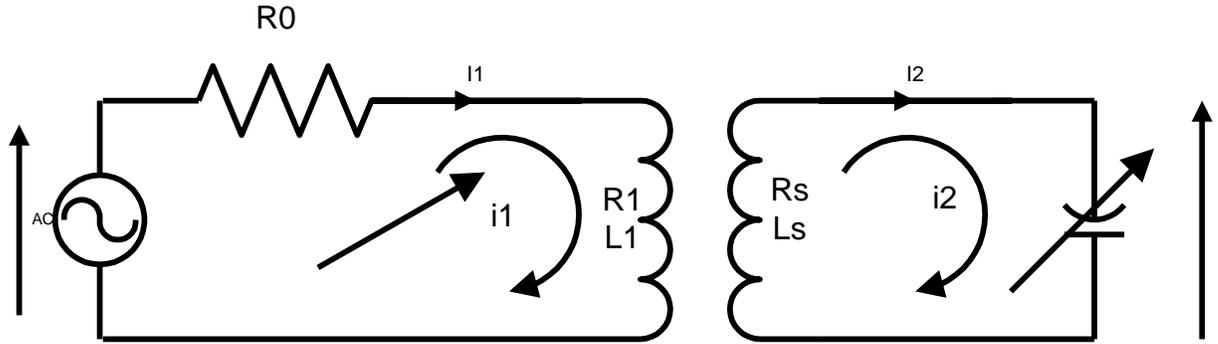


Fig.9

$$Z_{11}I_1 - Z_{12}I_2 = E$$

$$-Z_{21}I_1 + Z_{22}I_2 = 0$$

$$I_2 = \frac{\begin{bmatrix} Z_{11} & E \\ -Z_{21} & 0 \end{bmatrix}}{\begin{bmatrix} Z_{11} & -Z_{12} \\ -Z_{21} & Z_{22} \end{bmatrix}} = \frac{EZ_{21}}{Z_{11}Z_{22} - Z_{12}Z_{21}}$$

$$I_2 = \frac{E(j\omega M)}{R_1(R_2 + jX_2) + \omega^2 M^2}$$

For

$$\omega L \ll R$$

$$I_2 = \frac{E(j\omega M)}{R_1 \left[R_2 + j \left(\omega L_s - \frac{1}{\omega C_s} \right) \right] + \omega^2 M^2}$$

$$V_o = \frac{EM}{C_s \left\{ R_1 \left[R_2 + j \left(\omega L_s - \frac{1}{\omega C_s} \right) \right] + \omega^2 M^2 \right\}}$$

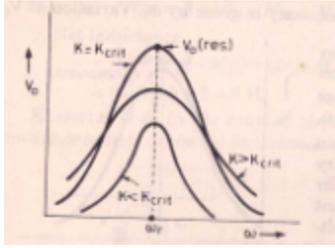


FIG. variation of V_0 with ω for different values of K

$$A = \frac{V_0}{E} = \frac{EM}{EC_s \left\{ R_1 \left[R_2 + j \left(\omega L_s - \frac{1}{\omega C_s} \right) \right] + \omega^2 M^2 \right\}}$$

At resonance

$$I_{2r} = \frac{EM}{R_1 R_2 + \omega_r^2 M^2}$$

$$V_{2r} = \left(\frac{E(\omega_r M)}{R_1 R_2 + \omega_r^2 M^2} \right) \times \frac{1}{C_s}$$

$$A_r = \left(\frac{M}{R_1 R_2 + \omega_r^2 M^2} \right) \times \frac{1}{C_s}$$

$$M_{opt} = \frac{\sqrt{R_1 R_2}}{\omega_r}$$

1.7.Double tuned coupled circuit.(lecture-4)

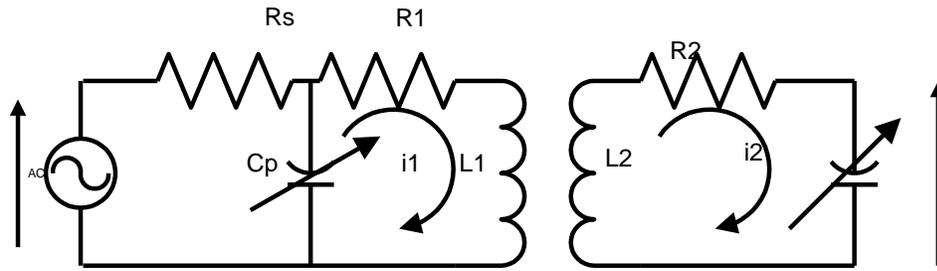


Fig.10

In this circuit the tuning capacitors are placed both in primary as well as secondary side. From the figure the eq. impedance on primary side is, Z_{11} ,
Given by

$$Z_{11} = R_0 + R_p + j \left(\omega_p L_p - \frac{1}{\omega C_p} \right)$$

$$= R_1 + jX_1$$

$$Z_{22} = R_s + j \left(\omega L_s - \frac{1}{\omega C_s} \right)$$

$$= R_2 + jX_2$$

$$Z_{12} = Z_{21} = j\omega M$$

Applying loop analysis

$$I_2 = \frac{E_1 Z_{12}}{Z_{11} Z_{12} - Z_{12}^2}$$

$$V_0 = \frac{I_2}{j\omega C_s} = \frac{E_1 M}{C_s (R_1 + jX_1)(R_2 + jX_2) + \omega^2 M^2}$$

$$= \frac{E_1 M}{C_s ((R_1 + jX_1)(R_2 + jX_2) + \omega^2 M^2)}$$

$$A = \frac{V_0}{E_1} = \frac{E_1 M}{C_s ((R_1 + jX_1)(R_2 + jX_2) + \omega^2 M^2)} \div E_1$$

$$= \frac{E_1 M}{C_s ((R_1 + jX_1)(R_2 + jX_2) + \omega^2 M^2)} \times \frac{1}{E_1}$$

$$= \frac{M}{C_s ((R_1 + jX_1)(R_2 + jX_2) + \omega^2 M^2)}$$

At resonance

$$I_{2_{res}} = \frac{E_1 \omega_r M}{R_1 R_2 + \omega_r^2 M^2}$$

$$V_{o_{res}} = \frac{E_1 \omega_r M}{R_1 R_2 + \omega_r^2 M^2} \times \frac{1}{\omega_r C_s}$$

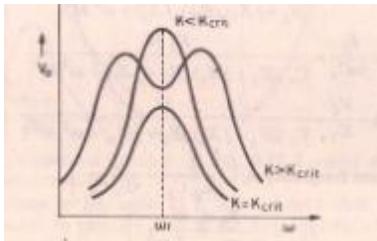


FIG. variation of V_o with ω for different values of K

Or,

$$= \frac{E_1 M / C_s}{R_1 R_2 + \omega_r^2 M^2}$$

$$A_{res} = \frac{V_{o_{res}}}{E_1} = \frac{E_1 M / C_s}{R_1 R_2 + \omega_r^2 M^2} \div E_1$$

$$= \frac{E_1 M}{C_s (R_1 R_2 + \omega_r^2 M^2)} \times \frac{1}{E_1}$$

$$= \frac{M}{C_s (R_1 R_2 + \omega_r^2 M^2)}$$

For maximum out put voltage at resonance,

The denominator should be minimum

As

$$\frac{R_1 R_2}{M} = \omega_r^2 M$$

$$M_{opt} = \frac{\sqrt{R_1 R_2}}{\omega_r} = K_{crit} \sqrt{L_p L_s}$$

This value will give the optimum M

M_{opt}

2. Two-Port Networks

2.1. Terminals and Ports (lecture-5)

In a two-terminal network, the terminal voltage is related to the terminal current by the impedance

$$\mathbf{Z} = \mathbf{V} / \mathbf{I}.$$

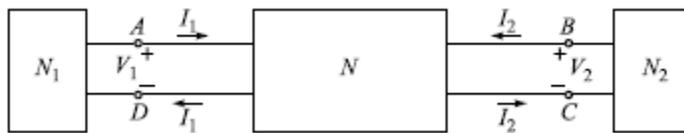


Fig.1

In a four-terminal network, if each terminal pair (or port) is connected separately to another circuit as in Fig. , the four variables i_1 , i_2 , v_1 , and v_2 are related by two equations called the terminal characteristics. These two equations, plus the terminal characteristics of the connected circuits, provide the necessary and sufficient number of equations to solve for the four variables.

2.2. Z-PARAMETERS (open circuit parameters)

The terminal characteristics of a two-port network, having linear elements and dependent sources,

may be written in the s-domain as

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad (1)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad (2)$$

$$Z_{11} = V_1 / I_1 \quad (\text{for } I_2 = 0)$$

$$Z_{21} = V_2/I_1 \quad (\text{for } I_2 = 0)$$

$$Z_{12} = V_1/I_2 \quad (\text{for } I_1 = 0)$$

$$Z_{22} = V_2/I_2 \quad (\text{for } I_1 = 0)$$

2.3.Y-PARAMETERS(short circuit parameters)

The terminal characteristics may also be written as , where I1 and I2 are expressed in terms of V1 and V2.

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad (3)$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad (4)$$

this yields

$$Y_{11} = I_1/V_1 \quad \text{for } V_2 = 0$$

$$Y_{12} = I_1/V_2 \quad \text{For } V_1 = 0$$

$$Y_{21} = I_2/V_2 \quad \text{for } V_1 = 0$$

$$Y_{22} = I_2/V_2 \quad \text{for } V_1 = 0$$

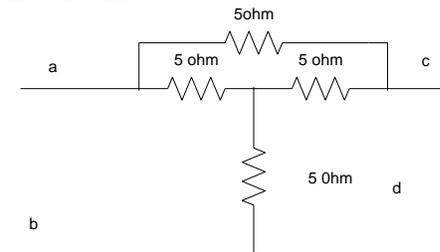
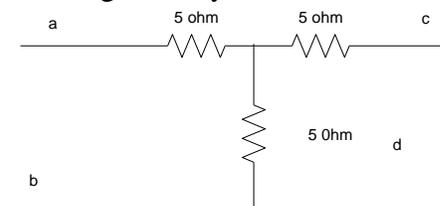


Fig.2

Solution:

We can make two separate networks one a T network comprising R1,R2,R3 and a network containing R4 only.



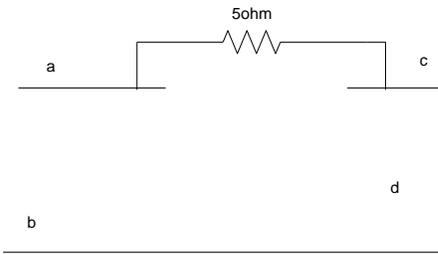
$$Y_{11} = \frac{1}{7.5} \text{ mho}$$

$$Y_{12} = -\frac{1}{15} \text{ mho}$$

$$Y_{21} = \frac{1}{15} \text{ mho}$$

$$Y_{22} = \frac{1}{7.5} \text{ mho}$$

Similarly for the next network



$$Y_{11} = \frac{1}{5} \text{ mho}$$

$$Y_{21} = -\frac{1}{5} \text{ mho}$$

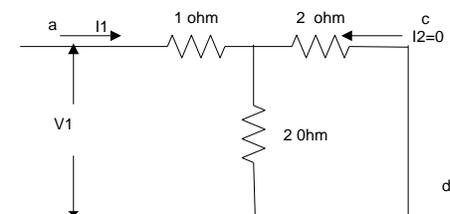
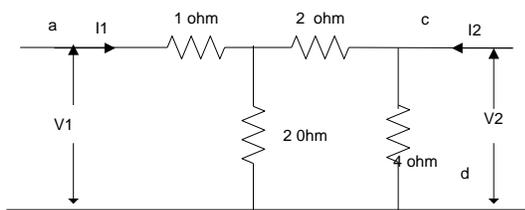
$$Y_{12} = -\frac{1}{5} \text{ mho}$$

$$Y_{22} = \frac{1}{5} \text{ mho}$$

$$[Y_{net}] = [Y_T] + [Y_B]$$

$$Y_{net} = \begin{bmatrix} 1/3 & -4/15 \\ -4/15 & 1/3 \end{bmatrix} \quad (\text{ans})$$

Example. Find the Y parameters for the given circuit

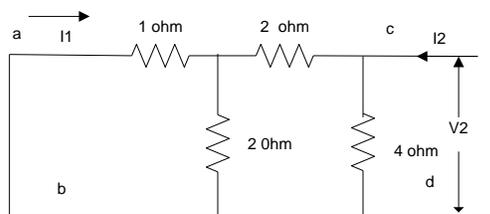


$$Y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0}$$

$$Y_{11} = \frac{1}{1 + \frac{2 \times 2}{2+2}} = \frac{1}{2} \text{ mho}$$

$$I_2 = \frac{-I_1 \times 2}{2} + 2 = \frac{-\frac{V_1}{2} \times 2}{2+2} = \frac{-V_1}{4}$$

$$Y_{21} = \left. \frac{I_2}{V_1} \right|_{I_2=0} = -\frac{1}{4} \text{ mho}$$



$$I_2 = \frac{V_2}{\left[\frac{1 \times 2}{1+2} + 2 \right] \times 4} + 2 = \frac{5V_2}{8}$$

$$Y_{22} = \frac{5}{8} \text{ mho}$$

$$Y_{12} = -\frac{1}{4} \text{ mho}$$

2.4. Transmission Parameters (ABCD parameters)

The transmission parameters A, B, C, and D express the required source variables V_1 and I_1 in terms of the existing destination variables V_2 and I_2 . They are called ABCD or T-parameters and are defined by

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

$$A = V_1/V_2 \quad (\text{for } I_2=0)$$

$$B = - V_1/I_2 \quad (\text{for } V_2=0)$$

$$C = I_1/V_2 \quad (\text{for } I_2=0)$$

$$D = - I_1/I_2 \quad (\text{for } V_2=0)$$

2.5. Hybrid parameters (lecture-6)

Short circuit and open circuit terminal conditions are used for determining the hybrid parameters. H – parameters representation used in modeling of electronic components and circuits.

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

$$h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2=0}, \quad h_{21} = \left. \frac{I_2}{I_1} \right|_{V_2=0}$$

Are called input impedance and forward current gain

$$h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1=0}, \quad \text{and} \quad h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1=0}$$

Are called reverse voltage gain and out put admittance

2.6. Condition for reciprocity and symmetry in two port parameters

A network is termed to be reciprocal if the ratio of the response to the excitation remains unchanged even if the positions of the response as well as the excitation are interchanged.

A two port network is said to be symmetrical if the input and the output port can be interchanged without altering the port voltages or currents.

In Z parameters

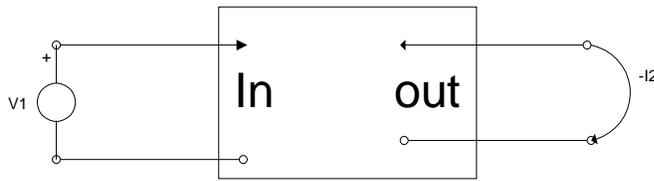


Fig.3

For short circuit and current direction output side is negative.

$$V_1 = Z_{11}I_1 - Z_{12}I_2$$

$$0 = Z_{21}I_1 - Z_{22}I_2$$

From the above two equations

$$I_2 = \frac{V_1 Z_{21}}{Z_{11} Z_{22} - Z_{12} Z_{21}}$$

Let us now interchange the input and output

$$0 = -Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = -Z_{21}I_1 + Z_{22}I_2$$

From these equations we find

$$I_1 = \frac{V_2 Z_{12}}{Z_{11} Z_{22} - Z_{12} Z_{21}}$$

Assume

$$V_1 = V_2$$

Then

$$Z_{12} = Z_{21}$$

Symmetry in Z parameters

Keeping the output port open supplying V in the input side

We get,

$$V = Z_{11}I_1$$

$$Z_{11} = \frac{V}{I_1}$$

And applying voltage at the output port and keeping the input port open

We get,

$$V = Z_{22}I_2$$

$$Z_{22} = \frac{V}{I_2} \text{ so condition of symmetry is achieved for}$$

$$Z_{22} = Z_{11}$$

Similarly the other parameters can be studied and reported in table

| parameter | Condition for reciprocity | Condition for symmetry |
|-----------|---------------------------|------------------------|
|-----------|---------------------------|------------------------|

| | | |
|------|--------------------|-------------------|
| Z | $Z_{12} = Z_{21}$ | $Z_{22} = Z_{11}$ |
| Y | $Y_{12} = Y_{21}$ | $Y_{11} = Y_{22}$ |
| h | $h_{11} = -h_{21}$ | $\Delta h = 1$ |
| ABCD | $AD - BC = 1$ | $A = D$ |

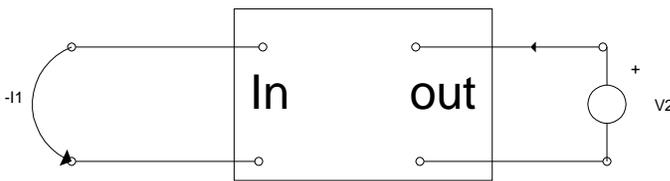


Fig.4

2.7. Interconnecting Two-Port Networks (lecture-7)

Two-port networks may be interconnected in various configurations, such as series, parallel, or cascade connection, resulting in new two-port networks. For each configuration, certain set of parameters may be more useful than others to describe the network.

2.7.1. Series Connection

A series connection of two two-port networks a and b with open-circuit impedance parameters Z_a and Z_b , respectively. In this configuration, we use the Z-parameters since they are combined as a series connection of two impedances.

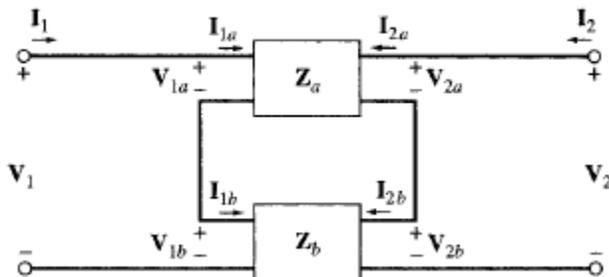


Fig.4

The Z-parameters of the series connection are

$$Z_{11} = Z_{11A} + Z_{11B}$$

$$Z_{12} = Z_{12A} + Z_{12B}$$

$$Z_{21} = Z_{21A} + Z_{21B}$$

$$Z_{22} = Z_{22A} + Z_{22B}$$

Or in the matrix form

$$[Z] = [Z_A] + [Z_B]$$

2.7.2. Parallel Connection

Fig.6

Fig.6

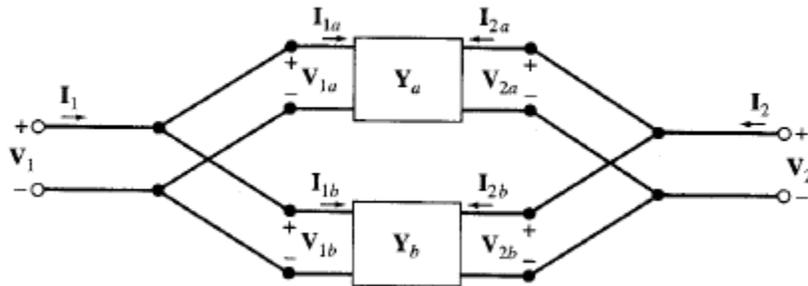


Fig.6

Figure shows a parallel connection of two-port networks a and b with short-circuit admittance parameters Y_a and Y_b . In this case, the Y-parameters are convenient to work with. The Y-parameters of the parallel connection are (see Problem 13.11):

$$Y_{11} = Y_{11A} + Y_{11B}$$

$$Y_{12} = Y_{12A} + Y_{12B}$$

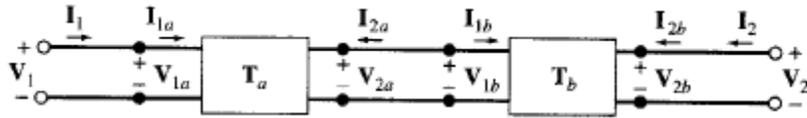
$$Y_{21} = Y_{21A} + Y_{21B}$$

$$Y_{22} = Y_{22A} + Y_{22B}$$

or, in the matrix form

$$[Y] = [Y_A] + [Y_B]$$

2.7.3. Cascade Connection



The cascade connection of two-port networks a and b is shown in Figure above. In this case the transmission-parameters are particularly convenient. The transmission-parameters of the cascade combination are

$$\begin{aligned}
 A &= A_a A_b + B_a C_b \\
 B &= A_a B_b + B_a D_b \\
 C &= C_a A_b + D_a C_b \\
 D &= C_a B_b + D_a D_b
 \end{aligned}$$

or, in the matrix form,

$$[T] = [T_a][T_b]$$

2.8. Y- parameters in terms of z- parameters (lecture-8)

$$V_1 = Z_{11}I_1 + Z_{12}I_2,$$

Or,

$$I_1 = \frac{(V_1 - Z_{12}I_2)}{Z_{11}}$$

similarly

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

$$I_1 = \frac{(V_2 - Z_{22}I_2)}{Z_{21}}$$

Equating the two equations R.H.S

$$\frac{(V_1 - Z_{12}I_2)}{Z_{11}} = \frac{V_2 - Z_{22}I_2}{Z_{21}}$$

$$(V_1 - Z_{12}I_2)Z_{21} = (V_2 - Z_{22}I_2)Z_{11}$$

$$(V_1 Z_{21} - Z_{12} Z_{21} I_2) = (V_2 Z_{11} - Z_{22} Z_{11} I_2)$$

$$(Z_{22} Z_{11} - Z_{12} Z_{21}) I_2 = (-V_1 Z_{21} + V_2 Z_{11})$$

$$I_2 = -\left(\frac{Z_{21}}{Z_{22} Z_{11} - Z_{12} Z_{21}}\right) V_1 + \left(\frac{Z_{11}}{Z_{22} Z_{11} - Z_{12} Z_{21}}\right) V_2$$

$$Y_{21} = \frac{-Z_{11}}{Z_{22}Z_{11} - Z_{12}Z_{21}}$$

$$Y_{22} = \frac{Z_{11}}{Z_{22}Z_{11} - Z_{12}Z_{21}}$$

Also from the two equations

$$I_2 = \frac{V_1 - Z_{11}I_1}{Z_{12}}$$

$$I_2 = \frac{V_2 - Z_{21}I_1}{Z_{22}}$$

And equating the both we get

$$\frac{V_1 - Z_{11}I_1}{Z_{12}} = \frac{V_2 - Z_{21}I_1}{Z_{22}}$$

$$(V_1 - Z_{11}I_1)Z_{22} = (V_2 - Z_{21}I_1)Z_{12}$$

$$(Z_{11}Z_{22} - Z_{21}Z_{12})I_1 = V_1Z_{22} - V_2Z_{12}$$

$$I_1 = \frac{Z_{22}}{(Z_{11}Z_{22} - Z_{21}Z_{12})}V_1 - \frac{Z_{12}}{(Z_{11}Z_{22} - Z_{21}Z_{12})}V_2$$

$$Y_{11} = \frac{Z_{22}}{(Z_{11}Z_{22} - Z_{21}Z_{12})}$$

$$Y_{12} = \frac{-Z_{12}}{D_{ZZ}}$$

Hence the y parameters can be expressed in z parameters as given below

$$\boxed{\begin{array}{l} Y_{11} =, \quad Y_{12} = \\ Y_{12} =, \text{ and } Y_{22} = \end{array}}$$

2.9.Z parameters in terms of Y parameters

$$\left. \begin{array}{l} I_1 = Y_{11}V_1 + Y_{12}V_2 \\ I_2 = Y_{21}V_1 + Y_{22}V_2 \end{array} \right\}$$

(2.9.1)

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

From the above equation we find

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$V_1 = \frac{I_1 - Y_{12}V_2}{Y_{11}}$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

$$V_1 = \frac{I_2 - Y_{22}V_2}{Y_{21}}$$

$$\frac{I_2 - Y_{22}V_2}{Y_{21}} = \frac{I_1 - Y_{12}V_2}{Y_{11}}$$

$$I_2Y_{11} - Y_{11}Y_{22}V_2 = Y_{21}I_1 - Y_{21}Y_{12}V_2$$

$$Y_{21}Y_{12}V_2 - Y_{11}Y_{22}V_2 = Y_{21}I_1 - I_2Y_{11}$$

$$(Y_{11}Y_{22} - Y_{21}Y_{12})V_2 = Y_{21}I_1 - I_2Y_{11}$$

$$V_2 = \frac{Y_{21}}{(Y_{11}Y_{22} - Y_{21}Y_{12})} I_1 - \frac{Y_{11}}{(Y_{11}Y_{22} - Y_{21}Y_{12})} I_2 \quad (2.9.2)$$

$$V_2 = \frac{Y_{21}I_1 - I_2Y_{11}}{(Y_{11}Y_{22} - Y_{21}Y_{12})}$$

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad (2.9.3)$$

$$V_2 = \frac{I_1 - Y_{11}V_1}{Y_{12}}$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad (2.9.4)$$

$$V_2 = \frac{I_2 - Y_{21}V_1}{Y_{22}}$$

$$\frac{I_1 - Y_{11}V_1}{Y_{12}} = \frac{I_2 - Y_{21}V_1}{Y_{22}}$$

$$Y_{22}I_1 - Y_{22}Y_{11}V_1 = Y_{12}I_2 - Y_{12}Y_{21}V_1$$

$$(Y_{22}Y_{11} - Y_{12}Y_{21})V_1 = Y_{22}I_1 - Y_{12}I_2$$

$$V_1 = \frac{Y_{22}}{(Y_{22}Y_{11} - Y_{12}Y_{21})} I_1 - \frac{Y_{12}}{(Y_{22}Y_{11} - Y_{12}Y_{21})} I_2 \quad (2.9.6)$$

$$Z_{11} = \frac{Y_{22}}{(Y_{22}Y_{11} - Y_{12}Y_{21})}$$

$$Z_{12} = \frac{Y_{12}}{(Y_{22}Y_{11} - Y_{12}Y_{21})}$$

Hence,

$$\begin{aligned} Z_{11} &= \frac{Y_{22}}{\Delta Y}, Z_{12} = \frac{Y_{12}}{\Delta Y} \\ Z_{21} &= \frac{Y_{21}}{\Delta Y}, \text{ and } Z_{22} = \frac{Y_{11}}{\Delta Y} \end{aligned}$$

where, $\Delta Y = Y_{11}Y_{22} - Y_{21}Y_{12}$

2.10.Z parameters in terms of ABCD parameters

$$I_1 = CV_2 - DI_2$$

$$V_2 = \frac{1}{C}I_1 + \frac{D}{C}I_2$$

$$V_1 = AV_2 - BI_2$$

$$V_1 = \left(\frac{1}{C}I_1 + \frac{D}{C}I_2 \right) A - BI_2$$

$$V_1 = \frac{A}{C}I_1 + \frac{AD - BC}{C}I_2$$

$$\begin{aligned} Z_{11} &= \frac{A}{C}, Z_{12} = \frac{AD - BC}{C} \\ Z_{21} &= \frac{1}{C}, \text{ and } Z_{22} = \frac{D}{C} \end{aligned}$$

2.11.Z parameters in terms of hybrid- parameters

$$V_1 = h_{11}I_1 + h_{12}V_2 \quad (2.11.1)$$

$$I_2 = h_{21}I_1 + h_{22}V_2 \quad (2.11.2)$$

Using the above equation we get,

$$V_2 = -\frac{h_{21}}{h_{22}}I_1 + \frac{1}{h_{22}}I_2$$

Putting the value of V_2 in the first equation, we get,

$$V_1 = h_{11}I_1 + h_{12} \left(-\frac{h_{21}}{h_{22}}I_1 + \frac{1}{h_{22}}I_2 \right)$$

$$V_1 = \left(\frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}} \right) I_1 + \frac{h_{12}}{h_{22}} I_2$$

$$V_1 = \left(\frac{\Delta h}{h_{22}} \right) I_1 + \frac{h_{12}}{h_{22}} I_2$$

Comparing the equations with governing equation of Z-parameters

$$\boxed{Z_{21} = -\frac{h_{21}}{h_{22}}, \quad Z_{22} = \frac{1}{h_{22}}}$$

$$\boxed{Z_{11} = \frac{\Delta h}{h_{22}}, \quad \text{and } Z_{12} = \frac{h_{12}}{h_{22}}}$$

2.12.Y- parameters in terms of ABCD parameters

$$V_1 = AV_2 - BI_2 \quad (1)$$

$$I_2 = -\frac{1}{B}V_1 + \frac{A}{B}V_2 \quad (2)$$

$$I_1 = CV_2 - DI_2 \quad (3)$$

$$I_1 = CV_2 - D \left[\left(-\frac{1}{B} \right) V_1 + \frac{A}{B} V_2 \right]$$

$$I_1 = \left[\left(\frac{D}{B} \right) V_1 - \frac{AD - BC}{B} V_2 \right]$$

Hence,

$$\boxed{Y_{11} = \frac{D}{B}, \quad Y_{12} = -\left(\frac{AD - BC}{B} \right)}$$

$$\boxed{Y_{21} = -\frac{1}{B}, \quad \text{and } Y_{22} = \frac{A}{B}}$$

2.13. ABCD parameters in terms of Y- parameters

The governing equation for Y parameters

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad (2.13.1)$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad (2.13.2)$$

Or,

$$V_1 = -\frac{Y_{22}}{Y_{21}}V_2 + \left(-\frac{1}{Y_{21}}\right)I_2$$

For the above equation

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

Putting the value of V_1

$$I_1 = Y_{11} \left(-\frac{Y_{22}}{Y_{21}}V_2 + \left(-\frac{1}{Y_{21}}\right)I_2 \right) + Y_{12}V_2$$

$$I_1 = \left(-\frac{Y_{11}Y_{22} - Y_{21}Y_{12}}{Y_{21}} \right) V_2 - \left(-\frac{Y_{11}}{Y_{21}} \right) I_2$$

Comparing the equation with governing equation for ABCD parameters we get

$$A = -\frac{Y_{22}}{Y_{21}}, \quad B = -\frac{1}{Y_{21}}$$
$$C = -\frac{Y_{11}Y_{22} - Y_{21}Y_{12}}{Y_{21}}, \quad \text{and } D = -\frac{Y_{11}}{Y_{21}}$$

3.Transients in DC circuits and AC circuits

3.1.What do you mean by transient? (lecture-9)

Whenever a circuit is switched from one condition to another, either by a change in the applied source or a change in the circuit elements, there is a transitional period during which the branch currents and element voltages change from their former values to new ones. This period is called the transient. After the transient has passed, the circuit is said to be in the steady state. Now, the linear differential equation that describes the circuit will have two parts to its solution, the complementary function (or the homogeneous solution) and the particular solution. The complementary function corresponds to the transient, and the particular solution to the steady state.

3.2.Transients in R-L circuit D.C and A.C source

3.2.1.Consider a series R-L circuit supplied by an DC source with a switch . Step response of dc circuit.

Take the initial condition of the circuit before closing the switch. Applying KVL to the given circuit. Taking laplace transform

$$RI(s) + L\{sI(s) - i(0^-)\} = \frac{V}{s}$$

$$(Ls + R)I(s) = \frac{V}{s}$$

$$I(s) = \frac{V}{s(Ls + R)}$$

$$I(s) = \frac{V/L}{s(s + R/L)}$$

$$= \left(\frac{A}{s} + \frac{B}{s + R/L} \right)$$

$$A = V/R$$

$$B = -V/R$$

$$I(s) = \left(\frac{V/R}{s} + \frac{-V/R}{s + R/L} \right)$$

$$i(t) = \frac{V}{R} - \frac{V}{R} e^{-\frac{R}{L}t}$$

$$i(t) = \frac{V}{R} (1 - e^{-\frac{R}{L}t})$$

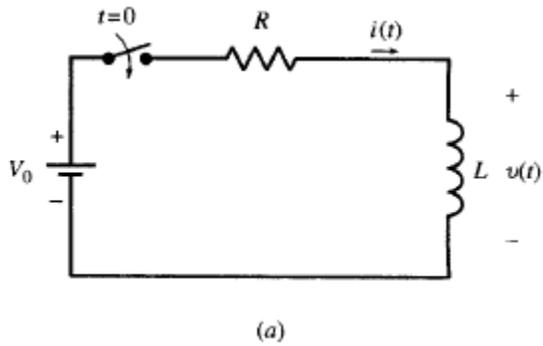
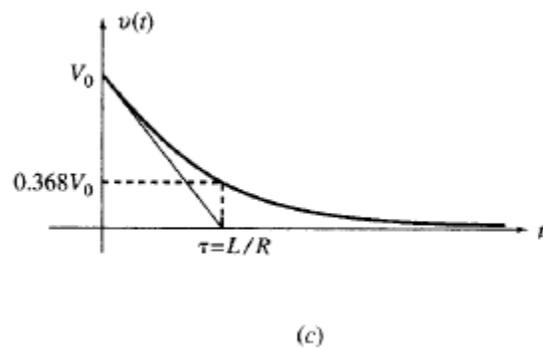
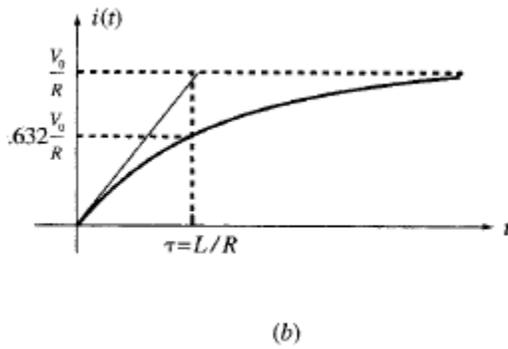
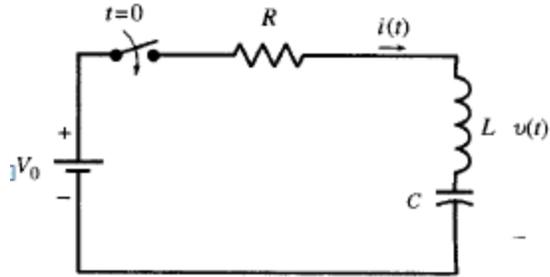


Fig.1



3.2.2. Transient response of series R-L –C circuit with step input.



(a)

Fig.2.

$$I(s) = \frac{1/s}{R + Ls + 1/Cs}$$

$$I(s) = \frac{1/L}{s^2 + (R/L)s + 1/LC}$$

$$I(s) = \frac{1/s}{(s + \alpha)(s + \beta)}$$

$$\text{where } \alpha, \beta = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Case 1

Both roots are real and not equal

$$\frac{R}{2L} > \frac{1}{LC}$$

$$I(s) = \frac{K_1}{(s + \alpha)} + \frac{K_2}{(s + \beta)}$$

$$K_1 = \left. \frac{1/L}{(s + \beta)} \right|_{s = -\alpha} = \frac{1}{L(\beta - \alpha)}$$

$$K_2 = \left. \frac{1/L}{(s + \alpha)} \right|_{s = -\beta} = \frac{1}{L(\alpha - \beta)}$$

$$I(s) = \frac{1}{L(\beta - \alpha)(s + \alpha)} + \frac{1}{L(\alpha - \beta)(s + \beta)}$$

$$i(t) = \left[\frac{1}{L(\beta - \alpha)} e^{-\alpha t} + \frac{1}{L(\alpha - \beta)} e^{-\beta t} \right]$$

Case 2

$$\frac{R}{2L} = \frac{1}{LC}$$

$$i(t) = \frac{1}{L}te^{-\gamma t}$$

Case3

$$\frac{R}{2L} < \frac{1}{\sqrt{LC}}, \alpha = \beta^*$$

Let

$$\alpha = -A_0 + jB^*$$

$$\beta = -A_0 - jB^*$$

$$I(s) = \frac{1/L}{(s + A_0 - jB)(s + A_0 + jB)}$$

$$I(s) = \frac{K_3}{(s + A_0 - jB)} + \frac{K_3^*}{(s + A_0 + jB)}$$

$$K_3 = (s + A_0 - jB)I(s) \Big|_{s=-A_0+jB}$$

$$K_3 = \frac{1/L}{(s + A_0 + jB)} \Big|_{s=-A_0+jB} = \frac{1/L}{j2B}$$

$$K_3^* = -\frac{1/L}{j2B}$$

So,

$$I(s) = \frac{\frac{1/L}{j2B}}{(s + A_0 - jB)} + \frac{\left(-\frac{1/L}{j2B}\right)}{(s + A_0 + jB)}$$

Taking inverse Laplace transform

We get,

$$i(t) = \frac{1}{j2BL} e^{-A_0 t} e^{jBt} - \frac{1}{j2BL} e^{-A_0 t} e^{-jBt}$$

Or,

$$i(t) = \frac{e^{-A_0 t}}{BL} \left(\frac{e^{jBt} - e^{-jBt}}{j2} \right)$$

$$i(t) = \frac{e^{-A_0 t}}{BL} (\sin Bt)$$

3.2.3. Pulse response of RC series circuit (lecture-10)

In this section we will derive the response of a first-order circuit to a rectangular pulse. The derivation applies to RC or RL circuits where the input can be a current or a voltage. As an example, we use the series RC circuit in Fig. 7-17(a) with the voltage source delivering a pulse of duration T and height V_0 . For $t < 0$, v and i are zero. For the duration of the pulse, we use (6b) and (6c) in Section

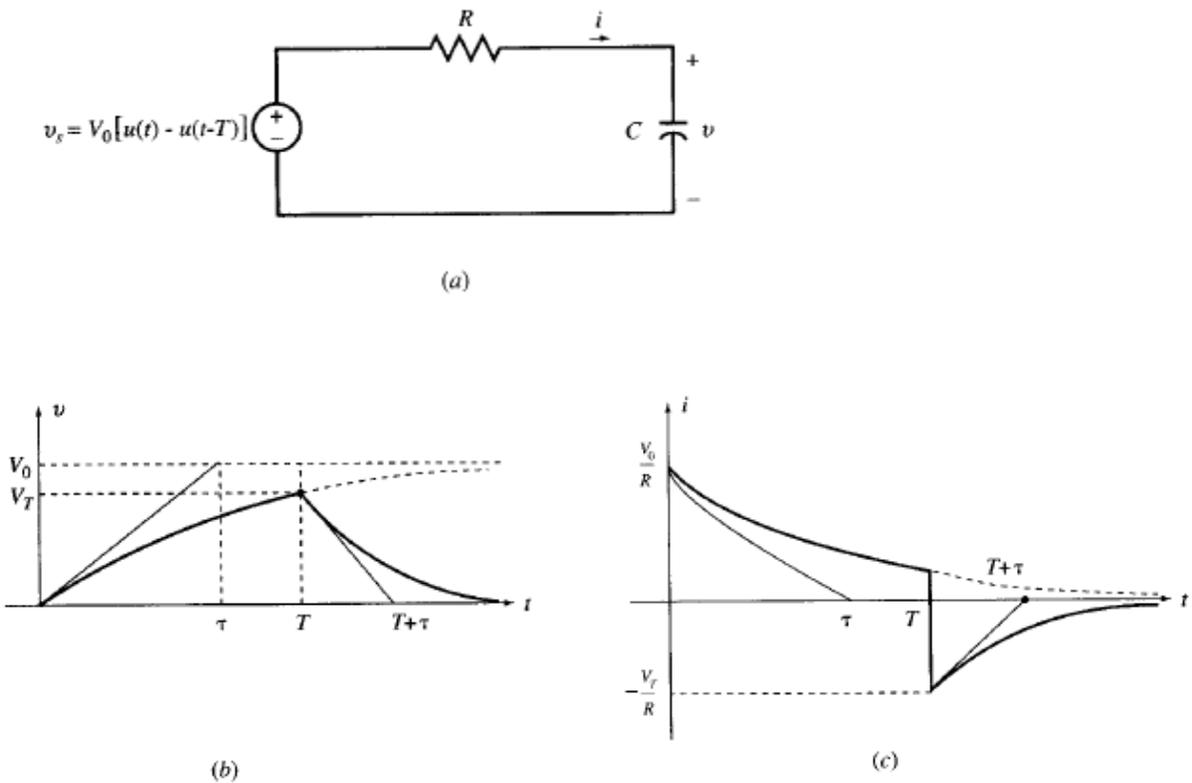


Fig.3

$$v = V_0(1 - e^{-t/RC}) \quad 0 < t < T$$

$$i = \frac{V_0}{R} e^{-t/RC} \quad 0 < t < T$$

When the pulse ceases, the circuit is source-free with the capacitor at an initial voltage V_T .

$$V_T = V_0(1 - e^{-T/RC}) \quad t > T$$

taking into account the time shift T , we have

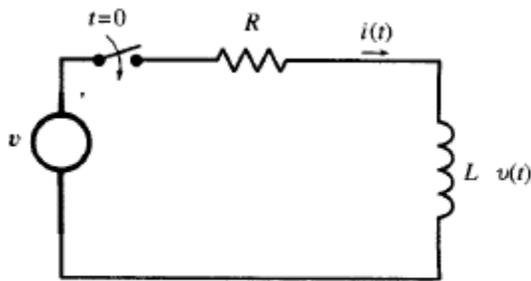
$$v = V_T(1 - e^{-(t-T)/RC}) \quad t > T$$

$$i = -\frac{V_T}{R} e^{-(t-T)/RC} \quad t > T$$

The capacitor voltage and current are plotted in Figs.(b) and (c)

3.2.4.R-L series with sine wave

Applying KVL to the circuit



(a)

Fig.4.

Applying KVL to the circuit

$$i(t)R + L \frac{di}{dt} = E_m \sin \omega t$$

Taking Laplace transform

$$I(s)R + LsI(s) - sI(0) = E_m \left(\frac{\omega}{s^2 + \omega^2} \right)$$

Or

$$I(s)R + LsI(s) - 0 = E_m \left(\frac{\omega}{s^2 + \omega^2} \right)$$

$$I(s)(R + Ls) = E_m \left(\frac{\omega}{s^2 + \omega^2} \right)$$

$$I(s) = \frac{E_m}{(R + Ls)} \left(\frac{\omega}{s^2 + \omega^2} \right)$$

Or

$$I(s) = \frac{E_m}{L(s + R/L)} \left(\frac{\omega}{(s + j\omega)(s - j\omega)} \right)$$

$$I(s) = \frac{K_1}{(s + R/L)} + \frac{K_2}{(s + j\omega)} + \frac{K_3}{(s - j\omega)}$$

$$K_1 = \left[\frac{E_m}{L(s + R/L)} \left(\frac{\omega}{(s + j\omega)(s - j\omega)} \right) \right]_{s=-R/L} = \frac{E_m \omega}{R^2 + \omega^2 L^2}$$

$$K_2 = \left[\frac{E_m}{L(s + R/L)} \left(\frac{\omega}{(s + j\omega)(s - j\omega)} \right) \right]_{s=-j\omega} = \frac{E_m}{-2j(R - j\omega L)}$$

$$K_3 = \left[\frac{E_m}{L(s + R/L)} \left(\frac{\omega}{(s + j\omega)(s - j\omega)} \right) \right]_{s=j\omega} = \frac{E_m}{2j(R - j\omega L)}$$

Thus

$$I(s) = \frac{E_m \omega L}{(R^2 + \omega^2 L^2)(s + R/L)} + \frac{E_m}{-2j(R - j\omega L)(s + j\omega)} + \frac{E_m}{2j(R - j\omega L)(s - j\omega)}$$

Taking inverse Laplace Transform

$$i(t) = \frac{E_m \omega L}{(R^2 + \omega^2 L^2)} e^{-Rt/L} - \frac{E_m (R + j\omega L)}{2j(R^2 + \omega^2 L^2)} e^{-j\omega t} + \frac{E_m (R - j\omega L)}{2j(R^2 - j\omega^2 L^2)} e^{j\omega t}$$

*****End of module –I *****

MODULE-II(10 HOURS)

CHAPTER - 4

NETWORK FUNCTIONS & RESPONSES

4.1 INTRODUCTION(lecture-11)

In engineering, a transfer function (also known as the system function or network function and, when plotted as a [graph](#), transfer curve) is a mathematical representation for [fit](#) or to describe the inputs and outputs of [black box models](#).

All the systems are designed to produce a particular output, when the input is applied to it. The system parameters, perform some operation on the applied input, in order to produce the required output. The mathematical indication of cause and effect relationship existing between input and output of a system is called the system function or transfer function of the system. The Laplace transform plays an important role in defining the system function.

4.2 CONCEPT OF COMPLEX FREQUENCY

A complex number used to characterize exponential and damped sinusoidal motion in the same way that an ordinary frequency characterizes the simple harmonic motion; designated by the constants corresponding to a motion whose amplitude is given by Ae^{st} , where A is a constant and t is time.

A type of frequency that depends on two parameters; one is the “ σ ” which controls the magnitude of the signal and the other is “ ω ”, which controls the rotation of the signal; is known as “complex frequency”.

A complex exponential signal is a signal of type 1

$$X(t) = X_m e^{st} \dots\dots\dots (4.2.1)$$

Where X_m and s are time independent complex parameter and $S = \sigma + j\omega$ where X_m is the magnitude of $X(t)$, sigma (σ) is the real part in S and is called neper frequency and is expressed in Np/s. “ ω ” is the radian frequency and is expressed in rad/sec. “ S ” is called complex frequency and is expressed in complex neper/sec.

Now put the value of S in equation (4.2.1), we get

$$X(t) = X_m e^{\sigma t + j\omega t}$$

$$X(t) = X_m e^{\sigma t} e^{j\omega t}$$

By using Euler’s theorem

i.e. $e^{i\theta} = \cos\theta + i \sin\theta$

$$X(t) = X_m e^{\sigma t} [\cos(\omega t) + j \sin(\omega t)]$$

The real part is

$$X(t) = X_m e^{\sigma t} \cos(\omega t)$$

And imaginary part is

$$X(t) = X_m e^{\sigma t} \sin(\omega t)$$

The physical interpretation of complex frequency appearing in the exponential form will be studied easily by considering a number of special cases for the different value of S.

Case no 1: When $\omega = 0$ and σ has a certain value, then, the real part is

$$X(t) = X_m e^{\sigma t} \cos(\omega t)$$

$$X(t) = X_m e^{\sigma t} \cdot 1$$

$$X(t) = X_m e^{\sigma t}$$

The imaginary part is zero (0),
 Since $S = \sigma + j\omega$, $S = \sigma$ as $\omega = 0$

Now there are also three cases in above case no 1.

- (i) If the neper frequency is positive i.e. $\sigma > 0$ the curve obtains is exponentially increasing curve as shown below.

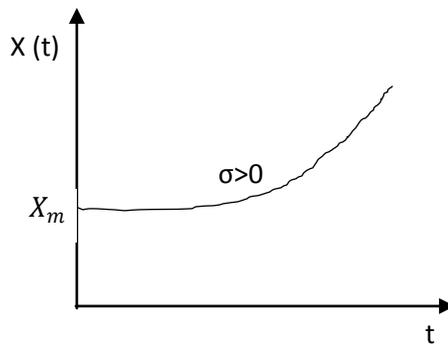


Fig. 4.1

- (ii) If $\sigma < 0$ then the curve obtain is exponentially decreasing curve as shown below.

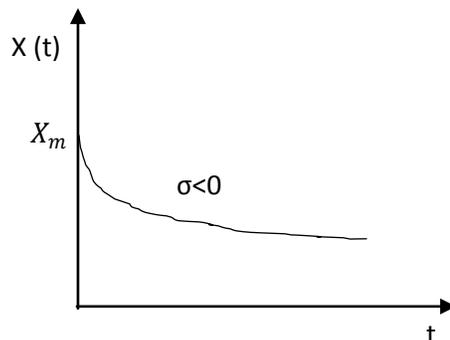


Fig. 4.2

(iii) If $\sigma = 0$ then the curve obtain is the steady state DC curve as shown below fig. 4.3

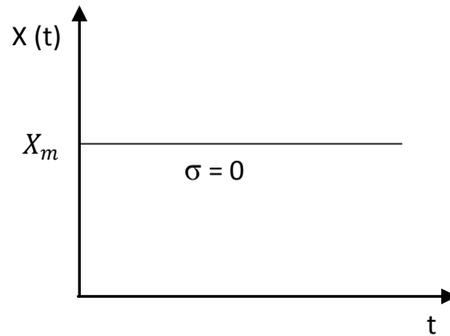


Fig. 4.3

Case no 2: When $\sigma = 0$ and ω has some value then, the real part is

$$X(t) = X_m e^{0.t} \cos(\omega t)$$

$$X(t) = X_m \cos(\omega t)$$

And the imaginary part is

$$X(t) = X_m \sin(\omega t)$$

Hence the curve obtained is a sinusoidal steady state curve, as shown in the below figure.

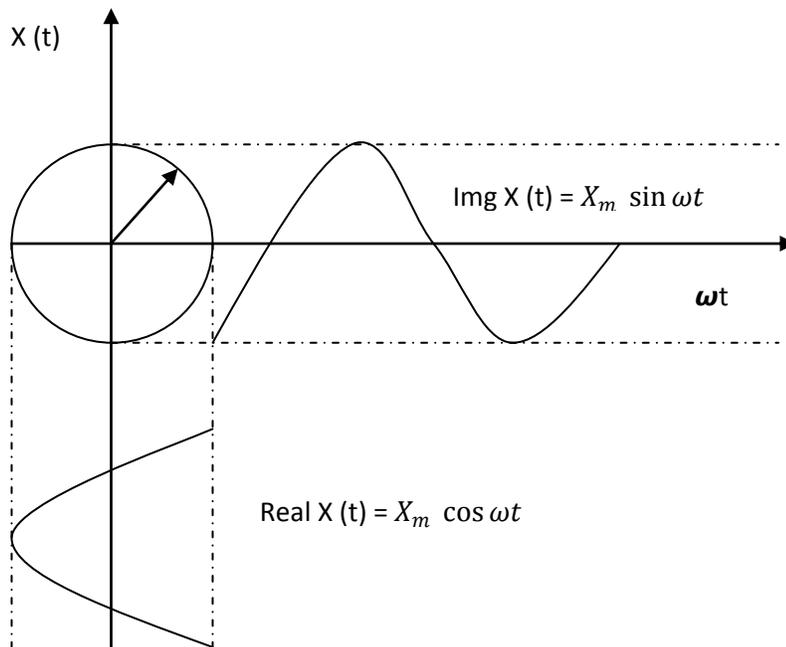


Fig. 4.4

Case no 3: When σ and ω both have some value, then the real part is

$$X(t) = X_m e^{\sigma t} \cos(\omega t)$$

And the imaginary part is

$$X(t) = X_m e^{\sigma t} \sin(\omega t)$$

So, the curve obtained is time varying sinusoidal signal. These case no 3 is also has some two cases

(a) When $\sigma > 0$

Real Part

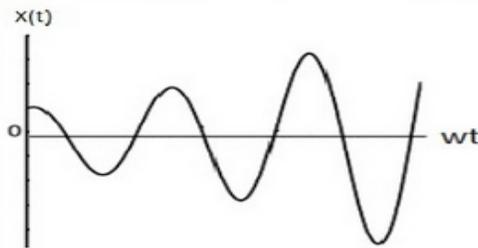


Fig. 4.5

Imaginary Part

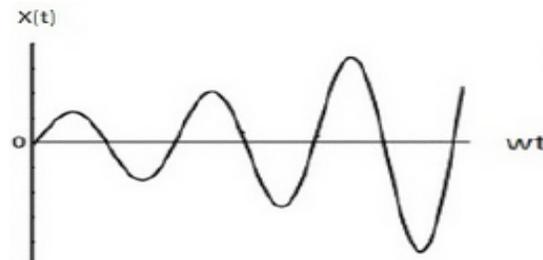


Fig. 4.6

(b) When $\sigma < 0$

Real Part

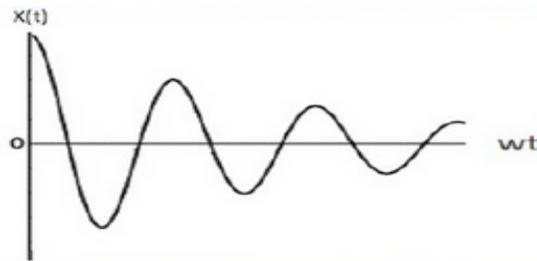


Fig. 4.7

Imaginary Part

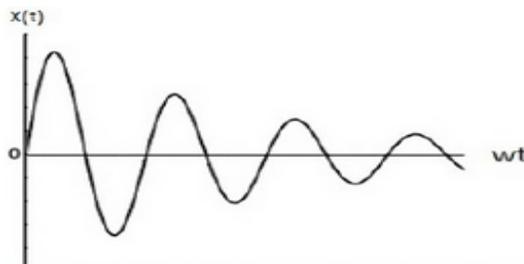


Fig. 4.8

4.3 NETWORK FUNCTIONS FOR ONE PORT NETWORK: (Lecture -12)

(a) Driving Point Impedance Function

The ratio of the Laplace transform of the voltage at the port to the Laplace transform of the current at the same port, neglecting initial conditions is called driving point impedance function, denoted as $Z(s)$

$$Z(s) = \frac{V(s)}{I(s)}$$

(b) Driving Point Admittance Function

The ratio of the Laplace transform of the current at the port to the Laplace transform of the voltage at the same port, neglecting initial conditions is called driving point admittance function.

$$Y(s) = \frac{I(s)}{V(s)}$$

Example 4.3.1 For the given one port network shown in fig., find driving point impedance function.

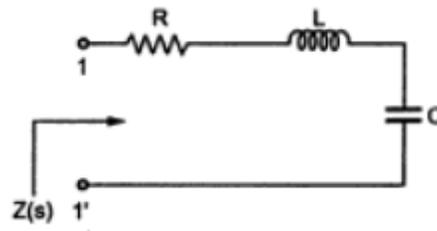


Fig.4.9

Solution: To find the driving point impedance for the given network, first convert the network into s domain which is called transformed network.

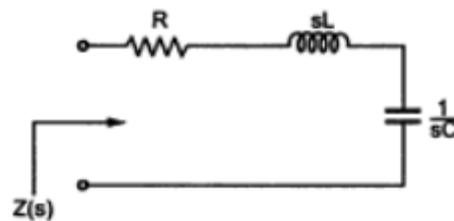


Fig. 4.10

The transformed network is as shown in fig

Then,

$$Z(s) = R + sL + \frac{1}{sC}$$

$$= \frac{R(sC) + (sL)(sC) + 1}{sC} = \frac{sRC + s^2LC + 1}{sC}$$

Hence, the driving point impedance function of a given network is

$$Z(s) = \frac{s^2LC + sRC + 1}{sC}$$

Example 4.3.2 Find driving point admittance function for the given network having only one port.

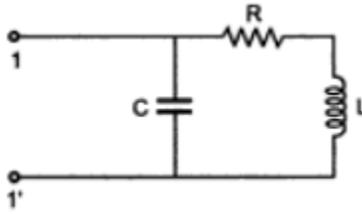


Fig. 4.11

Solution: First transform the given network as shown in fig. 4.12

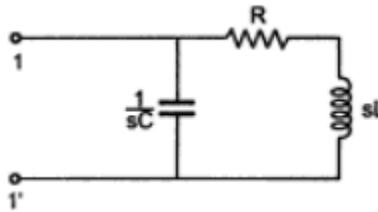


Fig. 4.12 Transformed network

When the branches are connected in parallel it is very easy to find admittance of an overall function by adding admittances of individual branches which are connected in parallel.

Admittance of capacitive branch is given as

$$Y_1(s) = \frac{1}{1/sC} = sC$$

Admittance of the inductive branch is given as

$$Y_2(s) = \frac{1}{R + sL}$$

∴ Total admittance is given as

$$\begin{aligned} Y(s) &= Y_1(s) + Y_2(s) \\ &= sC + \frac{1}{R + sL} = \frac{sRC + s^2LC + 1}{R + sL} \end{aligned}$$

Hence, the driving point admittance function of a given network is,

$$Y(s) = \frac{s^2LC + sRC + 1}{R + sL}$$

4.4 NETWORK FUNCTIONS FOR TWO PORT NETWORK: (Lecture -13)

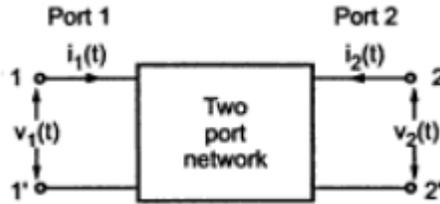


Fig. 4.13

For the two port network, there are two ports. The variables measured at port 1 are $v_1(t)$ and $i_1(t)$ while the variables measured at port 2 are $v_2(t)$ and $i_2(t)$. This is shown in above fig. 4.13. The ratio of the variables measured at the same port either port 1 or port 2 defines driving point function. Hence, for two port networks, we can define,

(a) Driving Point Impedance Function

This is the ratio of $V_1(s)$ and $I_1(s)$ at port 1, denoted as $Z_{11}(s)$ or the ratio of $V_2(s)$ to $I_2(s)$ at port 2, denoted as $Z_{22}(s)$. Both are driving impedance functions.

$$Z_{11}(s) = \frac{V_1(s)}{I_1(s)} \text{ and } Z_{22}(s) = \frac{V_2(s)}{I_2(s)}$$

(b) Driving Point Admittance Function

The reciprocals of the driving point impedance functions at the ports give the driving point admittance functions at the respective ports. These are denoted as $Y_{11}(s)$ and $Y_{22}(s)$ at the two ports respectively.

$$Y_{11}(s) = \frac{I_1(s)}{V_1(s)} \text{ and } Y_{22}(s) = \frac{I_2(s)}{V_2(s)}$$

4.5 TRANSFER FUNCTION OF TWO PORT NETWORK:

The function defined as the ratio or Laplace of two variables such that one variable is defined at one port while the other variable is defined at the second port. Accordingly, four transfer functions can be defined for two port network as,

1. Voltage Ratio Transfer Function

It is the ratio of Laplace transforms of voltage at one port to voltage at other port. It is denoted as $G(s)$.

$$\therefore G(s) = \frac{V_2(s)}{V_1(s)}$$

2. Current Ratio Transfer Function

It is the ratio of Laplace transforms of current at one port to current at other port. It is denoted as $\alpha(s)$.

$$\therefore \alpha(s) = \frac{I_2(s)}{I_1(s)} \text{ or } \frac{I_1(s)}{I_2(s)}$$

3. Transfer Impedance Function

It is the ratio of Laplace transforms of voltage at one port to current at other port. It is denoted as $Z_{12}(s)$ or $Z_{21}(s)$.

$$\therefore Z_{21}(s) = \frac{V_2(s)}{I_1(s)} \text{ and } Z_{12}(s) = \frac{V_1(s)}{I_2(s)}$$

4. Transfer Admittance Function

It is the ratio of Laplace transforms of current at one port to voltage at other port. It is denoted as $Y_{12}(s)$ or $Y_{21}(s)$.

$$\therefore Y_{21}(s) = \frac{I_2(s)}{V_1(s)} \text{ and } Y_{12}(s) = \frac{I_1(s)}{V_2(s)}$$

Example 4.5.1 For given two port network find driving point impedance function and voltage ratio transfer function.

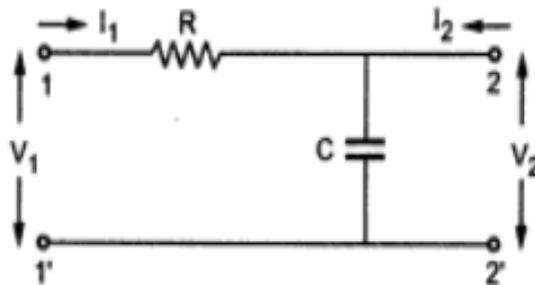


Fig. 14

Solution: Transforming given network in s domain, the network will be as shown in below fig.

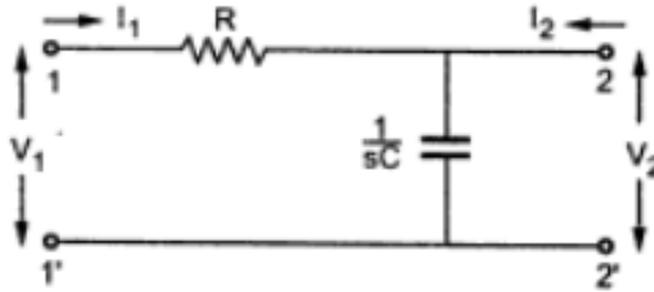


Fig 15

To find voltage ratio transfer function, find V_2 using potential divider rule as follows,

$$V_2 = V_1 \left[\frac{1/sC}{R + 1/sC} \right]$$

$$\therefore \frac{V_2}{V_1} = \frac{1/sC}{sRC + 1}$$

$$\frac{V_2}{V_1} = \frac{1}{sRC + 1}$$

As port 2 is open circuited, $I_2 = 0$.

Hence current I_1 will flow through R and C.

$$V_1 = I_1 \left[R + \frac{1}{sC} \right]$$

$$Z(s) = \frac{V_1}{I_1} = \frac{sRC + 1}{sC}$$

4.6 POLES & ZEROS OF NETWORK FUNCTIONS : (lecture-14)

In pole/zero analysis, a network is described by its network transfer function which, for any linear time-invariant network, can be written in the general form:

$$H(s) = \frac{N(s)}{D(s)} = \frac{a_0 s^m + a_1 s^{(m-1)} + \dots + a_m}{b_0 s^n + b_1 s^{(n-1)} + \dots + b_n}$$

In the factorized form, the general function is:

$$H(s) = \frac{a_0}{b_0} \cdot \frac{(s + z_1)(s + z_2) \dots (s + z_i) \dots (s + z_m)}{(s + p_1)(s + p_2) \dots (s + p_j) \dots (s + p_n)}$$

The roots of the numerator $N(s)$ (that is, z_i) are called the zeros of the network function, and the roots of the denominator $D(s)$ (that is, p_j) are called the poles of the network function. S is a complex frequency.

The dynamic behavior of the network depends upon the location of the poles and zeros on the network function curve. The poles are called the natural frequencies of the network. In general, you can graphically assume the magnitude and phase curve of any network function from the location of its poles and zeros.

Example 4.6.1 A function given by $Z(s) = \frac{s+4}{s}$. Find the pole-zero plot.

Solution: In the above function the zero is at $s = -4$ and the pole is at $s = 0$. The pole-zero plot is shown below in fig. 16

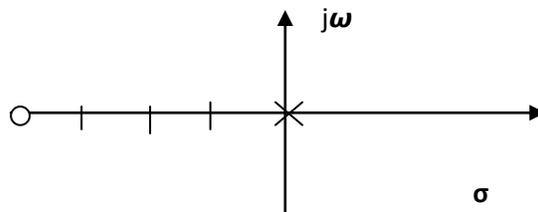


Fig. 16

Example 4.6.2 A function given by $Z(s) = \frac{2s}{(s+2)(s^2+2s+2)}$. Obtain its pole-zero plot.

Solution: The zero is at $s = 0$ and the pole is at $s = -2, (-1+j1), (-1-j1)$. The pole-zero plot is shown below in fig. 17

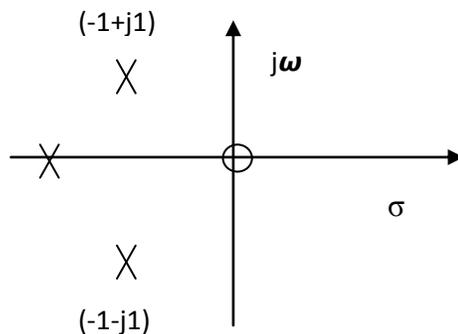


Fig. 17

4.7 RESTRICTION ON POLE AND ZERO LOCATIONS OF NETWORK FUNCTION

(Lecture -15)

The following are the restrictions on pole and zero location of network function:

- (a) The co-efficient of the polynomials of numerator and denominator of the network function $H(s)$ must be real and positive.
- (b) Poles and zeros, if complex or imaginary, must occur in conjugate pairs.
- (c) The real parts of all poles and zeros must be zero or negative.
- (d) The polynomial of the numerator or denominator cannot have any missing term between those of highest and lowest order values unless all even order or all odd order terms are missing.
- (e) The degree of the numerator or the denominator polynomial may differ by zero or one only.
- (f) The lowest degree in numerator and denominator may differ in degree by at the most one.

Example 4.7.1 Check if the impedance function $Z(s)$ given by $Z(s) = \frac{s^4+s^2+1}{s^3+2s^2-2s+10}$ can represent a passive one port network.

Solution: The given function is not suitable to represent the impedance of a one port network due to the following reasons:

- (i) In the numerator one co-efficient is missing.
- (ii) In the denominator one co-efficient is negative.

4.8 IMPULSE RESPONSE

The system function is defined as,

$$H(s) = \frac{\text{Laplace of output response}}{\text{Laplace of input excitation}} = \frac{R(s)}{E(s)}$$

$$\therefore R(s) = H(s).E(s)$$

Let the input impulse function $\delta(t)$.

$$\therefore e(t) = \delta(t)$$

$$\therefore E(s) = L\{\delta(t)\} = 1$$

$$\therefore R(s) = H(s).1 = H(s)$$

Thus the Laplace of the response is same as the system function, for unit impulse input. Thus, if $r(t)$ is the impulse response of the network then as

$$R(s) = H(s)$$

$$\text{Impulse response} = \mathcal{L}^{-1}[R(s)] = \mathcal{L}^{-1}[H(s)] = h(t)$$

Thus $h(t) = \mathcal{L}^{-1}[H(s)]$ is nothing but the impulse response of the network.

Example 4.8.1 Find the impulse response of the network shown in below fig. 16. The excitation is the voltage $v(t)$ while the response is $i(t)$.

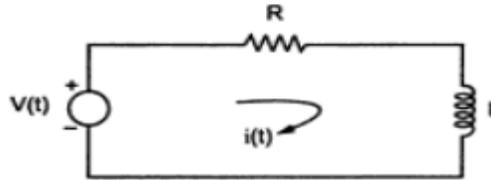


Fig. 16

Solution: The excitation is $v(t)$ and $i(t)$ is response hence,

$$H(s) = \frac{I(s)}{V(s)}$$

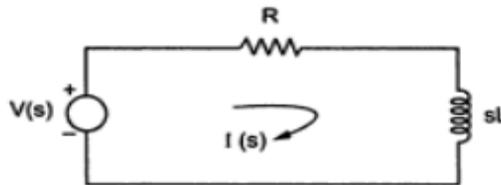


Fig. 17

As the initial condition is zero, the circuit can be transformed in the Laplace domain as shown in above fig.

After applying KVL,

$$-I(s)R - I(s)Ls + V(s) = 0$$

$$I(s)[R + Ls] = V(s)$$

$$H(s) = \frac{I(s)}{V(s)} = \frac{1}{R + sL}$$

Example 4.8.2 Impulse response of a circuit is given as $h(t) = -te^{-t} + 2e^{-t}, t > 0$. Find the transfer function of the network.

Solution: L [Impulse Response] = Transfer Function

$$\begin{aligned} H(s) &= -\frac{1}{(s+1)^2} + \frac{2}{(s+1)} \\ &= \frac{-1+2(s+1)}{(s+1)^2} \\ &= \frac{2s+1}{(s+1)^2} \end{aligned}$$

Example 4.8.3 The transfer function of a network is given by $(S) = \frac{1}{1+sRC}$. Find its impulse response.

$$\begin{aligned} \text{Solution: } \delta(t) &= L^{-1} \left[\frac{1}{1+sRC} \right] \\ &= L^{-1} \left[\frac{\frac{1}{RC}}{\frac{1}{RC}+s} \right] \\ &= \frac{1}{RC} e^{-t/RC} \end{aligned}$$

4.9 COMPLETE RESPONSE

Complete response is the sum of forced response (steady state response) and source free response (natural response). The response of a circuit or network with presence of source is called forced or source response. This response is independent of the nature of passive elements and their initial condition but this response completely depends on type of input. This response is different for different types of input. Response of any circuit or network without any source is called as source free response or natural response.

Example 4.9.1 For the circuit given below, find complete response for $i(t)$ if $v(0) = 15$ V.

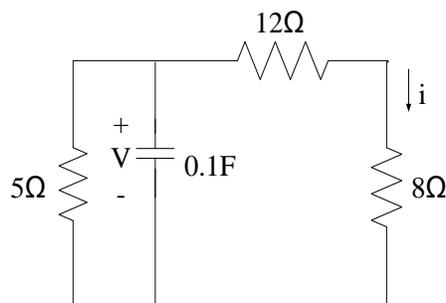


Fig.18

Solution: It is natural response due to the capacitor internal stored energy. This is a source free first order RC circuit.

$$v(t) = v_o e^{-\frac{t}{\tau}}$$

$$R_{eq} = 5 \parallel 20 = 4 \Omega$$

$$\tau = RC = 4 * 0.1 = 0.4 \text{ sec}$$

$$v(t) = 15 e^{-\frac{t}{0.4}} = 15 e^{-2.5t} \text{ V}$$

$$i(t) = \frac{v(t)}{R} = \frac{15 e^{-2.5t}}{12 + 8} = 0.75 e^{-2.5t} \text{ A}$$

4.10 TIME DOMAIN BEHAVIOR FROM POLE-ZERO PLOT (Lecture -16)

In time-domain analysis the response of a dynamic system to an input is expressed as a function of time. It is possible to compute the time response of a system if the nature of input and the mathematical model of the system are known.

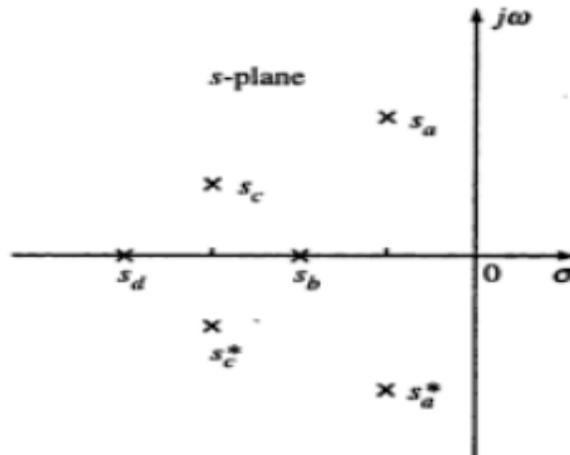


Fig.19 Array of poles

The poles s_b and s_d are quite different from the conjugate pole pairs. Further, s_b and s_d are the real poles and the response due to the poles s_b and s_d converges monotonically. The poles s_b and s_d may correspond to the quadratic function,

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0; \zeta > 1$$

And the roots

$$s_b, s_d = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}; \quad \zeta > 1 \quad \dots\dots\dots (1)$$

The contribution to the total response, due to poles s_b and s_d is

$$K_b e^{s_b t} + K_d e^{s_d t}$$

The contribution to response due to the pole s_b is predominant compared to that of s_d as $|s_d| \gg |s_b|$. In this case, s_b is dominant compared to s_d . The pole s_b is the dominant pole amongst these two real poles. The response due to pole at s_d dies down faster compared to that due to pole at s_b . The complex conjugate poles s_a and s_a^* which belong to the quadratic factor

$$s^2 + 2\zeta\omega_n s + \omega_n^2; \quad \zeta > 1$$

The roots are

$$s_a, s_a^* = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

Contribution to the total response by the complex conjugate poles s_a and s_a^* is

$$K_a e^{-\zeta\omega_n t} e^{j\omega_n\sqrt{1-\zeta^2}t} + K_a^* e^{-\zeta\omega_n t} e^{-j\omega_n\sqrt{1-\zeta^2}t}$$

The factor $\exp(-\zeta\omega_n t)$ gives the decreasing function, whereas the factor $\exp(j\omega_n\sqrt{1-\zeta^2}t)$ gives sustained oscillation. The resultant will give the damped sinusoid waveform, as shown in fig. 20

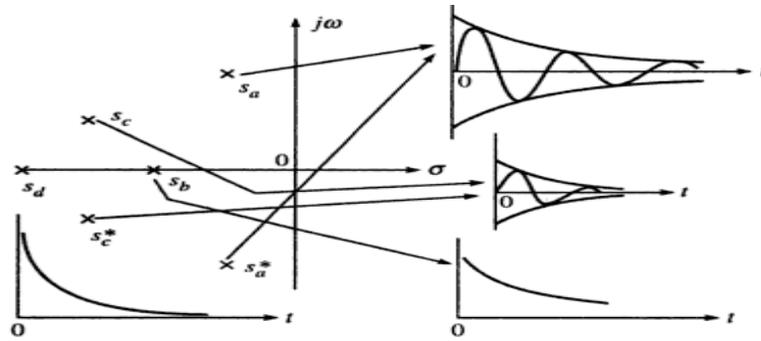


Fig. 20 Nature of response for arbitrary magnitude for all poles.

Similarly, s_c and s_c^* are also the complex conjugate pole pair. The response due to s_c and s_c^* will die down faster than due to s_a and s_a^* . Hence s_a and s_a^* are the dominant complex conjugate pole pair compared to the complex pole pair s_c and s_c^* . The time domain response can be obtained by taking the inverse Laplace transform after the partial fraction expansion,

$$\mathcal{L}^{-1} \left[\frac{K_a}{s-s_a} + \frac{K_a^*}{s-s_a^*} + \frac{K_b}{s-s_b} + \frac{K_c}{s-s_c} + \frac{K_c^*}{s-s_c^*} + \frac{K_d}{s-s_d} \right]$$

Where the residues K_a and K_a^* are complex conjugate, as also the residues K_c and K_c^* . Any residues K_r can be obtained as

$$K_r = H \times \frac{(s - s_1) \dots (s - s_n)}{(s - s_a) \dots (s - s_r) \dots (s - s_m)} (s - s_r)$$

The term $(s_r - s_n)$ is also a complex number expressed in polar co-ordinates as

$$s_r - s_n = M_{rn} \exp(j\phi_{rn})$$

$$K_r = H \times \frac{M_{r1} \dots M_{rn}}{M_{ra} \dots M_{rm}} \exp(j(\phi_{r1} + \phi_{r2} \dots \phi_{rn} - \phi_{ra} \dots \phi_{rm}))$$

Example 4.10.1 A transfer function is given by $Y(s) = \frac{10s}{(s+5+j15)(s+5-j15)}$. Find the time domain response.

Solution: $Y(s) = k_1 e^{-(5+j15)t} + k_2 e^{-(5-j15)t}$

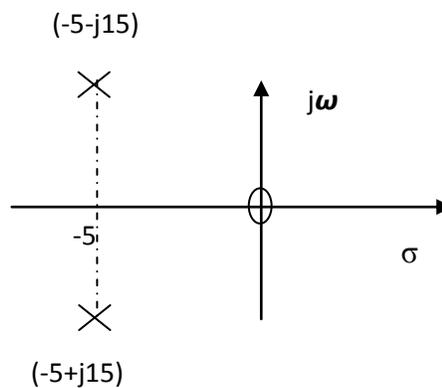


Fig.21

$$k_1 = \frac{10 \sqrt{5^2 + 15^2} \angle 90 + \tan^{-1}\left(\frac{5}{15}\right)}{30 \angle 90} = 5.267 \angle 18.4$$

$$k_2 = \frac{10 \sqrt{5^2 + 15^2} \angle -90 - \tan^{-1}\left(\frac{5}{15}\right)}{30 \angle -90} = 5.267 \angle -18.4$$

$$Y(t) = 5.267 \angle 18.4 e^{-(5+j15)t} + 5.267 \angle -18.4 e^{-(5-j15)t}$$

CHAPTER - 5

THREE PHASE CIRCUITS

5.1 INTRODUCTION (Lecture -17)

There are two types of system available in electric circuit, single phase and **three phase system**. In single phase circuit, there will be only one phase, i.e. the [current](#) will flow through only one wire and there will be one return path called neutral line to complete the circuit. So in the single phase minimum amount of power can be transported. Here the generating station and load station will also be single phase. This is an old system using from previous time.

In 1882, a new invention has been done on polyphase system, that more than one phase can be used for generating, transmitting and for load system. **Three phase circuit** is the polyphase system where three phases are sent together from the generator to the load. Each phase is having a phase difference of 120° , i.e. 120° angle electrically. So from the total of 360° , three phases are equally divided into 120° each. The power in **three phase system** is continuous as all the three phases are involved in generating the total power. The sinusoidal waves for 3 phase system is shown below

The three phases can be used as single phase each. So if the load is single phase, then one phase can be taken from the **three phase circuit** and the neutral can be used as ground to complete the circuit.

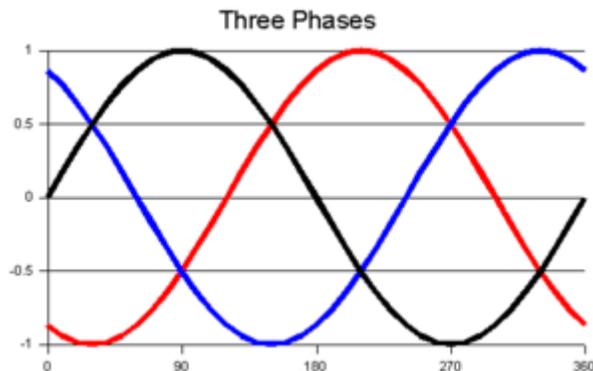


Fig. 1

5.2 IMPORTANCE OF THREE PHASE OVER SINGLE PHASE

There are various reasons for this because there are a number of advantages over single phase circuit. The [three phase system](#) can be used as three single phase line so it can act as three

single phase system. The three phase generation and single phase generation are the same in the generator except the arrangement of coil in the generator to get 120° phase difference. The conductor needed in three phase circuit is 75% that of conductors needed in a single phase circuit. And also the instantaneous power in a single phase system falls down to zero as in single phase, we can see from the sinusoidal curve, but in [three phase system](#) the net power from all the phases gives a continuous power to the load.



Till now we can say that there are three [voltage source](#) connected together to form a three phase circuit. And actually it is inside the generator. The generator is having three [voltage sources](#) which are acting together in 120° phase difference. If we can arrange three single phase circuit with 120° phase difference, then it will become a three phase circuit. So 120° phase difference is must otherwise the circuit will not work, the three phase load will not be able to get active and it may also cause damage to the system.

The size or metal quantity of three phase devices is not having much difference. Now if we consider the transformer, it will be almost same size for both single phase and three phase because transformer will make only the linkage of flux. So the [three phase system](#) will have a higher efficiency compared to single phase because of the same or little difference in mass of transformer, three phase line will be out whereas in single phase it will be only one. And losses will be minimized in three phase circuit. So overall in conclusion the [three phase system](#) will have better and higher efficiency compared to the single phase system.

In a three phase circuit, connections can be given in two types:

1. Star connection
2. Delta connection

5.2.1 STAR CONNECTION

In star connection, there are four wires, three wires are phase wire and fourth is neutral which is taken from the star point. Star connection is preferred for long distance power transmission because it is having the neutral point. In this we need to come to the concept of balanced and unbalanced [current](#) in power system.

When equal [current](#) will flow through all the three phases, then it is called as balanced current. And when the [current](#) will not be equal in any of the phases, then it is unbalanced current. In this case, during balanced condition, there will be no [current](#) flowing through the

neutral line and hence there is no use of the neutral terminal. But when there will be an unbalanced current flowing in the three phase circuit, neutral is having a vital role. It will take the unbalanced [current](#) through to the ground and protect the transformer.

Unbalanced [current](#) affects transformer and it may also cause damage to the transformer and for this star connection is preferred for long distance transmission. The star connection is shown below-

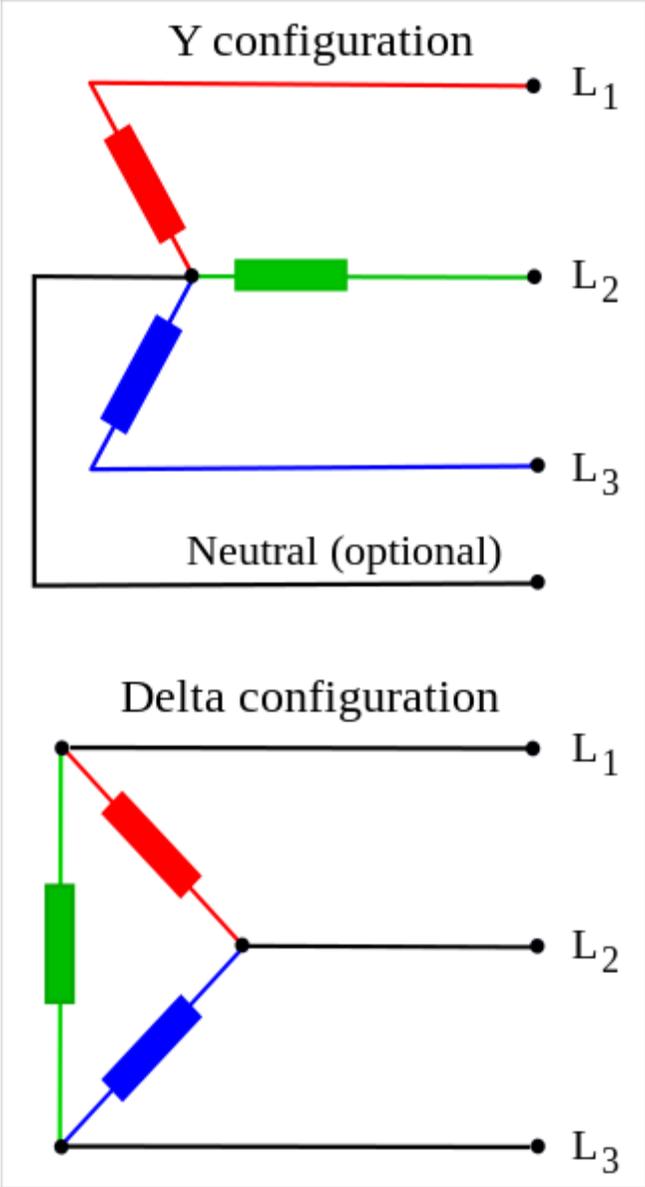


Fig. 2, 3

In star connection, the line [voltage](#) is $\sqrt{3}$ times of phase voltage. Line [voltage](#) is the [voltage](#) between two phases in three phase circuit and phase [voltage](#) is the [voltage](#) between one phase to the neutral line. And the [current](#) is same for both line and phase. It is shown as expression below

$$E_{line} = \sqrt{3} E_{phase} \text{ and } I_{Line} = I_{phase}$$

5.2.2 DELTA CONNECTION

In delta connection, there are three wires alone and no neutral terminal is taken. Normally delta connection is preferred for short distance due to the problem of unbalanced [current](#) in the circuit. The figure is shown below for delta connection. In the load station, ground can be used as the neutral path if required.

In delta connection, the line [voltage](#) is same with that of phase voltage. And the line [current](#) is $\sqrt{3}$ times of the phase current. It is shown as expressed below,

$$E_{line} = E_{phase} \text{ and } I_{Line} = \sqrt{3} I_{phase}$$

In a three phase circuit, star and delta connection can be arranged in four different ways-

1. Star-Star connection
2. Star-Delta connection
3. Delta-Star connection
4. Delta-Delta connection

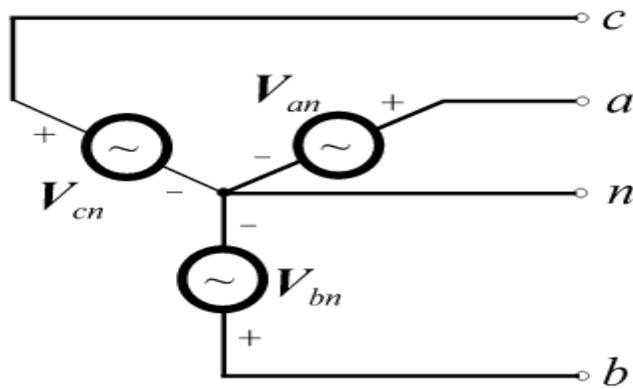
But the power is independent of the circuit arrangement of the [three phase system](#). The net power in the circuit will be same in both star and delta connection. The power in three phase circuit can be calculated from the equation below,

$$P_{Total} = 3 \times E_{phase} \times I_{phase} \times PF$$

Since there are three phases, so the multiple of 3 is made in the normal power equation and the PF is the power factor. Power factor is a very important factor in [three phase system](#) and sometimes due to certain error, it is corrected by using capacitors.

5.3 ANALYSIS OF BALANCED THREE PHASE CIRCUITS(Lecture -18)

In a balanced system, each of the three instantaneous voltages has equal amplitudes, but is separated from the other voltages by a phase angle of 120. The three voltages (or phases) are typically labeled a, b and c. The common reference point for the three phase voltages is designated as the neutral connection and is labeled as n. The three sources V_{an} , V_{bn} and V_{cn} are designated as the line-to-neutral voltages in the three-phase system. An alternative way of defining the voltages in a balanced three-phase system is to define the voltage differences between the phases. These voltages are designated as line-to-line voltages. The line-to-line voltages can be expressed in terms of the line-to-neutral voltages by applying Kirchoff's voltage law to the generator circuit, which yields



$$V_{ab} = V_{an} - V_{bn}$$

$$V_{bc} = V_{bn} - V_{cn}$$

$$V_{ca} = V_{cn} - V_{an}$$

Fig.4

Either a positive phase sequence (abc) or a negative phase sequence(acb) as shown below.

Positive Phase Sequence

Negative Phase Sequence

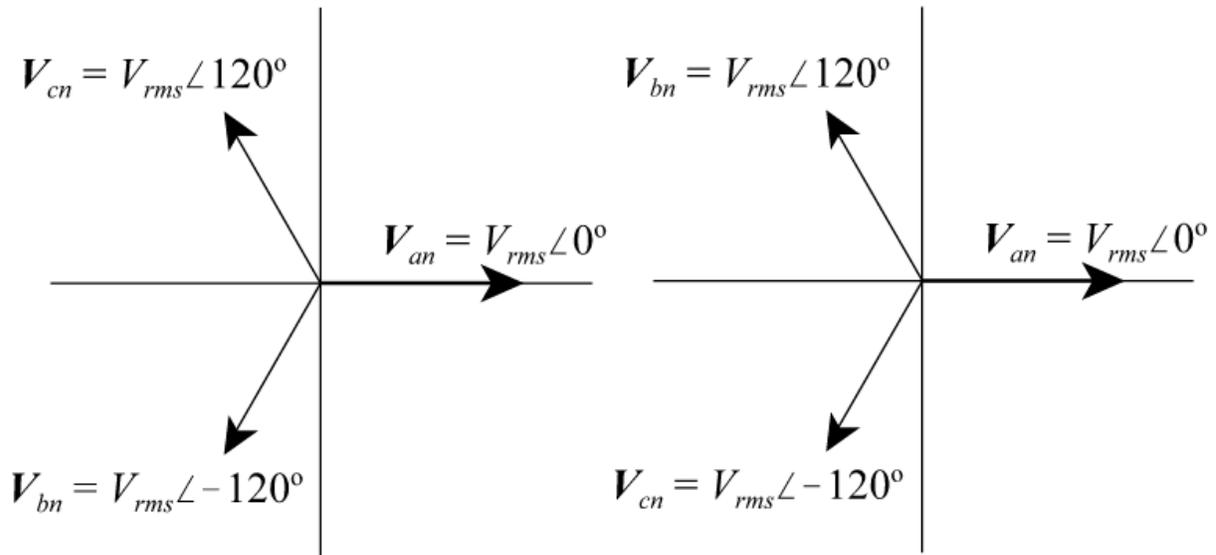


Fig.5

Inserting the line-to-neutral voltages for a positive phase sequence into the line-to-line equations yields

$$V_{ab} = V_{an} - V_{bn} = V_{rms} \angle 0^\circ - V_{rms} \angle -120^\circ$$

$$= V_{rms} [e^{j0^\circ} - e^{-j120^\circ}] = V_{rms} \{1 - [\cos(120^\circ) - j\sin(120^\circ)]\}$$

$$\begin{aligned}
&= V_{rms} \left[1 + \frac{1}{2} + j \frac{\sqrt{3}}{2} \right] = V_{rms} \left[\frac{3 + j\sqrt{3}}{2} \right] = \sqrt{3}V_{rms} \left[\frac{\sqrt{3} + j1}{2} \right] \\
&= \sqrt{3}V_{rms} [1 \angle 30^\circ] \\
&= \sqrt{3}V_{rms} [\angle 30^\circ]
\end{aligned}$$

$$\begin{aligned}
V_{bc} &= V_{bn} - V_{cn} = V_{rms} \angle -120^\circ - V_{rms} \angle 120^\circ \\
&= V_{rms} [e^{-j120^\circ} - e^{j120^\circ}] = V_{rms} \{[\cos(120^\circ) - j\sin(120^\circ)] - [\cos(120^\circ) + j\sin(120^\circ)]\}
\end{aligned}$$

$$\begin{aligned}
&= V_{rms} [-2j\sin(120^\circ)] = V_{rms} \left[-2j \frac{\sqrt{3}}{2} \right] = \sqrt{3}V_{rms} [1 \angle -90^\circ] \\
&= \sqrt{3}V_{rms} [\angle -90^\circ]
\end{aligned}$$

$$\begin{aligned}
V_{ca} &= V_{cn} - V_{an} = V_{rms} \angle 120^\circ - V_{rms} \angle 0^\circ \\
&= V_{rms} [e^{j120^\circ} - e^{j0^\circ}] = V_{rms} \{[\cos(120^\circ) + j\sin(120^\circ) - 1]\} \\
&= V_{rms} \left[-\frac{1}{2} + j \frac{\sqrt{3}}{2} - 1 \right] = V_{rms} \left[\frac{-3 + j\sqrt{3}}{2} \right] = \sqrt{3}V_{rms} \left[\frac{-\sqrt{3} + j1}{2} \right] \\
&= \sqrt{3}V_{rms} [1 \angle 150^\circ] \\
&= \sqrt{3}V_{rms} [\angle 150^\circ]
\end{aligned}$$

If we compare the line-to-neutral voltages with the line-to-line voltages, we find the following relationships,

| Line-to-neutral voltages | Line-to-line voltages |
|--------------------------------------|---|
| $V_{an} = V_{rms} \angle 0^\circ$ | $V_{ab} = \sqrt{3}V_{rms} \angle 30^\circ$ |
| $V_{bn} = V_{rms} \angle -120^\circ$ | $V_{bc} = \sqrt{3}V_{rms} \angle -90^\circ$ |
| $V_{cn} = V_{rms} \angle 120^\circ$ | $V_{ca} = \sqrt{3}V_{rms} \angle 150^\circ$ |

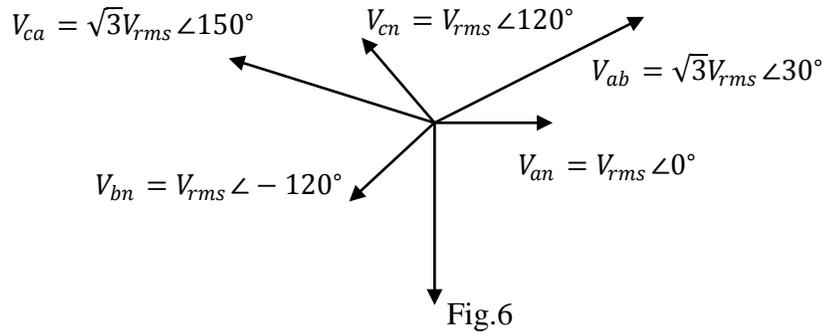
Line-to-line voltages in terms of line-to-neutral voltages:

$$V_{ab} = \sqrt{3}V_{an} e^{j30^\circ}$$

$$V_{bc} = \sqrt{3}V_{bn} e^{j30^\circ}$$

$$V_{ca} = \sqrt{3}V_{cn} e^{j30^\circ}$$

The equations above show that the magnitudes of the line-to-line voltages in a balanced three-phase system with a positive phase sequence are $\sqrt{3}$ times the corresponding line-to-neutral voltages and lead these voltages by 30° .



Balanced three-phase circuits consist of a balanced three-phase source and a balanced three-phase load can be of four varieties like Y-Y, Y- Δ , Δ -Y or Δ - Δ . Common analysis procedure for all the possible configurations are done.

5.3.1 EQUIVALENCE BETWEEN A Y AND Δ CONNECTED SOURCE (Lecture -19)

Let $V_{RY} = V_L \angle 0^\circ$, $V_{YB} = V_L \angle -120^\circ$, and $V_{BR} = V_L \angle 120^\circ$ be the three line voltages observed in a three-phase circuit and let $I_R = I_L \angle -(\theta + 30^\circ)$, $I_Y = I_L \angle -(\theta + 150^\circ)$, and $I_B = I_L \angle -(\theta - 90^\circ)$ be the observed line currents where the observed phase lag of first line current is expressed as 30° plus some angle designated by θ . If the source is within a black box and have to guess whether it is a Y-connected source or a Δ -connected source, it becomes difficult to resolve the using the observed line voltage phasors and line current phasors. This is so because both configurations shown in fig.7 will result in these observed line voltages and line currents.

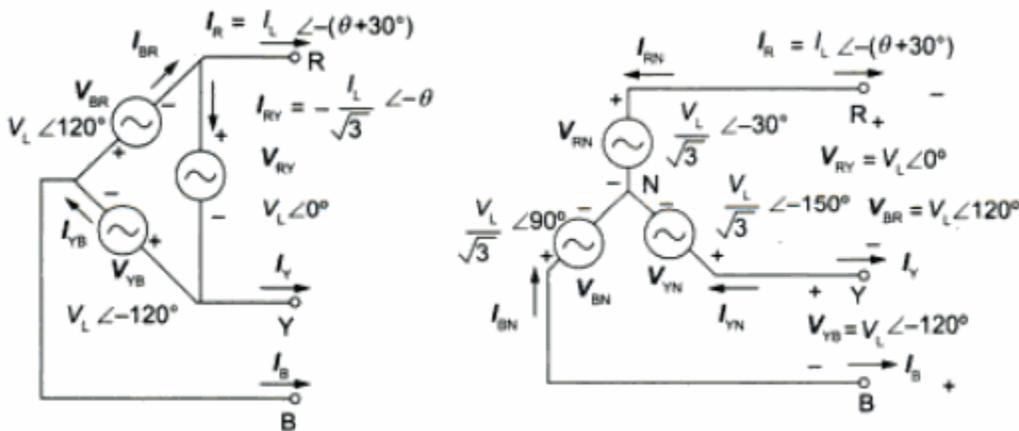


Fig.7

Therefore if a balanced three-phase source is actually Δ - connected, it may be replaced by an equivalent Y-connected source for purposes of analysis. The details of phase – source currents in the actual Δ - connected source can be obtained after the circuit problem is solved using the star equivalent source.

5.3.2 EQUIVALENCE BETWEEN A Y AND Δ CONNECTED LOAD

The individual branch currents in the branches of delta in the actual Δ -connected load can be obtained after line quantities have been obtained by using its equivalent Y-connected model. Three-phase symmetry and $\frac{1}{\sqrt{3}}$ factor connecting line currents and phase currents in a Δ -connected system can be used for this purpose. The impedance to be used in Y-connected load is $\frac{1}{3}$ times the impedance present in Δ -connected load as shown in fig. 8.

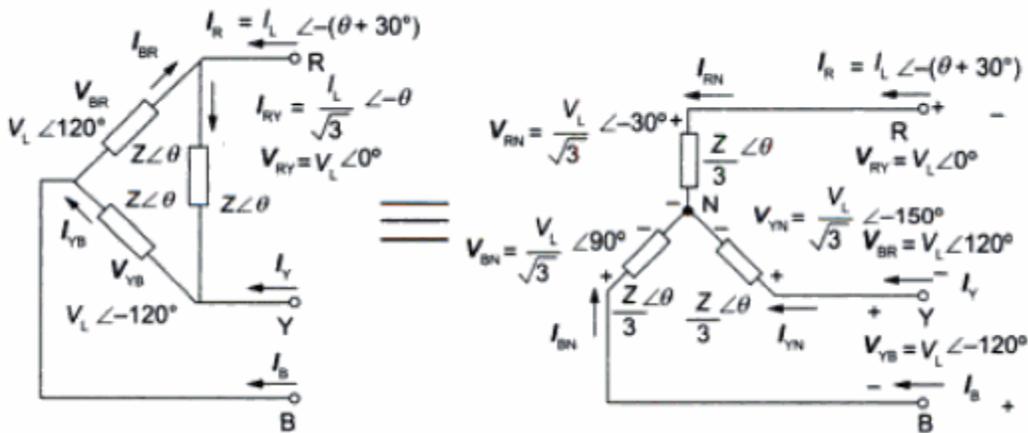


Fig.8

5.4 ANALYSIS OF UNBALANCED LOADS

Three-phase systems deliver power in enormous amounts to single-phase loads such as lamps, heaters, air-conditioners, and small motors. It is the responsibility of the power systems engineer to distribute these loads equally among the three-phases to maintain the demand for power fairly balanced at all times. While good balance can be achieved on large power systems, individual loads on smaller systems are generally unbalanced and must be analyzed as unbalanced three phase systems.

5.4.1 UNBALANCED DELTA CONNECTED LOAD

An unbalanced condition is due to unequal delta connected load.

Example 5.4.1.1 A delta connected load as shown in fig. 9 is connected across three-phase 100V supply. Determine all the line currents. Also draw the relevant phasor diagram showing all voltages and currents. Given $V_{AB} = 100\angle 0^\circ$, $V_{BC} = 100\angle -120^\circ$, $V_{CA} = 100\angle -240^\circ$

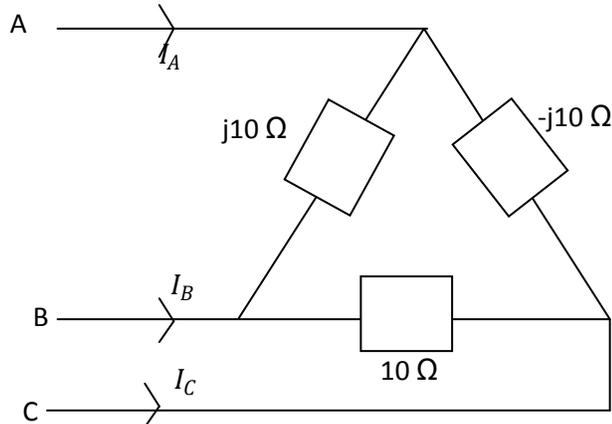


Fig. 9

Solution: Given: $V_{AB} = 100\angle 0^\circ$, $V_{BC} = 100\angle -120^\circ$, $V_{CA} = 100\angle -240^\circ$

$$I_{AB} = \frac{V_{AB}}{Z_{AB}} = \frac{100\angle 0^\circ}{j10} = 10\angle 0^\circ \text{ A,}$$

$$I_{BC} = \frac{V_{BC}}{Z_{BC}} = \frac{100\angle -120^\circ}{10} = 10\angle -120^\circ \text{ A,}$$

$$I_{CA} = \frac{V_{CA}}{Z_{CA}} = \frac{100\angle -240^\circ}{-j10} = 10\angle -150^\circ \text{ A}$$

$$I_A = I_{AB} - I_{CA} = 10\angle -90^\circ - 10\angle -150^\circ = 10\angle -30^\circ \text{ A}$$

$$I_B = I_{BC} - I_{AB} = 10\angle -120^\circ - 10\angle -90^\circ = 5.17\angle 165^\circ \text{ A}$$

$$I_C = I_{CA} - I_{BC} = 10\angle -150^\circ - 10\angle -120^\circ = 5.17\angle 135^\circ \text{ A}$$

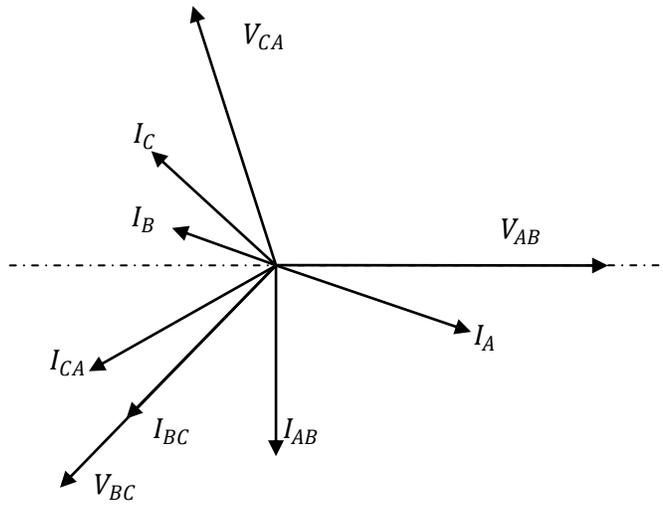


Fig.9 phasor diagram

5.4 NEUTRAL SHIFT

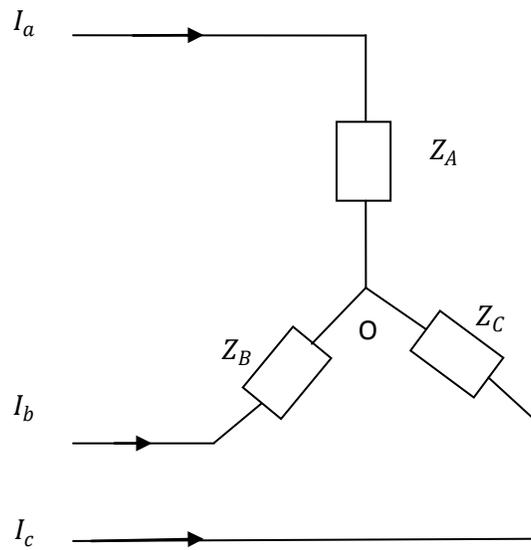


Fig. 10

For balanced three-phase, $I_N = 0$. For unbalanced system, I_N has some finite value, due to flow of unequal currents in each of the phases, voltage appears at the neutral and its value can be found as follows:

$$\begin{aligned}
 I_a + I_b + I_c &= V_{a_0} Y_a + V_{b_0} Y_b + V_{c_0} Y_c \\
 &= (V_{an} - V_{on}) Y_a + (V_{bn} - V_{on}) Y_b + (V_{cn} - V_{on}) Y_c \\
 &= V_{an} Y_a + V_{bn} Y_b + V_{cn} Y_c - V_{on} (Y_a + Y_b + Y_c)
 \end{aligned}$$

Applying KCL at O, $I_a + I_b + I_c = 0$

$$\begin{aligned}
 V_{an} Y_a + V_{bn} Y_b + V_{cn} Y_c - V_{on} (Y_a + Y_b + Y_c) &= 0 \\
 V_{on} &= \frac{V_{an} Y_a + V_{bn} Y_b + V_{cn} Y_c}{Y_a + Y_b + Y_c} \dots \dots \dots (5.4.1)
 \end{aligned}$$

The equation 5.4.1 gives the neutral shift.

5.5 SYMMETRICAL COMPONENTS (Lecture -20)

When an unbalanced three-phase fault occurs, we can solve the three-phase circuit using circuit theory. This is much more numerically complicated than the single phase circuit normally used in balanced three phase circuits. The degree of difficulty increases with the third power of the system size. For this reason, it is apparent that if we were to solve three different single-phase circuits, it would be numerically simpler than solving the one three-phase circuit in one set of equations. The purpose of this chapter is to break up the large three-phase circuit into three circuits, each one third the size of the whole system. Next, we solve the three components individually, and then combine the results to obtain the total system response.

5.5.1 FUNDAMENTALS OF SYMMETRICAL COMPONENTS

It was Fortescue in 1918 who developed the idea of breaking up asymmetrical three-phase voltages and currents into three sets of symmetrical components. These three basic components are:

- (1) The positive sequence currents and voltages (known also as the "abc" and often denoted by the superscript "1" or "+",) shown on the fig 9. There is a phase shift 120° between any two voltages.

$$|V_{a_1}| = |V_{b_1}| = |V_{c_1}|$$

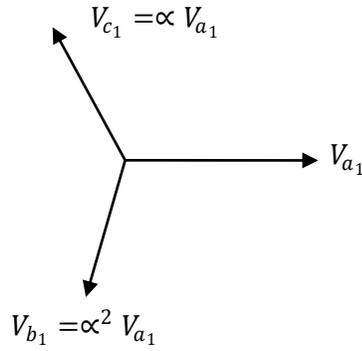


Fig. 9

- (2) The negative sequence currents and voltages (known also as the "acb" and often denoted by the superscript "2" or " - ") . Note the sequence of the phasors is the opposite direction from the positive sequence (acb instead of abc). There is a phase shift 120° between any two voltages.

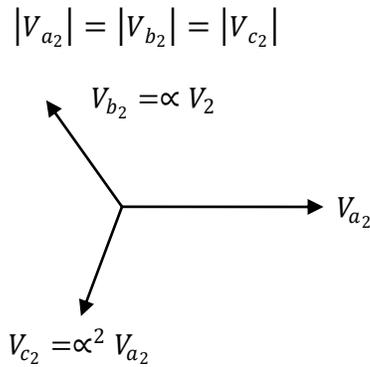


Fig. 10

- (3) The zero sequence components of currents and voltages (often denoted by the superscript "0") .Note that these zero sequence phasors are all in-phase and equal in magnitude.

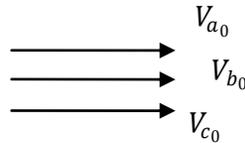


Fig. 11

Here the phase shift operator is α . α the vector by 120° without changing magnitude. We will need the vector $\alpha = 1\angle 120^\circ$, which is a unit vector at an angle of 120 degrees. It is easy to see that $\alpha^2 = 1\angle 240^\circ = 1\angle -120^\circ$ and $\alpha^3 = 1\angle 360^\circ = 1\angle 0^\circ$.

It is also clear that $1 + \alpha + \alpha^2 = 0$.

$$V_a = V_{a_0} + V_{a_1} + V_{a_2}$$

$$V_b = V_{b_0} + V_{b_1} + V_{b_2}$$

$$V_b = V_{a_0} + \alpha^2 V_{a_1} + \alpha V_{a_2}$$

$$V_c = V_{c_0} + V_{c_1} + V_{c_2}$$

$$V_c = V_{c_0} + \alpha V_{a_1} + \alpha^2 V_{a_2}$$

In matrix form,

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} V_{a_0} \\ V_{a_1} \\ V_{a_2} \end{bmatrix} \dots\dots\dots(5.5.1.1)$$

In one line we can write the above equation 5.5.1.1.

$$V^{abc} = [A][V^{012}]$$

$$V^{012} = [A^{-1}][V^{abc}]$$

$$\begin{bmatrix} V_{a_0} \\ V_{a_1} \\ V_{a_2} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}^{-1} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$$\begin{bmatrix} V_{a_0} \\ V_{a_1} \\ V_{a_2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$$V_{a_0} = \frac{1}{3}(V_a + V_b + V_c)$$

$$V_{a_1} = \frac{1}{3}(V_a + \alpha V_b + \alpha^2 V_c)$$

$$V_{a_2} = \frac{1}{3}(V_a + \alpha^2 V_b + \alpha V_c)$$

Similarly, for current

$$I_{a_0} = \frac{1}{3}(I_a + I_b + I_c)$$

$$I_{a_1} = \frac{1}{3}(I_a + \alpha I_b + \alpha^2 I_c)$$

$$I_{a_2} = \frac{1}{3}(I_a + \alpha^2 I_b + \alpha I_c)$$

5.5.2 POWER IN TERMS OF SYMMETRICAL COMPONENTS

Suppose in abc sequence, 3Φ complex power, $S^{abc} = V_a I_a^* + V_b I_b^* + V_c I_c^*$

$$= [V_a + V_b + V_c] \begin{bmatrix} I_a^* \\ I_b^* \\ I_c^* \end{bmatrix}$$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = V^{abc}, \quad \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = I^{abc}$$

$$S^{abc} = [V^{abc}]^T [I^{abc}]^* \dots \dots \dots (5.5.2.1)$$

5.2.2.1 RELATION BETWEEN ACTUAL QUANTITIES (a,b,c) AND SEQUENCE QUANTITIES

$$[V^{abc}] = [A][V^{012}]$$

$$[V^{abc}]^T = [A]^T [V^{abc}]^T$$

$$[I^{abc}] = [A][I^{012}]$$

From equation 5.2.2.1,

$$S^{abc} = [V^{012}]^T [A]^T [A]^* [I^{012}]^*$$

$$[A]^T \cdot [A]^* = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}^T \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}^* = 3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 3U$$

$$S^{abc} = [V^{012}]^T 3U [I^{012}]^* = 3[V^{012}]^T [I^{012}]^*$$

$$= 3[V_{a_0} \quad V_{a_0} \quad V_{a_0}] \begin{bmatrix} I_{a_0}^* \\ V_{a_0}^* \\ V_{a_0}^* \end{bmatrix}$$

$$= 3[V_{a_0} I_{a_0}^* + V_{a_1} I_{a_1}^* + V_{a_2} I_{a_2}^*]$$

$$S^{abc} = 3.S^{012}$$

*****END OF MODULE -II *****

Reference books:

- (1) Hayt Kemerly Durbin, "Engineering Circuit Analysis", Tata McGraw Hill Education Private Limited, New Delhi, 7th Edition
- (2) Van Valkenburg, "Network Analysis", PHI Learning Private Limited, New Delhi, 3rd Edition
- (3) Chakrabarti, "Circuit Theory", Educational and Technical Publishers, 5th Revised Edition

MODULE -III

Introduction to Realizability concept, Hurwitz property, positive realness, properties of real functions, synthesis of R-L, R-C and L-C driving point functions, Foster and Cauer forms

3.1. Network Function (lecture -21)

It is the relationship between the Laplace Transform of excitation to the Laplace Transform of the response in an electrical network. This network function can be analyzed using the “pole-zero” concept. A network function is characterized or depends on following factors:

- Driving point impedance and admittance
- Transfer impedance and admittance
- Voltage and current transfer ratio

(a) Driving point impedance and admittance

For one-port network,

$$\text{Driving point impedance, } Z(s) = \frac{V(s)}{I(s)}$$

$$\text{Driving point admittance, } Y(s) = \frac{I(s)}{V(s)}$$

For two-port network,

Driving point impedance,

$$Z_{11}(s) = \frac{\text{input voltage}}{\text{input current}} = \frac{V_1(s)}{I_1(s)}$$

$$Z_{22}(s) = \frac{\text{output voltage}}{\text{output current}} = \frac{V_2(s)}{I_2(s)}$$

Driving point admittance,

$$Y_{11}(s) = \frac{\text{input current}}{\text{input voltage}} = \frac{I_1(s)}{V_1(s)}$$

$$Y_{22}(s) = \frac{\text{output current}}{\text{output voltage}} = \frac{I_2(s)}{V_2(s)}$$

(b) Transfer impedance and admittance

$$\text{Transfer impedance, } Z_{12}(s) = \frac{V_2(s)}{I_1(s)}$$

$$\text{Transfer admittance, } Y_{12}(s) = \frac{I_2(s)}{V_1(s)}$$

(c) Voltage and Current Transfer Ratio

$$\text{Voltage transfer ratio, } G_{12}(s) = \frac{V_2(s)}{V_1(s)}$$

$$\text{Current transfer ratio, } \alpha_{12}(s) = \frac{I_2(s)}{I_1(s)}$$

3.2. Passive circuits (lecture -22)

A network can be represented by a network function. Some of the common properties of passive RLC functions.

- (i) They are real, rational functions.
- (ii) They do not have poles (or zeros) on the right-half s-plane, nor do they have multiple order poles on the $j\omega$ axis.
- (iii) The resulting matrices are symmetric.

The LC network functions have these additional properties.

- (i) They are simple, that is there are no higher order poles or zeros.
- (ii) All poles and zeros lie on the $j\omega$ axis.
- (iii) Poles and zeros alternate.
- (iv) The origin and infinity are always critical frequencies, that is, there will be either a pole or a zero at both the origin and at infinity.
- (v) The multiplicative constant is positive.

The properties of RC network functions are:

- (i) The poles and zeros of an RC driving point function lie on the non-positive real axis.
- (ii) They are simple.
- (iii) Poles and zeros alternate.
- (iv) The slopes of impedance functions are negative, those of admittance functions are positive.

3.2. Realizability Concept

This problem deals with the design of the network function when the excitation and response both are known. In this problem, the network function is usually realized as a physical passive network.

Examples:

1. Synthesize the PR impedance. $Z(s) = \frac{8s^3 + 4s^2 + 3s + 1}{8s^3 + 3s}$

Solution:

$$\text{Given, } Z(s) = 1 + \frac{4s^2 + 1}{8s^3 + 3s} = 1 + Z_1(s)$$

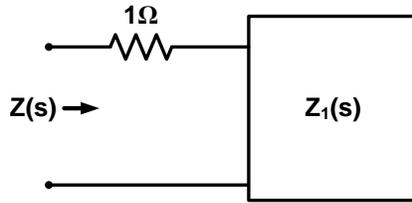


Fig.1

where

$$Z_1(s) = \frac{4s^2 + 1}{8s^3 + 3s}$$

$$\Rightarrow Y_1(s) = \frac{1}{Z_1(s)} = \frac{8s^3 + 3s}{4s^2 + 1} = 2s + \frac{s}{4s^2 + 1} = 2s + \frac{s}{Y_2(s)}$$

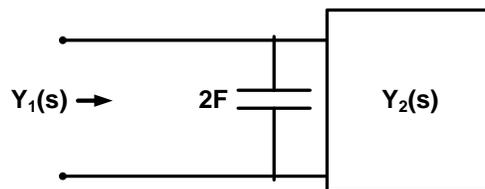


Fig.2

$$Z_2(s) = \frac{1}{Y_2(s)} = \frac{1}{4s^2 + 1} = 4s + \frac{1}{s}$$

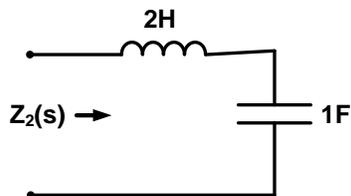


Fig.3

Now, the complete synthesis is

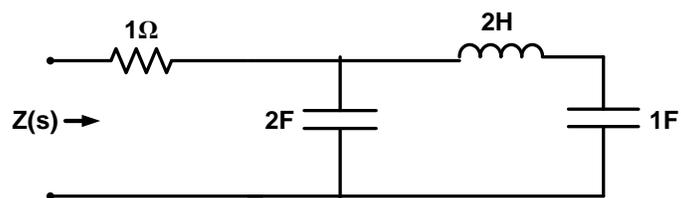


Fig.4

The network function can be realizable if it satisfies certain conditions such as

(a) Hurwitz property

(b) Positive realness property

(a) Hurwitz property (lecture -23)

Examples:

2. Test whether the following function is Hurwitz or not. $q(s) = s^4 + 3s^2 + 2$

Solution:

Given, $q(s) = s^4 + 3s^2 + 2 = q_1(s)$ (say)

Here, $q(s)$ consists of only even terms.

Then, $q_2(s) = \dot{q}(s) = 4s^3 + 6s$

Using continuous fraction method

$$\begin{aligned}
 & q_1(s) \left| \begin{array}{c} q_1(s) \\ \hline s^4 + 3s^2 + 2 \\ \hline s^4 + \frac{3}{2}s^2 + 0 \\ \hline - \quad - \\ \hline \frac{3}{2}s^2 + 2 \end{array} \right| \begin{array}{c} \\ \\ \\ \\ \frac{1}{4}s \\ \hline \frac{8}{3}s \\ \hline \frac{9}{4}s \\ \hline \frac{2}{3}s \\ \hline \frac{1}{3} \\ \hline 0 \end{array} \\
 & = 4s^3 + 6s \left| \begin{array}{c} s^4 + 3s^2 + 2 \\ \hline s^4 + \frac{3}{2}s^2 + 0 \\ \hline - \quad - \\ \hline \frac{3}{2}s^2 + 2 \end{array} \right| \begin{array}{c} \\ \\ \\ \\ \frac{1}{4}s \\ \hline \frac{8}{3}s \\ \hline \frac{9}{4}s \\ \hline \frac{2}{3}s \\ \hline \frac{1}{3} \\ \hline 0 \end{array} \\
 & \qquad \qquad \qquad \left| \begin{array}{c} \frac{3}{2}s^2 + 2 \\ \hline \frac{3}{2}s^2 \\ \hline - \\ \hline \frac{2}{3}s \end{array} \right| \begin{array}{c} \\ \\ \\ \\ \frac{9}{4}s \\ \hline \frac{2}{3}s \\ \hline \frac{1}{3} \\ \hline 0 \end{array} \\
 & \qquad \qquad \qquad \qquad \qquad \left| \begin{array}{c} \frac{2}{3}s \\ \hline \frac{2}{3}s \\ \hline - \\ \hline 0 \end{array} \right| \begin{array}{c} \\ \\ \\ \\ \frac{1}{3} \\ \hline 0 \end{array}
 \end{aligned}$$

Here, all the quotients are positive. So, the polynomial is Hurwitz.

3. Find the range of value of m in P(s) so that P(s) is Hurwitz.

$$P(s) = 2s^4 + s^3 + ms^2 + s + 2$$

Solution:

$$\text{Given, } P(s) = 2s^4 + s^3 + ms^2 + s + 2 = P_1(s) + P_2(s)$$

$$\text{Here, } P_1(s) = 2s^4 + ms^2 + 2, P_2(s) = s^3 + s$$

Using continuous fraction method

$$\begin{aligned}
 P_2(s) & \left| \begin{array}{c} P_1(s) \\ \hline 2s^4 + ms^2 + 2 \\ \hline 2s^4 + 2s^2 + 0 \\ \hline - \quad - \end{array} \right| \\
 & = s^3 + s \left| \begin{array}{c} (m-2)s^2 + 2 \\ \hline s^3 + s \\ \hline s^3 + \frac{2}{(m-2)}s \\ \hline - \quad - \end{array} \right| \\
 & \qquad \qquad \qquad \left| \begin{array}{c} \frac{1}{(m-2)}s \\ \hline \frac{(m-2)s^2 + 2}{(m-2)s^2} \\ \hline \frac{(m-2)}{(m-4)}s \\ \hline - \end{array} \right|
 \end{aligned}$$

For the polynomial is Hurwitz, all the quotients must be positive. So, $m > 4$.

(b) Positive Real (PR) Function (lecture -24)

A network function given by

$$N(s) = \frac{A(s)}{B(s)} = \frac{a_0s^n + a_1s^{n-1} + \dots + a_n}{b_0s^m + b_1s^{m-1} + \dots + b_m}$$

The function N(s) is called a PR function if

- (i) N(s) is real when s is real
- (ii) B(s) is Hurwitz polynomial
- (iii) If, B(s) consists of quadratic function. Which means in case there are some roots or poles on jw-axis, then they are simple and the residues are real and positive
- (iv) Real part of N(s) is positive for all values of w.

Examples:

4. Test whether the following function is PR or not.

$$Z(s) = \frac{2s^2 + 2s + 1}{s^3 + 2s^2 + s + 2}$$

Solution:

$$\text{Given, } Z(s) = \frac{A(s)}{B(s)} = \frac{2s^2 + 2s + 1}{s^3 + 2s^2 + s + 2}$$

Here,

Step-1: All coefficients of A(s) and B(s) are positive

Step-2: check residues of B(s)

$$\text{Characteristic equation: } B(s) = s^3 + 2s^2 + s + 2 = 0$$

$$\Rightarrow (s^2 + 1)(s + 2) = 0$$

$$\text{Now, } Z(s) = \frac{1}{s^2+1} + \frac{1}{s+2}$$

Both residues are positive

$$\text{Step-3: } Z(s) = \frac{A(s)}{B(s)} = \frac{2s^2+2s+1}{s^3+2s^2+s+2} = \frac{(2s^2+1)+(2s)}{(2s^2+2)+(s^3+s)} = \frac{M_1(s)+N_1(s)}{M_2(s)+N_2(s)}$$

$$D(s) = M_1(s)M_2(s) - N_1(s)N_2(s) = (2s^2+1)(2s^2+2) - (2s)(s^3+s) = 2(s^2+1)^2$$

$$D(j\omega) = 2(-\omega^2+1)^2$$

Here, $D(j\omega)$ is positive for all values of ω . Hence, the given polynomial is a PR function.

4.3.Procedure of Testing PR Function (lecture -25)

- (1) $A(s)$ and $B(s)$ must be Hurwitz
- (2) $N(s)$ must have simple poles (It has real and positive residues)

$$(3) N(s) = \frac{M_1(s)+N_1(s)}{M_2(s)+N_2(s)}$$

Where $M_1(s)$ and $M_2(s)$ are even terms and $N_1(s)$ and $N_2(s)$ are odd terms

$$\text{Re}[N(s)] = \text{Re} \left[\frac{M_1(s)+N_1(s)}{M_2(s)+N_2(s)} \right] = \frac{M_1(s)M_2(s) - N_1(s)N_2(s)}{M_2^2(s) - N_2^2(s)}$$

$M_2^2(s) - N_2^2(s)$ is always positive for any value of ω .

So, $N(s)$ is a PR function if $D(s) = M_1(s)M_2(s) - N_1(s)N_2(s) \Big|_{s=j\omega} \geq 0$

Examples:

5. Test whether the following function $Z(s) = \frac{4s+1}{s+2}$ is PR or not.

Solution:

$$\text{Given, } Z(s) = \frac{A(s)}{B(s)} = \frac{4s+1}{s+2}$$

Here,

Step-1: All coefficients of $A(s)$ and $B(s)$ are positive

Step-2: $B(s)$ has a simple root. So, $Z(s)$ has simple pole.

Step-3: Check residues of $B(s)$

$$\text{Characteristic equation: } Z(s) = \frac{4s+1}{s+2} = \frac{4s}{s+2} + \frac{1}{s+2}$$

$$\Rightarrow (s^2+1)(s+2) = 0$$

$$\text{Now, } Z(s) = \frac{A_1}{s^2+1} + \frac{A_2}{s+2} = \frac{1}{s^2+1} + \frac{1}{s+2}$$

Both residues A_1 and A_2 are positive

Step-4:

$$\operatorname{Re}[Z(j\omega)] = \operatorname{Re}\left[\frac{4(j\omega)+1}{(j\omega)+2}\right] = \operatorname{Re}\left[\frac{(1+j4\omega)(2-j\omega)}{4+\omega^2}\right] = \operatorname{Re}\left[\frac{2+4\omega^2+j7\omega}{4+\omega^2}\right] = \frac{2+4\omega^2}{4+\omega^2}$$

Here, for all values of ω , $\operatorname{Re}[Z(j\omega)] \geq 0$

Hence, the given function is a PR function.

6. Test whether the following function $Z(s) = \frac{s^2 - 3s + 5}{s - 2}$ is PR or not.

Solution:

$$\text{Given, } Z(s) = \frac{A(s)}{B(s)} = \frac{s^2 - 3s + 5}{s - 2}$$

Here,

Step-1: All coefficients of A(s) and B(s) are not positive

Hence, the given function is not a PR function.

3.4. LC Network Synthesis (lecture -26)

Foster and Cauer first proposed the realization of these networks in various canonical forms, in the 1920s. We will now examine each of these forms. There are four canonical forms, relating to the realization of LC, RC and RL networks as follows.

| | 1 st Foster form | 2 nd Foster form | 1 st Cauer form | 2 nd Cauer form |
|--|-----------------------------|-----------------------------|----------------------------|----------------------------|
| Partial fraction expansion of impedance function | ✓ | | | |
| Partial fraction expansion of admittance function | | ✓ | | |
| Continued fraction expansion about the point at infinity | | | ✓ | |
| Continued fraction expansion about the origin | | | | ✓ |

3.4.1. LC Network Synthesis

A LC network is synthesized in two forms, i.e. Foster and Cauer form

3.4.1.1 Foster Canonical Form

(a) First Foster Canonical Form

$$Z(s) = Z_1(s) + Z_2(s) + \dots + Z_i(s) + \dots + Z_{n-1}(s) + Z_n(s)$$

$$\Rightarrow Z(s) = \frac{A_0}{s} + \frac{2A_2s}{s^2 + \omega_2^2} + \dots + \frac{2A_i s}{s^2 + \omega_i^2} + \dots + \frac{2A_{n-1}s}{s^2 + \omega_{n-1}^2} + \frac{2A_n s}{s^2 + \omega_n^2}$$

Where,

(i) $C_0 = A_0 = \text{pole at origin}$

(ii) $\frac{2A_i s}{s^2 + \omega_i^2}$ has a pair of complex conjugate poles. From this we can find C_i and L_i as follows.

$$C_i = \frac{1}{2A_i}, L_i = \frac{2A_i}{\omega_i^2}$$

(iii) L_∞ represents $A_\infty s$ means a pole at infinity.

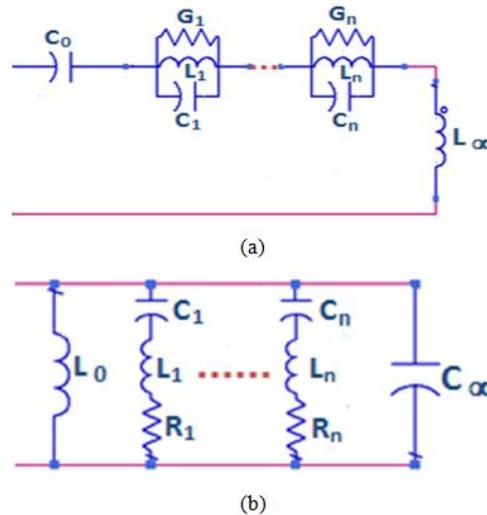


Fig.5 (a) First Foster's Canonical Form and (b) Second Foster's Canonical Form

(b) Second Foster's Canonical Form (lecture -27)

$$Y(s) = Y_1(s) + Y_2(s) + \dots + Y_i(s) + \dots + Y_n(s)$$

$$\Rightarrow Y(s) = \frac{B_0}{s} + \frac{2B_2 s}{s^2 + \omega_2^2} + \dots + \frac{2B_i s}{s^2 + \omega_i^2} + \dots + B_\infty s$$

Where,

(iv) $L_0 = B_0 =$ pole at origin

(v) $\frac{2B_i s}{s^2 + \omega_i^2}$ has a pair of complex conjugate poles. From this we can find C_i and L_i as follows.

$$C_i = \frac{1}{2A_i}, L_i = \frac{2A_i}{\omega_i^2}$$

(vi) C_∞ represents $B_\infty s$ means a pole at infinity.

N.B.:

1. [Order of Num] > [Order of Den]

So, one zero is present at $\omega = \infty$. The denominator has one 's' term, so one pole is present at $\omega = 0$. Therefore, both first and last terms are present.

2. [Order of Num] < [Order of Den]

So, no pole is present at $\omega = 0$. Therefore, both first and last terms are absent.

Example

7. The driving point impedance of LC network is given by

$$Z(s) = 10 \frac{(s^2 + 4)(s^2 + 16)}{s(s^2 + 9)}$$

Obtain first form Foster's network.

Solution:

Given,

$$Z(s) = 10 \frac{(s^2 + 4)(s^2 + 16)}{s(s^2 + 9)}$$

$$= \frac{A_0}{s} + \frac{A_2}{s + j3} + \frac{A_2^*}{s - j3} + A_\infty s$$

$$A_0 = 10 \left. \frac{(s^2 + 4)(s^2 + 16)}{(s^2 + 9)} \right|_{s=0} = 71.11$$

$$A_2 = 10 \left. \frac{(s^2 + 4)(s^2 + 16)}{s(s - j3)} \right|_{s=-j3} = 19.45$$

$$C_0 = \frac{1}{A_0} = 0.0141F$$

$$L_\infty = A_\infty = 10H$$

$$C_2 = \frac{1}{2A_2} = 0.0257F$$

$$L_2 = \frac{2A_2}{\omega_2^2} = 4.322H$$

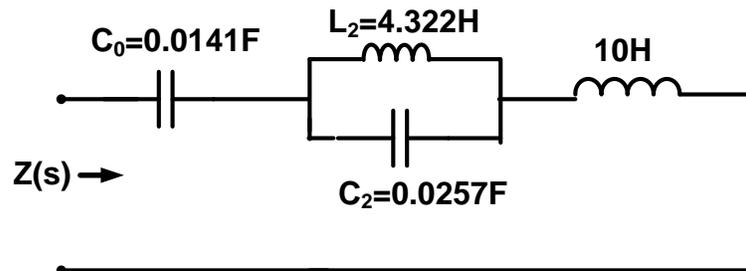


Fig.6

8. The driving point impedance of LC network is given by

$$Z(s) = 8 \frac{(s^2 + 4)(s^2 + 25)}{s(s^2 + 16)}$$

Obtain first form and second form of Foster's network.

Solution:

First form of Foster's network

Here, [Order of Num] > [Order of Den]

So, one zero is present at $\omega = \infty$. The denominator has one 's' term, so one pole is present at $\omega = 0$. Therefore, both first and last terms are present.

Given,

$$Z(s) = 8 \frac{(s^2 + 4)(s^2 + 25)}{s(s^2 + 16)}$$

$$= \frac{A_0}{s} + \frac{A_2}{s + j4} + \frac{A_2^*}{s - j4} + A_\infty s$$

$$A_0 = 8 \left. \frac{(s^2 + 4)(s^2 + 25)}{(s^2 + 16)} \right|_{s=0} = 50$$

$$A_2 = 8 \left. \frac{(s^2 + 4)(s^2 + 25)}{s(s - j4)} \right|_{s=-j4} = 27$$

$$A_0 = 8$$

$$C_0 = \frac{1}{A_0} = 0.02F$$

$$L_\infty = A_\infty = 8H$$

$$C_2 = \frac{1}{2A_2} = 0.0185F$$

$$L_2 = \frac{2A_2}{\omega_2^2} = 3.375H$$

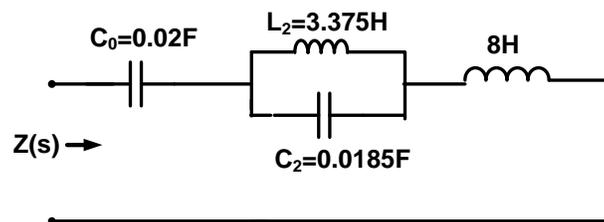


Fig.8

Second form of Foster's network

Given,

$$Y(s) = \frac{s(s^2 + 16)}{8(s^2 + 4)(s^2 + 25)}$$

Here, [Order of Num] > [Order of Den]

So, one zero is present at $\omega = \infty$. The denominator has one 's' term, so one pole is present at $\omega = 0$. Therefore, both first and last terms are present.

$$Y(s) = \frac{B_0}{s} + \frac{B_2}{s^2 + 4} + B_\infty s$$

$$B_0 = \frac{(s^2 + 1)(s^2 + 16)}{4(s^2 + 4)} \Big|_{s=0} = 1$$

$$B_2 = \frac{(s^2 + 1)(s^2 + 16)}{s(s - j2)} \Big|_{s=0} = 1.125$$

$$B_\infty = \frac{1}{4} = 0.25$$

$$L_0 = \frac{1}{B_0} = 1H$$

$$C_\infty = B_\infty = 0.25F$$

$$L_2 = \frac{1}{2B_2} = 0.5625H$$

$$C_2 = \frac{2B_2}{\omega_2^2} = 0.444F$$

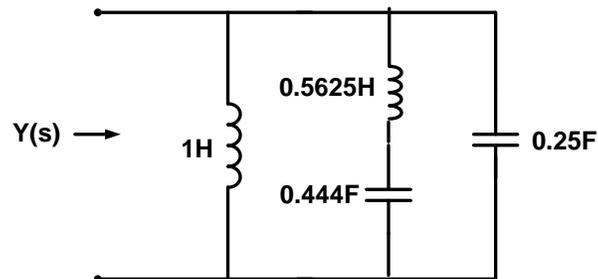


Fig.9

3.4.1.1 Cauer Form (lecture -28)

Step 1

For a given input impedance $Z(s)$, determine if the input impedance is LC- or RC-realizable. One way to identify if the input impedance is LC-impedance is to locate the pole/zero on the complex s-plane. If the input impedance is any rational function with a positive scale factor and with simple poles and zeros (conjugate pairs) arranged in alternating order along the imaginary axis, then this input impedance $Z(s)$ is realizable as a LC network. For a realizable input impedance of a RC network, the input impedance $Z(s)$ must be a rational function, that has simple, alternating poles and zeros located on the non-positive real axis with the critical frequency nearest to or at the origin being a pole.

Step 2

Determine the type of network (Cauer I or Cauer II) to be implemented. Fig. 10 and Fig. 11 show the Cauer I and Cauer IILC networks and Fig. 12 and Fig. 13 show the Cauer I and Cauer II RC networks, respectively.

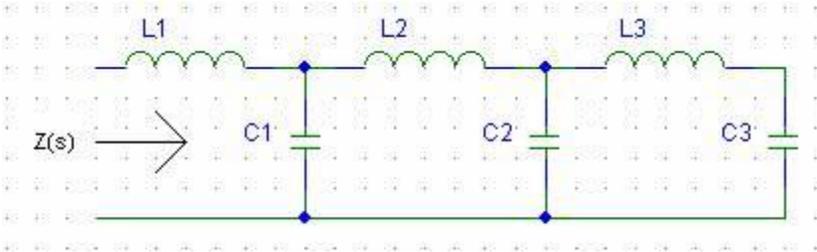


Fig.10 Cauer I LC network

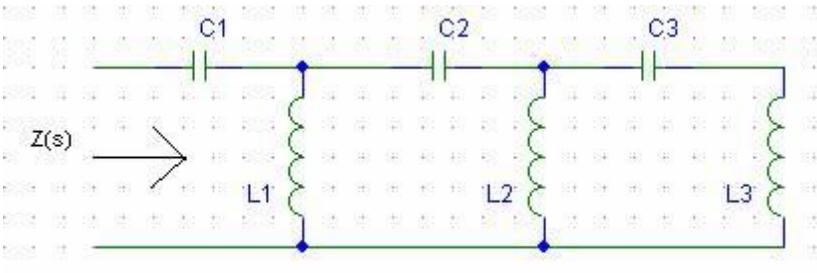


Fig. 11 Cauer II LC network

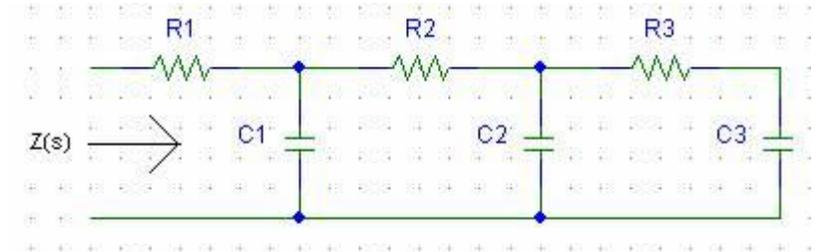


Fig. 12 Cauer I RC network

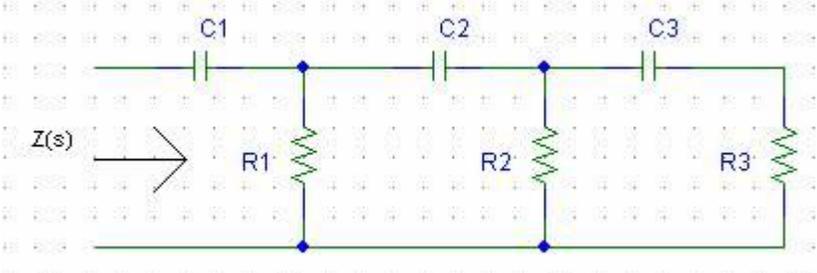


Fig. 13 Cauer II RC network

Step 3

Check asymptotic behavior of the input impedance.

- Cauer I LC: The first element is L, if $Z(s)$ has a pole at $s = \infty$. Otherwise the first element is C.
- Cauer II LC: The first element is C, if $Z(s)$ has a pole at $s = \text{origin}$. Otherwise the first element is L.
- Cauer I RC: The first element is R, if $Z(s)$ has no zero at $s = \infty$. Otherwise the first element is C.
- Cauer II RC: The first element is C, if $Z(s)$ has a pole at $s = \text{origin}$. Otherwise the first element is R.

Step 4

- If the first element is a shunt element, continued fraction expansion is done on input admittance $Y(s)$.
- If the first element is a series element, continued fraction expansion is done on input impedance $Z(s)$.

Step 5

- If Cauer I is chosen, arrange the numerator and denominator polynomials in descending power of s .
- If Cauer II is chosen, arrange the numerator and denominator polynomials in ascending power of s .

*****End of module III*****